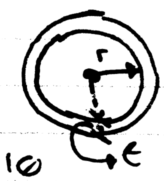
Stephanie Stangier - Lab instructor
CB1041
5 experiments - Groups assigned by instructor

Professor: Hao Bai
Office hours: Thursday, Friday
→ 10 ~ 11:30 AM

GIVE IT A WHIRL!

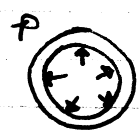
Chapter 8 - Combined Loading
8.1 - Thin-walled pressure vessels
Geometry: { cylindrical, spherical }

Dimension:
Cross-section:



For thin-wall: $r/t \geq 10$

Loading:

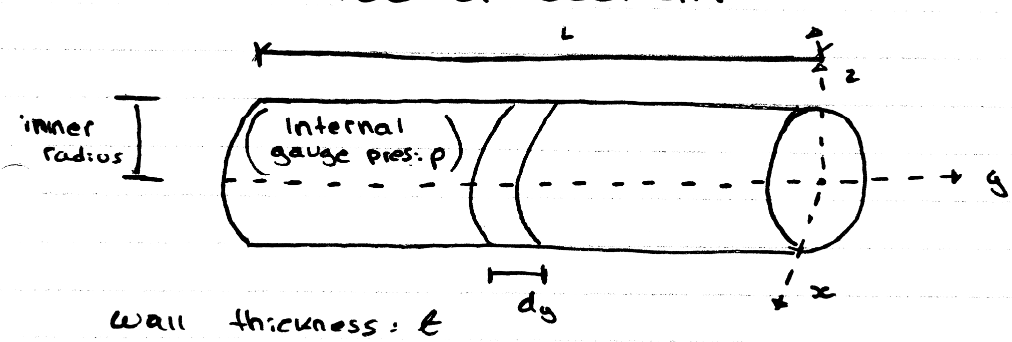


Materials: isotropic

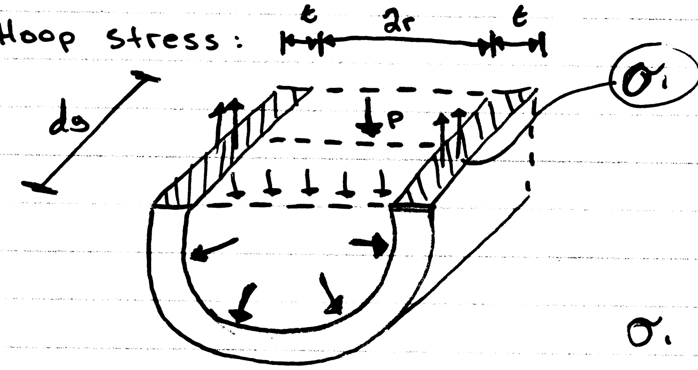
Assumptions:
Stresses: uniform throughout its' thickness

State of stress: 2D

Method of section:



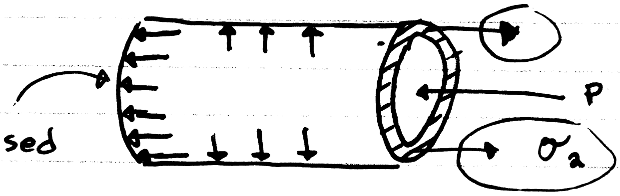
Hoop stress:



$p \cdot 2r \cdot dy = 2t\,dy \cdot \sigma_1$

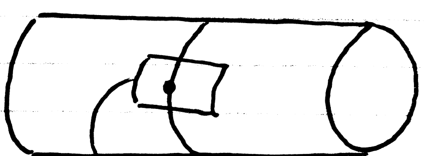
$\sigma_1 = \dfrac{pr}{t}$

Axial stress (or longitudinal stress)

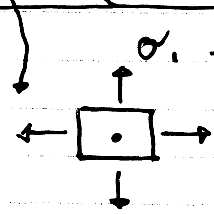


$p \cdot \pi r^2 = \sigma_2 \cdot 2\pi r t$

Method of Section:

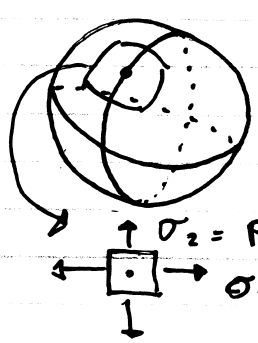


$\sigma_1 = pr/t$



$\sigma_2 = p \cdot r/2t = \frac{1}{2}\sigma_1$

A spherical pressure vessel:

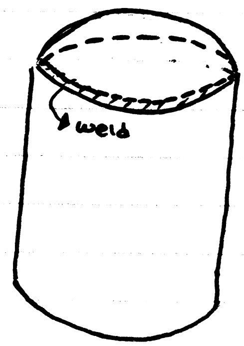


$\sigma_2 = p \cdot r/2t$

σ_2

Example:

$r = 4$ in
$t = 0.25$ in
$P = 2$ ksi



Determine

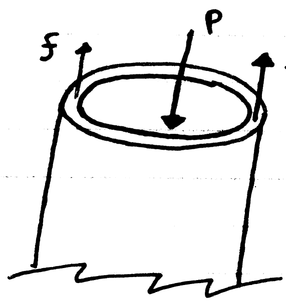
1) hoop stress σ_1 and axial stress σ_2

2) the tensile Force per inch length of the weld between the hemispherical head and the cylindrical body of the tank.

Solution:

$r/t = 4/0.25 = 16 > 10$

hoop stress $\sigma_1 = p \cdot r/t \Rightarrow 2(16) = 32$ ksi
axial stress $\sigma_2 = p \cdot r/2t \Rightarrow 2(8) = 16$ ksi



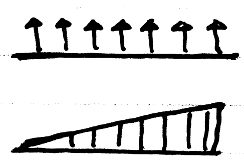
(force / unit length)

The resultant: $f \cdot 2\pi r = p \cdot \pi r^2$

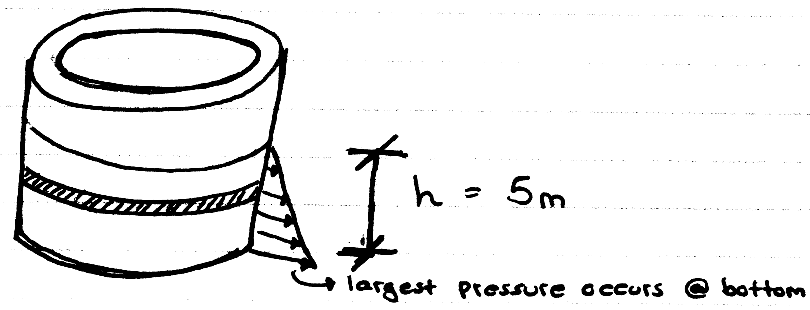
$\sigma_2 \cdot 2\pi r \cdot t = p \cdot \pi r^2$

$\hookrightarrow f = \sigma_2 t = 16(0.25) \Rightarrow 4$ kip/in

distributed Force:



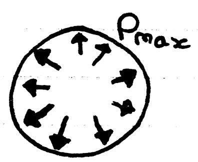
Example:
Inner radius
$r = 1.5\ m$



Allowable hoop stress is 80 MPa

What is the minimum wall thickness of the tank to the nearest millimeter?

Solution: $P_{max} = \gamma h$



$$= \left(1000\ \frac{kg}{m^3} \cdot 9.81\ m/s^2\right) \cdot 5\ m$$

$$= 49.05 \cdot 1000\ N/m^2$$

$$= 49.05\ kPa$$

Hoop stress

$$\sigma_1 = P_{max}\frac{r}{t}$$

$$\Rightarrow (80 \cdot 10^3) = 49.05 \times \frac{1.5}{t}$$

$$\Rightarrow t = \frac{(49.05 \cdot 1.5)}{(80 \cdot 10^3)} \quad (\text{in } m)$$

$$= 0.920\,(10^{-3})\,m$$

$$= 0.920\ mm$$

$$\therefore t = 1\ mm \quad (\text{minimum wall thickness})$$

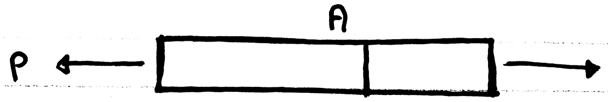
$$\frac{r}{t} = \frac{1.5\ m}{0.920(10^{-3})\ m} = 1.6\,(10^3) > 10$$

last step is to check.

Stresses by marginal loading (next topic)

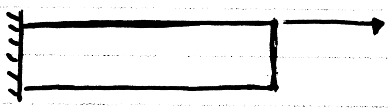
8.2 State of stress caused by combined loadings

* Axial Force



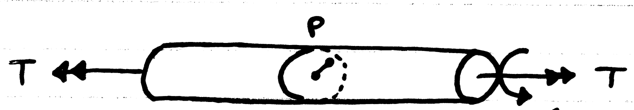
(must pass through centroid)

Normal stress $\sigma = P/A$



(can't be used in this scenario)

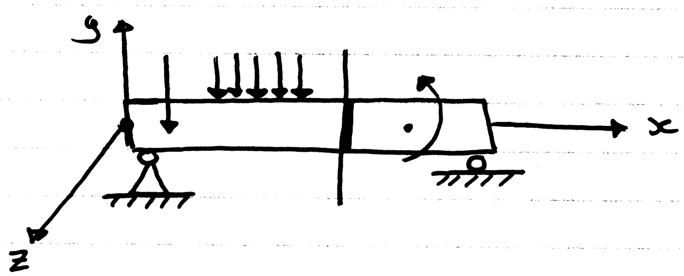
* Torsion of a circular shaft



(by RHR, thumb towards T)

$$\tau = \frac{T}{J} \cdot \rho$$

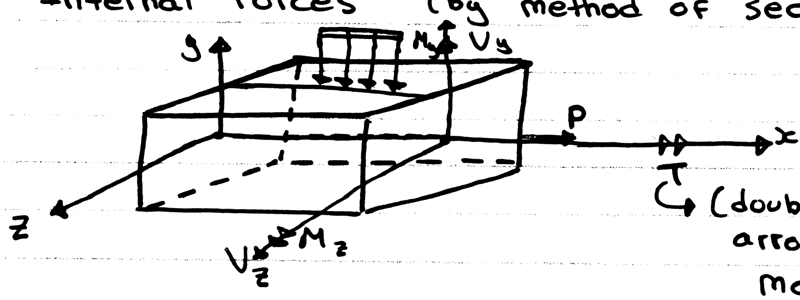
* Bending



$$\sigma = \frac{-M}{I} \cdot y \qquad \tau = \frac{VQ}{It}$$

* Thin-walled pressure vessels

Internal Forces (by method of section)



P : axial

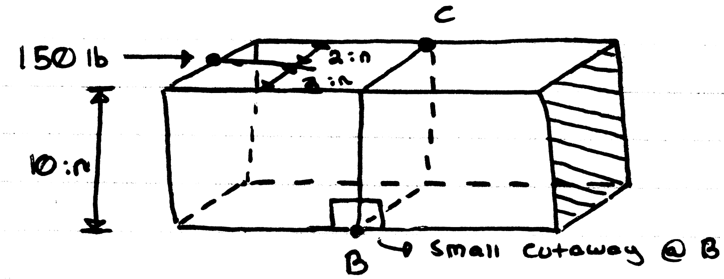
V_y, V_z : shear

(double-headed arrow to represent moment)

T : Torque

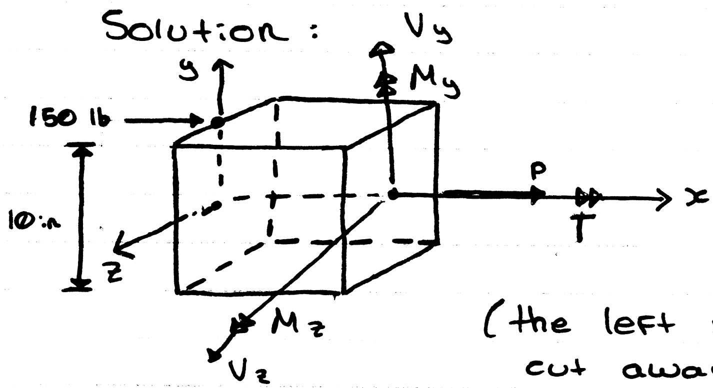
M_y, M_z : Moments

Example:



Determine the state of stress at B and C.

Solution:



(the left side cut away)

$\Sigma F_x = 0;$
$P + 150\,\text{lb} = 0$
$P = -150\,\text{lb}$

$\Sigma F_y = 0;$
$V_y = 0$

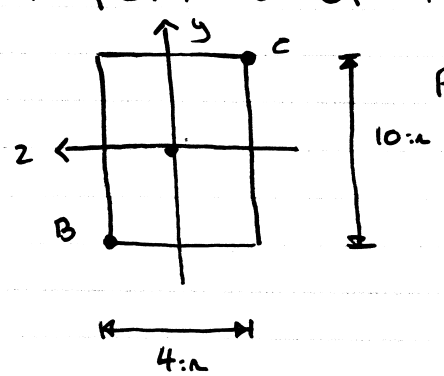
$\Sigma F_z = 0;$
$V_z = 0$

$\Sigma M_x = 0;$
$T = 0$

$\Sigma M_y = 0;$
$M_y = 0$

$\Sigma M_z = 0;$
$M_z - (150\,\text{lb})(5\,\text{in}) = 0$
$M_z = 750\,\text{in}\cdot\text{lb}$

Properties of the section



$A = 4 \times 10 = 40\,\text{in}^2$

$I_z = (\tfrac{1}{12})bh^3$
$I_z = (\tfrac{1}{12})(4)(10)^3$
$I_z = 333.3\,\text{in}^4$

Stress @ B:

$$\sigma = P/A - \frac{M_z}{I_z}\cdot y + \frac{M_y}{I_y}\cdot z$$

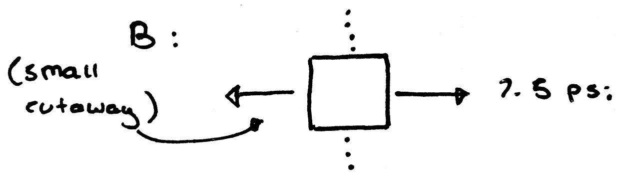
Stress at B : $y = -5$, $z = 2$

$$\sigma = P/A - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$\sigma = \frac{-150}{40} - \frac{750}{333.3}(-5)$$

$\sigma = 7.5$ psi (tensile)

B :



Stress at C : $y = 5$, $z = -2$

$$\sigma = P/A - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$\sigma = \frac{-150}{40} - \frac{750}{333.3}(5)$$

$\sigma = -7.5$ psi (compressive)

C :