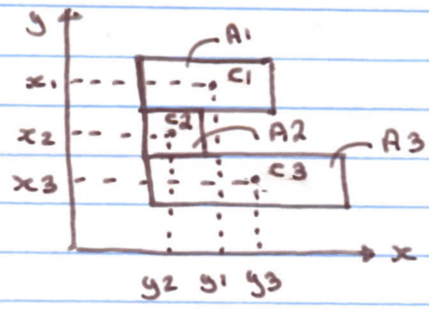
Nov. 1,

3) Composite Areas



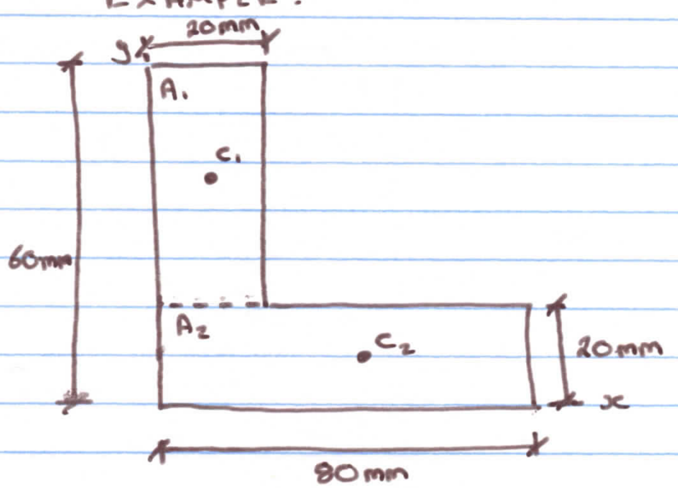
$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} \bar{x}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \bar{y}_i A_i}{\sum_{i=1}^{n} A_i}$$

Where n = # of simpler shapes.

EXAMPLE:



Shape 1:

$A_1 = (20mm) \times (60 - 20mm)$

$A_1 = 800mm^2$

$\bar{x}_1 = 10mm$

$\bar{y}_1 = 40mm$

Shape 2:

$A_2 = (20mm) \times (80mm)$

$A_2 = 1600mm^2$

$\bar{x}_2 = 40mm$

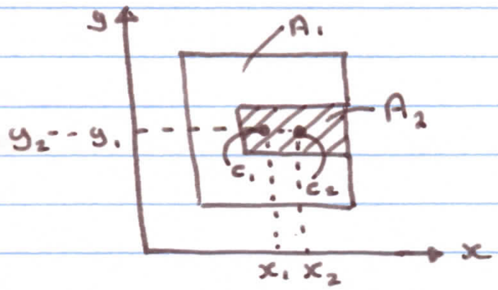
$\bar{y}_2 = 10mm$

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A_1 + A_2}$$

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2}$$

(1st order moment of an area)

4) Negative Area



If a shape has no material within an area, this region can be considered as a negative area.

$$A_1 = x_1 \cdot y_1$$
$$-A_2 = x_2 \cdot y_2$$

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2(-A_2)}{A_1 + (-A_2)}$$

$$\bar{y} = \frac{\bar{y}_1 A_1 + y_2(-A_2)}{A_1 + (-A_2)}$$

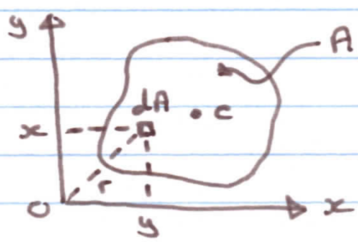
A.2 Moment of Inertia of an Area

1) Moment of Inertia

(2nd order moment of an area)

- Polar moment of inertia

$\int r^2 \cdot dA$ = moment of inertia (J_o)



$$J_o = \int r^2 \cdot dA = \int (x^2 + y^2)\, dA$$

Polar moment of inertia

$$J_o = \int_A r^2 dA = \underbrace{\int x^2 dA}_{I_y} + \underbrace{\int y^2 dA}_{I_x}$$

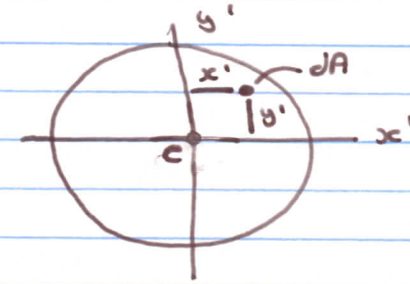
- J_o = polar moment of inertia
- $I_x = \int y^2 dA$ moment of inertia about x-axis
- $I_y = \int x^2 dA$ moment of inertia about y-axis

$$J_o = I_x + I_y$$

Units : m^4 : mm^4 : in^4

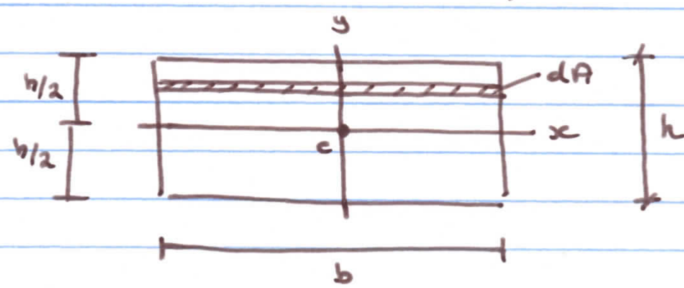
2) Moment of inertia about an axis passing through C

$$I_{x'} = \int y'^2 dA$$
$$I_{y'} = \int x'^2 dA$$
$$J_o = \int r^2 dA = \int (x^2 + y^2) dA$$
$$= I_{x'} + I_{y'}$$



$$I_{x'} = \int_A y'^2 dA$$

$$= \int_A y'^2 \underbrace{(bdy')}_{\varnothing}$$



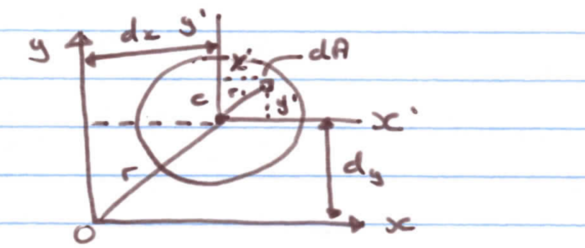
$$= \int_{h/2}^{b} y'^2 \, bdy'$$

$$= b\int_{-h/2}^{h/2} y'^2 dy' = b\tfrac{1}{3} y'^3 \Big|_{-h/2}^{h/2} = b/3 \left[\frac{(h/2)^3 - (h/2)^3}{\frac{h^3}{2} + \frac{h^3}{2}}\right]$$

$$\boxed{= \frac{bh^3}{12}}$$

3) Parallel axis method:

- Set up an axis x' Parallel to the x-axis Passing through the centroid
- Determine I_x



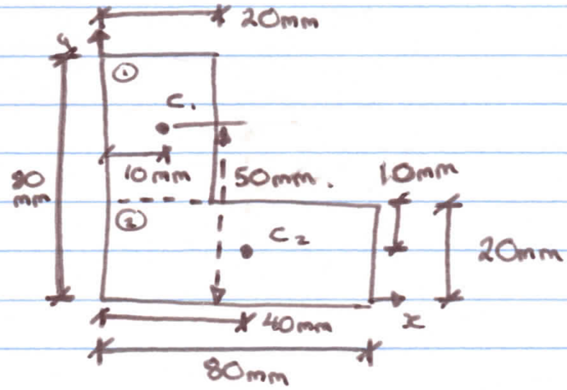
$$I_x = \int (d_1 + y')^2 dA$$
$$= \int (d_1^2 + y'^2 + 2d_1 y') dA$$
$$= \int (d_1 dA + y'^2 dA + 2d_1 y' dA)$$
$$= \int d_1 dA + \int y'^2 dA + \int d_1 y' dA$$
$$= \underbrace{d_1^2 \int dA}_{A} + I_{x'} + 2d_1 \underbrace{\int y' dA}_{\varnothing}$$

A = area of shape
d_1 = distance between x', x-axis
$C: (\bar{x}', \bar{y}') \Rightarrow (\bar{x}, \bar{y})$

$$I_x = I_{x'} + d_1^2 A$$
$$I_y = I_{y'} + d_2^2 A$$
$$J_o = I_x + I_y = (I_{x'} + d_1^2 A) + (I_{y'} + d_2^2 A)$$
$$= I_{x'} + I_{y'} \underbrace{(d_1^2 + d_2^2)}_{(r^2)} A$$

Solution

I_x, I_y, J_o



Area 1 : $C_1 (10, 50)$

$A_1 = 1200 mm^2$

Area 2 : $C_2 (40, 10)$

$A_2 = 1600 mm^2$

$I_x = \frac{1}{12} \cdot 20 \cdot 60^3 + \ldots$
$+ \ldots 50^2 \times 1200 + \ldots$
$+ \ldots \frac{1}{12} \cdot 80 \cdot 20^3 + \ldots$
$+ \ldots 10^2 \cdot 1600 \Rightarrow 3.57 \cdot 10^6 mm^4$

(For Rectangle : $I_x = \frac{1}{12} bh^3$, $I_y = \frac{1}{12} bh^3$)

$I_y = \frac{1}{12} \cdot 60 \cdot 20^3 + 10^2 \cdot 1200 + \frac{1}{12} \cdot 20 \cdot 80^3 + 80^2 \cdot 1600$
$\Rightarrow 3.57 \cdot 10^6 mm^4$

$J_o = I_x + I_y = 7.14 \cdot 10^6 mm^4$

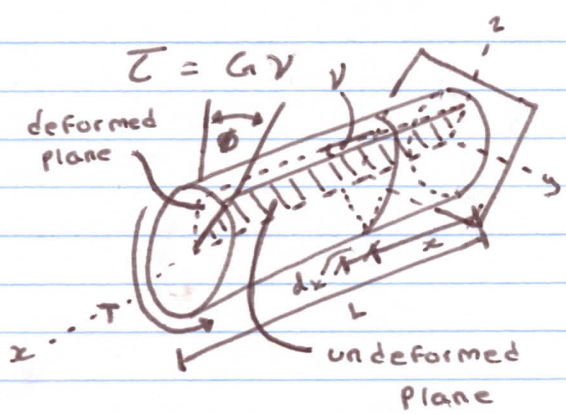
Chapter 5 - Torsion

5.1 - Torsional Deformation

Observations:

- each longitudinal line ~ helix
- A circle → a circle
- cross sections → Flat (no warping)

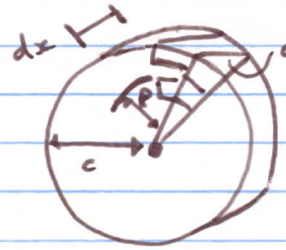
$\tau = G\gamma$

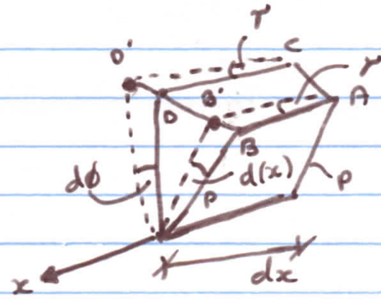


$$\boxed{\delta = \frac{Fl}{AE} \qquad \varepsilon = \frac{\sigma}{E}}$$

$x = 0$

$x = L : \phi(x)\Big|_{x=L} = \phi(L)$





$BB' = \gamma dx$

$= \rho d\phi$

$\gamma dx = \rho d\phi$

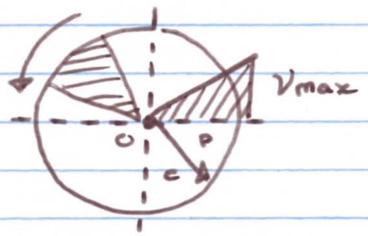
strain @ P

$\gamma = \rho \left(\dfrac{d\phi}{dx} \right)$ con strain

- the magnitude of shear strain γ, varies only with ρ (the distance from the axis of rotation)
- Shear strain γ varies linearly along any radial direction

$\gamma_{min} = 0$. if $\rho = \emptyset$

$\gamma_{max} = c \dfrac{d\phi}{dc}$



$$\gamma = \frac{\rho d\phi}{dx}$$

$$\gamma_{max} = \frac{c\, d\phi}{dx}$$

$$\frac{\gamma_{max}}{\gamma} = \frac{c \frac{d\phi}{dx}}{\rho \frac{d\phi}{dx}} = \frac{c}{\rho}$$

$\tau = G \cdot \gamma$

$$\gamma_{max} = \gamma \, c/\rho$$

5.2 - Tension Formula

if the material ~ linear elastic.

@P $\tau = G\gamma$

τ = shear stress @ P

G = shear modulus of elasticity

$\rho = c$ (at the outer surface)

$$\tau_{max} = G\gamma_{max}$$

$$\frac{\tau_{max}}{\tau} = \frac{G\left(\gamma_{max}\right)}{G\left(\gamma\right)}$$

$$\tau_{max} = \tau\left(\frac{\gamma_{max}}{\gamma}\right) = \tau\left(c/\rho\right)$$

Assignment #4
5-7,
5-9

- Shear stress τ varies linearly along any radia direction

$\tau = 0$ ~ τ_{max} => $\tau = \dfrac{T\rho}{J}$

$(\rho = 0)$

T = internal torque

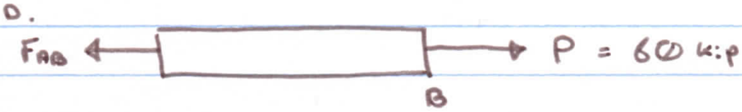
P = Polar moment of inertia

Example:

Solution: $\delta_B = \delta_{AA'} + \delta_{AB}$

Member AB

F.B.D.

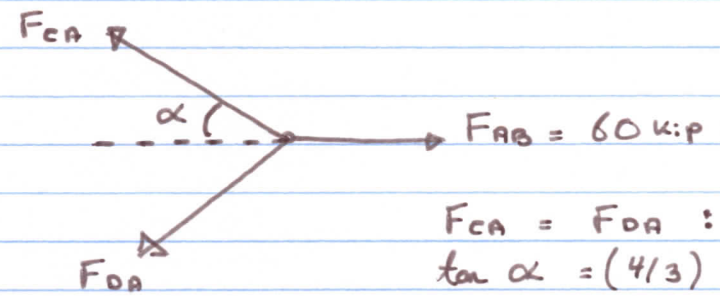


$F_{AB} = 60$ kip (T)

$$\delta_{AB} = \frac{F_{AB} \cdot 6 \cdot 12\text{ in}}{E_{AB} \cdot A_{AB}} \Rightarrow \frac{(60 \times 10^3\text{ lb})(6)(12)}{(29 \times 10^3\text{ psi}) \times \left(\frac{\pi}{4}(1\tfrac{1}{4})\right)}$$

$\delta_{AB} = 0.1214$ in

Point A:



$F_{CA} = F_{DA}$: $(+\uparrow \Sigma F_y = 0)$

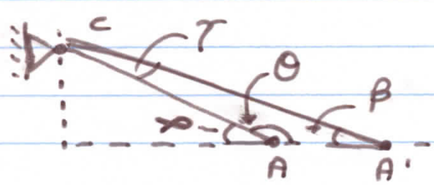
$\tan\alpha = (4/3)$ $\quad \alpha = \tan^{-1}(4/3) = 53.13°$

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{CA}\cos\alpha - F_{DA}\cos\alpha + F_{AB} = 0$

$-2F_{CA} \cdot (3/5) + 60\text{ kip} = 0$

$F_{CA} = F_{DA} = 50$ kip



$$\delta_{A/C} = \frac{F_{AC} \cdot l_{AC}}{E \cdot A_{AC}} = \frac{(50 \cdot 10^3)(60\text{ in})}{(29 \cdot 10^3)(\pi/4\,(5/4)^2)} \Rightarrow 0.0430\text{ in}$$

$l_{CA'} = 60\text{ in} + (0.0430\text{ in}) = 60.0430\text{ in}$

$\theta = 180° - \alpha \Rightarrow 180° - 53.130° = 126.87°$

$\sin\beta = \dfrac{(4 \cdot 12)}{60.0430} \Rightarrow \beta = 53.023°$

$\gamma = 180° - \theta - \beta \Rightarrow 0.107°$

$$\frac{\delta_{AA'}}{\sin\gamma} = \frac{l_{CA}}{\sin\beta}$$

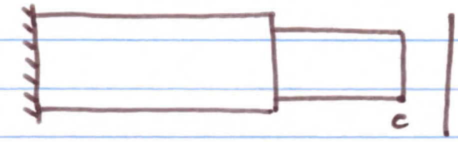
$\delta_{AA'} = 0.1403$ in

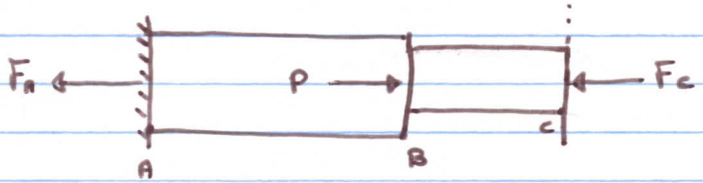
$\delta = \delta_{AB} + \delta_{AA'} = 0.1214 + 0.1403$

$\boxed{= 0.2617 \text{ in}}$

EXAMPLE 4-46

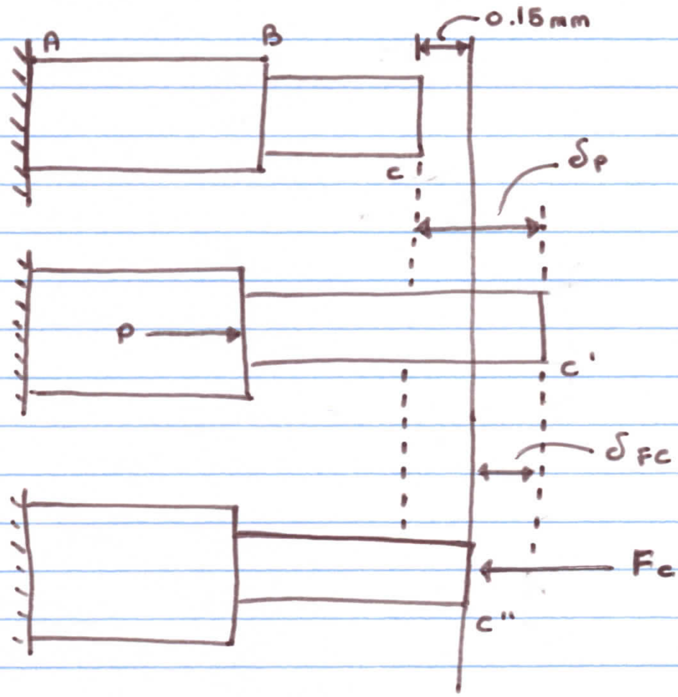
Solution:





$\Sigma F_x = 0$

$-F_A + 200\text{ kN} - F_c = 0$



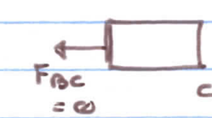
SUPERPOSITION METHOD

$$\delta_P = \frac{F_{AB}\, l_{AB}}{E\, A_{AB}}$$

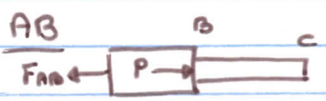
$$\Rightarrow \frac{(200\cdot10^3\text{ N})(0.6\text{ m})}{(200\cdot10^9\text{ Pa})(\pi/4(0.05)^2)}$$

CASE 1

BC



AB



(c)

Deformation compatability

$\delta_P = \delta_{FC} = 0.15$ mm $= 0.15\times10^{-3}$ m

$F_c = 20.365$ N

$F_A = 179.635$ N

($\leftarrow$)

Case 2

$F_{AB} = F_c$ (c)

$F_{BC''} = F_c$ (c)

$\delta_{FC} = \delta_{AB} + \delta_{BC''}$

$$= \frac{-F_c \times 0.6}{200\times10^9 \times \pi/4\,(0.05)^2} + \frac{-F_c \cdot 0.6}{200\cdot10^9 \times \pi/4\,(0.05)^2}$$