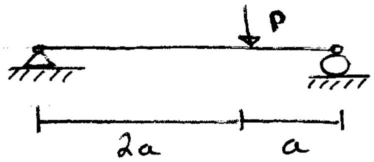
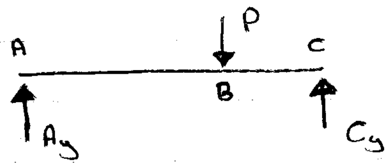
Example: Determine the deflection of a simply supported beam. EI = const.



Solution:

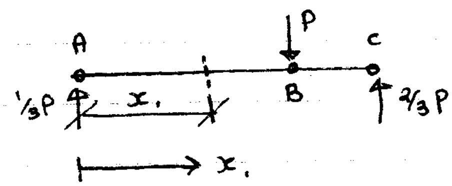


$\Sigma M_A = 0$

$C_y(3a) - P(2a) = 0$

$C_y = 2/3 P$

AB:

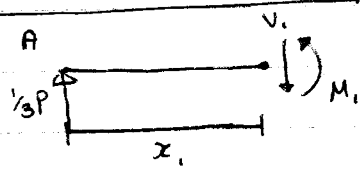


$\Sigma M_C = 0$

$P(a) - A_y(3a) = 0$

$A_y = 1/3 P$

$\boxed{0 \le x_1 \le 2a}$

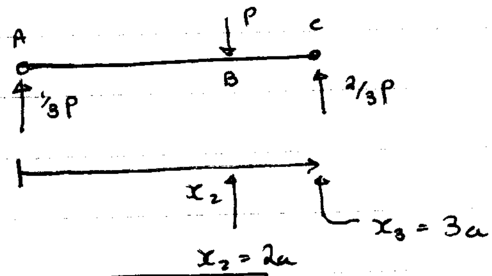


$\Sigma M = 0$

$M_1 - 1/3 P x_1 = 0$

$M_1(x_1) = 1/3 P x_1$

BC:

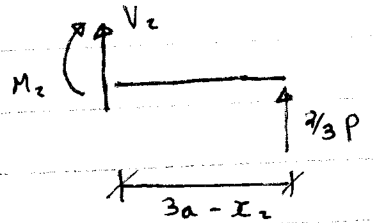


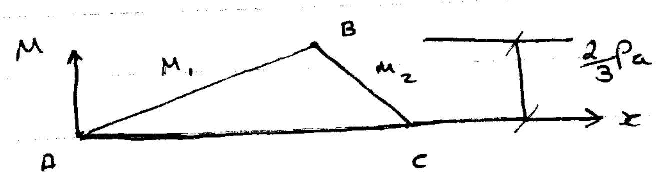
$\Sigma M = 0$

$-M_2 + 2/3 P(3a - x_2) = 0$

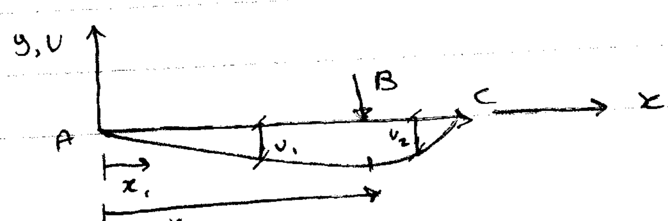
$\therefore M_2(x_2) = 2/3 P(3a - x_2)$

$\boxed{2a \le x_2 \le 3a}$





Elastic Curve:



$\boxed{AB}$ $\quad 0 \le x_1 \le 2a$

$$EI \frac{d^2 v_1}{dx_1^2} = M_1 = \frac{1}{3} P x_1 \qquad (1)$$

THEN $\Rightarrow$ $$EI \frac{dv_1}{dx_1} = \tfrac{1}{6} P x_1^2 + C_1 \qquad (2)$$

THEN $\Rightarrow$ $$EI\, v_1 = \tfrac{1}{18} P x_1^3 + C_1 x_1 + C_2 \qquad (3)$$

At A, $x_1 = 0$, $v_1 = 0$

Eq.(2) $\Rightarrow$: $\quad 0 = 0 + 0 + C_2$

$$C_2 = 0$$

$\boxed{BC}$: $\quad 2a \le x_2 \le 3a$

$$EI \frac{d^2 v_2}{dx_2^2} = M_2 = 2/3\, P(3a - x_2)$$

$$= -2/3\, P(x_2 - 3a)$$

$$\Rightarrow EI \frac{dv_2}{dx_2} = -\tfrac{1}{3} P (x_2 - 3a)^2 + C_3 \qquad (4)$$

$$\Rightarrow EI\, v_2 = -\tfrac{1}{9} P (x_2 - 3a)^3 + C_3 (x_2 - 3a) + C_4 \qquad (5)$$

$$z = x_2 - 3a$$

$$\frac{dv_2}{dx_2} = \frac{dv_2}{dz}$$

$$\frac{d^2 v_2}{dx_2^2} = \frac{d^2 v_2}{dz^2}$$

$$\Rightarrow EI \frac{d^2 v_2}{dz^2} = -\frac{2}{3} P z$$

$$EI \frac{dv_2}{dz} = -\frac{1}{3} P z^2 + C_3$$

$$EI\, v_2 = -\tfrac{1}{9} P z^3 + C_3 z + C_4$$

At C, $x_2 = 3a$, $v_2 = 0$

Eq (5) : $0 = 0 + 0 + C_4$

$C_4 = 0$ (6)

At B, $x_1 = 2a$ $x_2 = 2a$

AB : $EIv_1 = 1/18\,P(2a)^3 + C_1(2a)$

BC : $EIv_2 = -1/9\,P(2a-3a)^3 + C_3(2a-3a)$

$(v_1 = v_2) \to$ at point B

$\Rightarrow 1/18\,P(2a)^3 + C_1(2a) = -1/9\,P(2a-3a)^3 + C_3(2a-3a)$ (7)

At B; $x_1 = x_2 = 2a$

AB: $EI\dfrac{dv_1}{dx_1} = 1/6\,P(2a)^2 + C_1$

BC $EI\dfrac{dv_2}{dx_2} = -1/3\,P(2a-3a)^2 + C_3$

$$\boxed{\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}}$$

$\Rightarrow \frac{1}{6}P(2a)^2 + C_1 = -1/3\,(2a-3a)^2 + C_3$ (8)

Solving (7)(8) $\Rightarrow$ $\left.\begin{aligned} C_1 &= -4/9\,Pa^2 \\ C_3 &= 5/9\,Pa^2 \end{aligned}\right\}$ (9)

Deflection :

AB: $v_1 = \dfrac{P}{18EI}(x_1^3 - 8a^2x_1)$ $0 \le x_1 \le 2a$

BC: $v_2 = \dfrac{P}{9EI}(-x_2^3 + 9ax_2^2 - 22a^2x_2 + 12a^3)$ $2a \le x_2 \le 3a$

BC: $\dfrac{dv_2}{dx_2} = \dfrac{P}{9EI}\left(-3x_2^2 + 18ax_2 - 22a^2\right) = 0$

$x_2 = \left(3 \pm \sqrt{5/3}\right)a$

$x_2 = 4.291a \qquad x_2 = 1.709a$

No solution, max at B or C.

At B., $x_1 = x_2 = 2a$

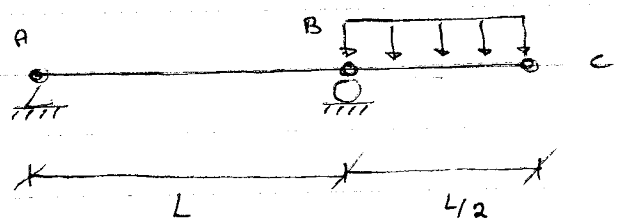
$$v_1 = \frac{P}{18EI}\left((2a)^3 - 8a^2 \cdot 2a\right)$$

$$= \frac{4\,Pa^3}{9EI}$$

$$= -0.444\,\frac{Pa^3}{EI}$$

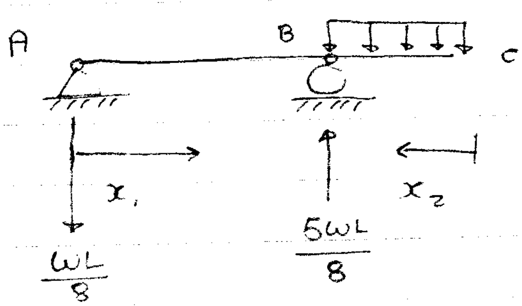
$\therefore\ v_{max} = -0.484\,\dfrac{Pa^3}{EI} \qquad \left(x_1 = \sqrt{8/3}\,a\right)$

Example:

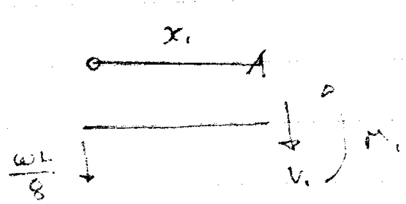


EI = const. Find the elastic curve.

Solution:



AB $\quad 0 \le x_1 \le L$



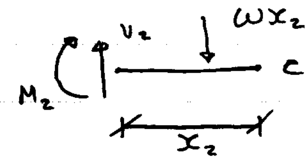
$M_1 + \dfrac{wL}{8}x_1 = 0$

$M_1 = -\dfrac{wL}{8}x_1$

BC: $\quad 0 \le x_2 \le L/2$

$$-M_2 - \omega x_2 \cdot \tfrac{1}{2} x_2 = 0$$

$$M_2 = \frac{-\omega}{2} x_2^2$$



<u>Elastic Curve</u>

AB :

$$EI \frac{d^2 v_1}{dx_1^2} = M_1 = \frac{-\omega L}{8} x_1$$

$$EI \frac{dv_1}{dx_1} = -\frac{\omega L}{16} x_1^2 + C_1$$

$$EI\, v_1 = \frac{-\omega L}{48} x_1^3 + C_1 x_1 + C_2$$

BC:

$$EI \frac{d^2 v_2}{dx_2^2} = M_2 = \frac{-\omega}{2} x_2^2$$

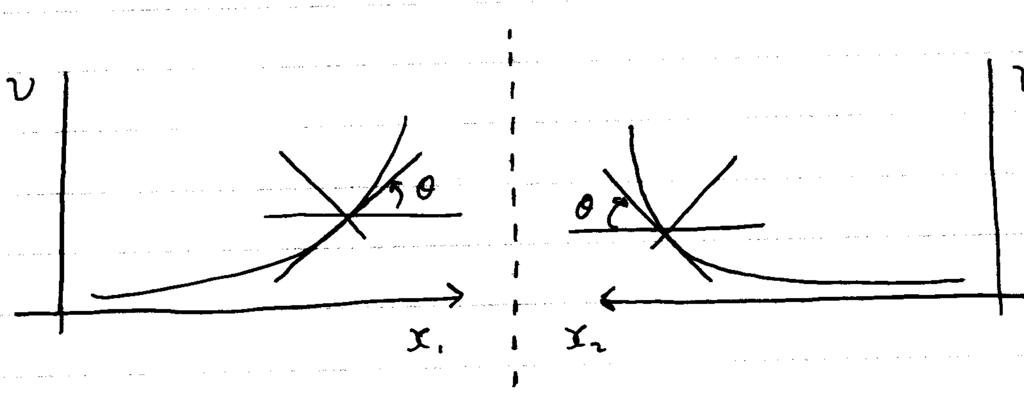
$$EI \frac{dv_2}{dx_2} = -\frac{\omega}{6} x_2^3 + C_3$$

$$EI\, v_2 = \frac{-\omega}{24} x_2^4 + C_3 x_2 + C_4$$

At A, $x_1 = 0$, $v_1 = 0$

At B, $x_1 = L$, $x_2 = L/2$

$v_1 = v_2 = 0$



At B : $x_1 = L$ $\quad x_2 = L/2$

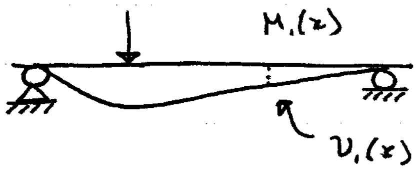
$$\boxed{\frac{dv_1}{dx_1}(L) = -\frac{dv_2}{dx_2}(L/2)}$$

Deflections :

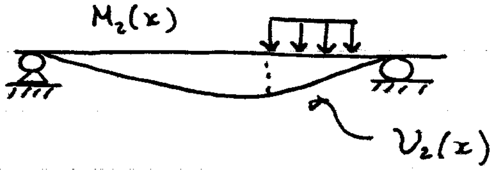
<u>AB</u>: $v_1 = \dfrac{wL}{48EI}\left(-x_1^3 + L^2 x_1\right)$

<u>BC</u> : $v_2 = \dfrac{w}{384EI}\left(-16x_2^4 + 24L^3 x_2 - 11L^4\right)$

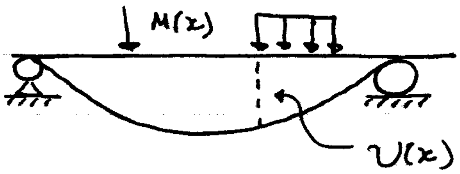
Method of Superposition



$$EI\frac{d^2v}{dx^2} = M_1(x)$$



$$EI\frac{d^2v}{dx^2} = M_2(x)$$

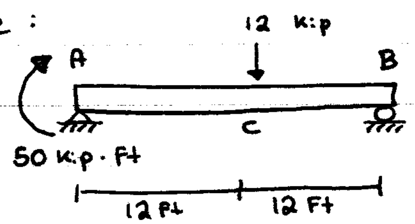


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M(x) = M_1(x) + M_2(x)$$

$$\Rightarrow v(x) = v_1(x) + v_2(x)$$

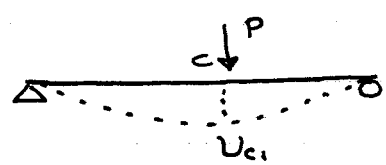
Example :

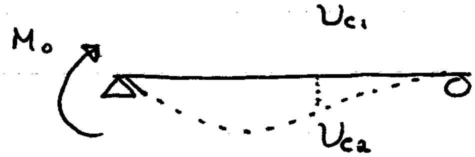


Determine the deflection @ point C

Given $E = 29 \times 10^3$ ksi , $I = 350$ in^4

Solution :





$$v_c = v_{c1} + v_{c2}$$

$$v_{c1} = \frac{-PL^3}{48EI}$$

$$v_{c2} = \frac{-M_o x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)\Big|_{x = L/2}$$

$$= -\frac{M_o L^2}{16EI}$$

$P = 12 \text{ kip} = 12\,(10^3)\text{ lb}$

$M_o = 50 \text{ kip}\cdot\text{ft} = 50\,(10^3) \times 12 \text{ in} = 600\,(10^3)\text{ lb}\cdot\text{in}$

$L = 24 \text{ ft} = 24 \times 12 \text{ in} = 288 \text{ in}$

$I = 350 \text{ in}^4$

$E = 29\,(10^3)\text{ ksi} = 29\,(10^6)\text{ psi}$

$$\therefore\; v_{c1} = \frac{-PL^3}{48EI} = -\frac{12(10^3)(288)^3}{48(29)(10^6)(350)}$$

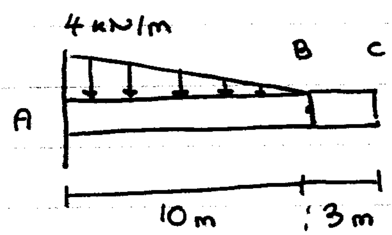
$$= -0.3064 \text{ in}$$

$$v_{c2} = \frac{-M_o L^2}{16EI} = -\frac{600(10^3)(288)^2}{16(29)(10^6)(350)}$$

$$= -0.5884 \text{ in}$$

$$\therefore\; v_c = v_{c1} + v_{c2} = -0.895 \text{ in}$$

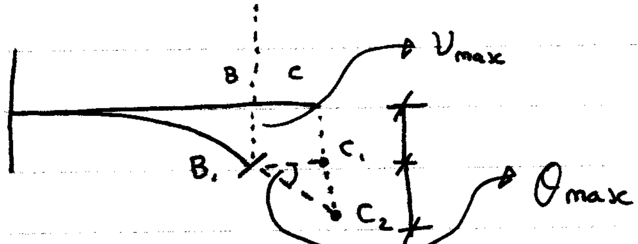
Example:



EI = const.

Find the deflection at C.

Solution:



$$v_c = \underset{\text{translation}}{CC_1} + \underset{\text{rotation}}{C_1C_2}$$

$$v_{max} = \frac{-w_o L^4}{30EI} \qquad \theta_{max} = \frac{-w_o L^3}{24EI}$$

$$C_1C_2 = BC \cdot \theta_{max}$$

$$\therefore v_c = v_{max} + BC \cdot \theta_{max}$$

$$= \frac{-w_o L^4}{30EI} + BC\left(\frac{-w_o L^3}{24EI}\right)$$

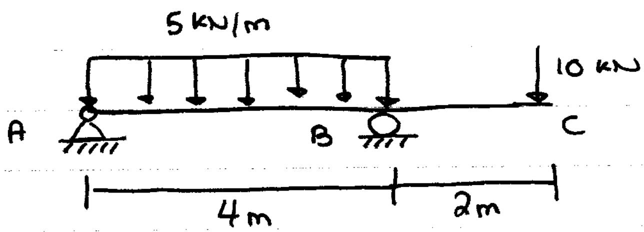
Here, $L = 10m$ $\quad BC = 3m$

$w_o = 4 \, kN/m = 4000 \, N/m$

$$\therefore v_L = -\frac{4000(10)^4}{30\,EI} - 3 \times \frac{4000(10^3)}{24\,EI}$$

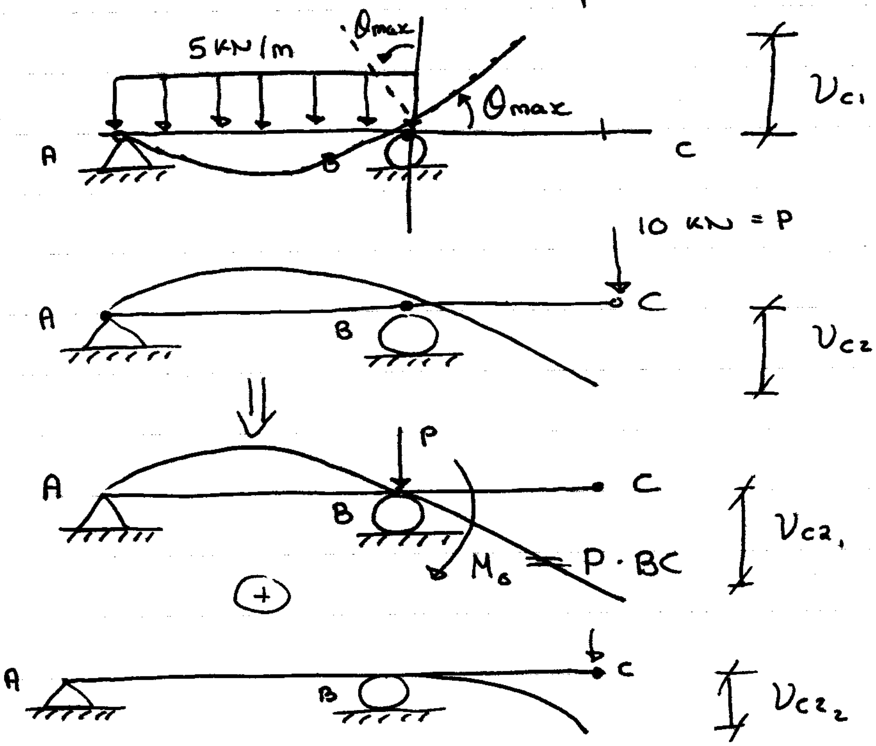
$$= -\frac{1.833(10^6)}{EI}$$

$\boxed{\text{Example :}}$



EI = const $\longrightarrow$ Find the displacement at point C.

Solution :



$$v_{c2} = \underbrace{v_{c21}}_{\text{rotation}} + \underbrace{v_{c22}}_{\text{bending}}$$

$$v_{c1} = BC \cdot \theta_{max} = 2 \times \frac{w_o L^3}{24EI}$$

$$= 2 \times \frac{5000(4)^3}{24EI}$$

$$26.67 = \frac{\cancel{166.67} \times 10^3}{EI}$$

$$\uparrow$$

$$v_{c2} = v_{c21} + v_{c22}$$

$$v_{c21} = BC \cdot \theta_1 = 2\left(-\frac{M_o L}{3EI}\right)$$

$$= -2 \times \frac{P \times BC \times L}{3EI}$$

$$= -2 \times \frac{10(10^3) \times 2 \times 4}{3EI}$$

$$= \frac{-53.3 \times 10^3}{EI}$$

$$v_{c22} = \frac{-PL^3}{3EI}$$

$$= -\frac{10(10^3)(2)^3}{3EI} \Rightarrow \frac{-26.7 \times 10^3}{EI}$$

$$\therefore v_c = v_{c1} + v_{c2}$$

$$= v_{c1} + v_{c21} + v_{c22}$$

$$= \frac{26.67 \times 10^3}{EI} \cancel{+} \frac{-53.3 \times 10^3}{EI} - \frac{26.7 \times 10^3}{EI}$$

$$= \frac{-53.3 \times 10^3}{EI}$$