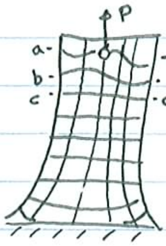
Mechanics

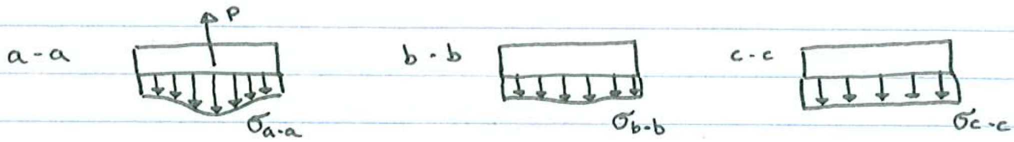
Oct. 18/16

Chapter 4 - Axial Load

4.1 Saint Venant's Principle



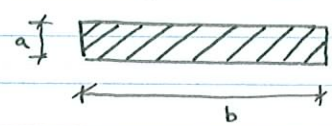
- load distorts lines located near load.
- lines located away from load and support remain straight.
- load distorts lines located near support.



The localized effects caused by any load acting on the body will smooth out regions that are sufficiently far away from the location of the load.

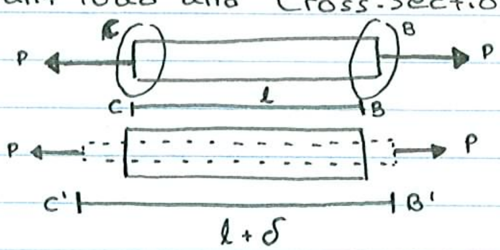
The resulting stress distribution at these regions will be the the same as that caused by any other statistically equivalent load applied to the body with the same localized area.

Localized deformation caused by $\vec{P}$ vanishes if the measurement > the largest dimension of the cross-section.



4.2 Elastic Deformation of an Axially Loaded Member

1) Constant load and Cross-sectional area



$\sigma = E\varepsilon$

$F/A = E\,\delta/l$

A = Area

E = Modulus of Elasticity

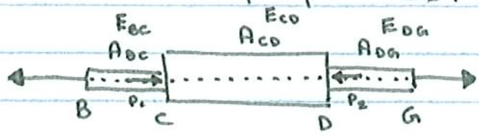
F = Internal Loading

Assume linear-elastic region:

$F = P$

$$\delta = \frac{Pl}{AE}$$

2) Non-constant load and/or cross-sectional area and/or Material properties.

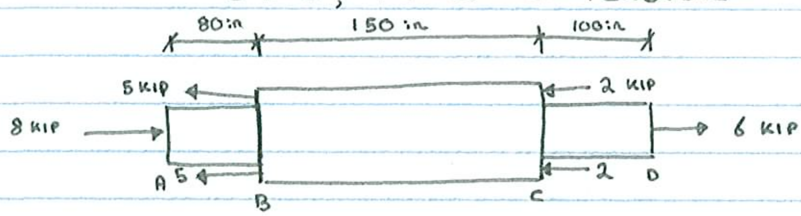


$\cancel{F_{BC} = P}$
$\cancel{F_{CD} = P}$
$\cancel{F_{DG} = P}$

$\delta = \dfrac{Pl}{EA}$ can be applied to each segment where these quantities are constant.

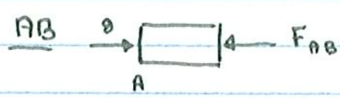
$$\delta_{B/G} = \delta_{B/C} + \delta_{C/D} + \delta_{D/G}$$
$$= \frac{F_{BC} \cdot l_{BC}}{E_{BC} \cdot A_{BC}} + \frac{F_{CD} \cdot l_{CD}}{E_{CD} \cdot A_{CD}} + \frac{F_{DG} \cdot l_{DG}}{E_{DG} \cdot A_{DG}}$$

Note: $\delta < 0$, $F < 0$ ~ compression
$\delta > 0$, $F > 0$ ~ tension

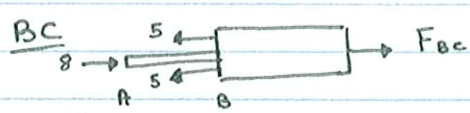


($d_{AB} = 0.75$ in
$d_{BC} = 1.00$ in
$d_{CD} = 0.5$ in)

Solution



$F_{AB} = 8$ kip (C)



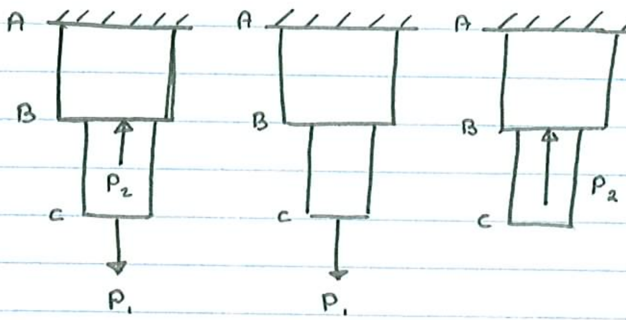
$F_{BC} = 2$ kip (T)

$F_{CD} = 6$ kip (T)

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$
$$\delta_{AD} = \frac{F_{AB} \cdot l_{AB}}{E_{AB} \cdot A_{AB}} + \frac{F_{BC} \cdot l_{BC}}{E_{BC} \cdot A_{BC}} + \frac{F_{CD} \cdot l_{DC}}{E_{CD} \cdot A_{CD}}$$
$$\delta_{AD} = \frac{(8 \times 10^3)(80)}{(18 \times 10^6)(\pi/4)(.75)^2} + \frac{(2 \times 10^3)(150)}{(18 \times 10^6)(\pi/4)(1)^2} + \frac{(6 \times 10^3)(100)}{(18 \times 10^6)(\pi/4)(0.5)^2}$$
$$\delta_{AD} = 0.111 \text{ in (total elongation)}$$

Principle of Superposition: The resulting stress or displacement at the point can be determined by first finding the stress or displacement caused by each component load acting separately on the member, and then algebraically adding the results.

4.3 Principle of Superposition



- material must be linear-elastic
- loading must be linearly related to stress and deformation
  $\sigma = P/A$, $\tau = \frac{V}{A}$, $\delta = \frac{F\ell}{EA}$
- loading does not significantly change the geometry of the ~~structure~~ member

ASSIGNMENT 3 QUESTIONS:

(4-5)

(4-13)