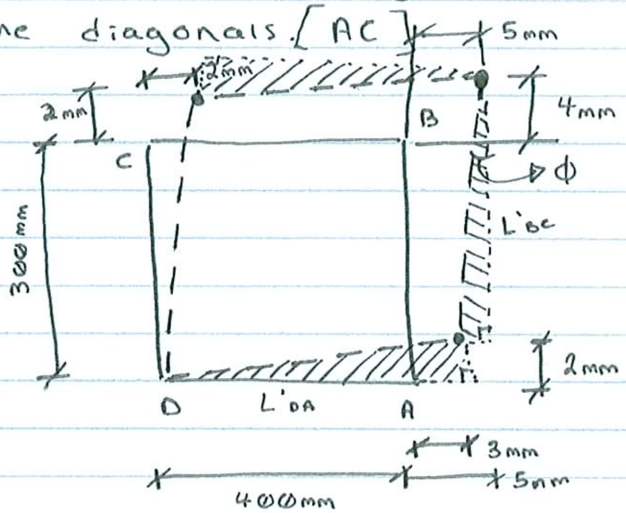
Oct. 4/16

Question 2.18 and 2-20

Determine the shear strain γ_{xy} at corners A and B, if the plate distorts as shown by the dashed lines.

Determine the average normal strain that occurs along the diagonals [AC]



$L'_{BC} = 300 + 4 - 2 = 302\ mm$

$\phi \approx \tan\phi = \dfrac{2mm}{302mm} = 0.00662$ rad

$0.00662 \times \dfrac{180°}{\pi} = (\deg)$

$L'_{DA} = 400\ mm + 3\ mm = 403mm$

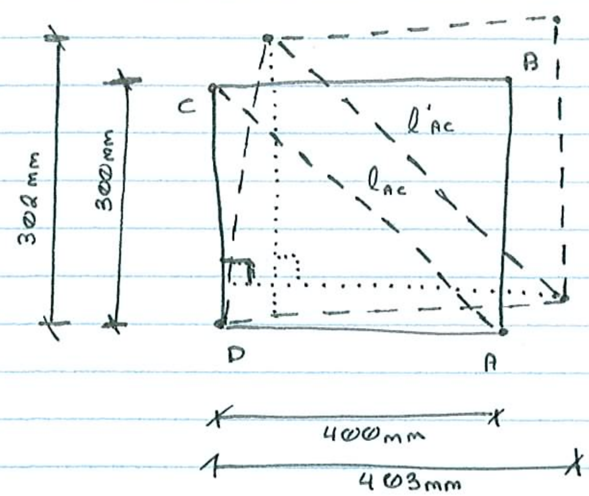
$\alpha \approx \tan\alpha = \dfrac{2mm}{400+3} = 0.00496$ rad

Shear strain @ A : $-(\alpha+\beta) = -(0.00662 + 0.00496)$

@ B : $+(\gamma + \phi) = +(0.00662 + 0.00496)$

Original length

$l_{AC} = \sqrt{300^2 + 400^2} = 500\ mm$



Original length

$l_{ac} = \sqrt{300^2 + 400^2} = 500$

$DA' = \sqrt{(400+3)^2 + 2^2} =$

$DC' = \sqrt{(300+2)^2 + 2^2} =$

$\overline{C'A'} = \sqrt{\overline{DA'}^2 + \overline{DC'}^2 - 2\overline{DA'} * \overline{DC'} \cdot \cos\frac{\pi}{2}}$

But you don't have to use cosine law.
(do it this way):

IF $\overline{DA} = 400 + 3 - 2 = 401$

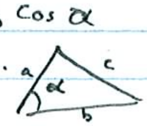
$\overline{DC} = 300 + 2 - 2 = 300$

$\varepsilon_{AC} = \dfrac{l_{AC} - \overline{C'A'}}{l_{AC}} = 0.00160\ mm/mm = 1.6\ mm/m$

$l'_{AC} = \sqrt{(401)^2 + (300)^2} = 500.8\ mm$

$C = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$

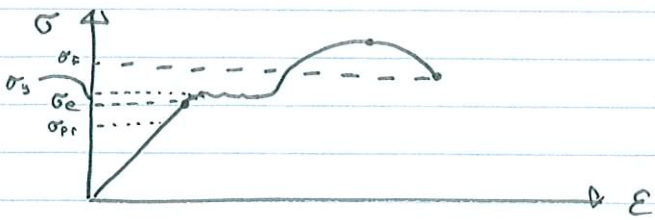
in terms of..



$\sigma - \varepsilon$ behavior of Materials

1) Ductile materials

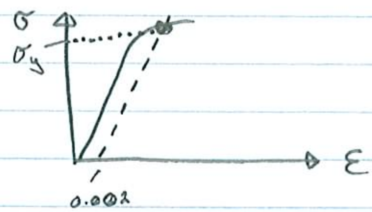
$$P.E. = \frac{L_f - L_o}{L_o} \times 100\% > 5\%$$



$$P.R.A. = \frac{A_o - A_F}{A_o} \times 100\%$$

- Carry impacts, overloading

0.2% strain offset method:



2) Brittle materials

$P.E. < 5\%$

- No clear yielding period (little or no yielding before Fracture)
  (or Failure)

3) Hooke's law

- linear elastic region

$\frac{\sigma}{\varepsilon} = E$ (modulus of elasticity)
  or Young's modulus. - if material behaves in linear elastic region then...

Notes:

- material must have linear elastic behavior
- $E \sim$ slope
- Units of E ( ) $\sim$ pa, Mpa, Mpa, psi, Msi
- E values $\sim$ hand books

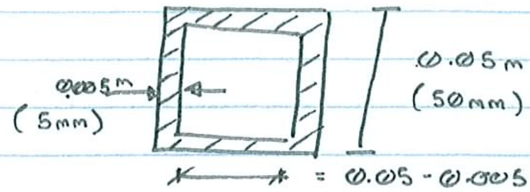
Example 2 (not in text)

Solution

when $P = 100$ kN

$\sigma = F/A$

$\downarrow$



$= 0.05 - 0.005 - 0.005 = 0.04m$

$$\sigma = \frac{F}{A} = \frac{100 \times 10^3 \text{ N}}{(0.05m \times 0.05m) - (0.04m)^2} = 111.11 \times 10^6 \text{ Pa}$$

- lies in the linear elastic region
- no permanent deformation

Assignment Questions:

Q3-2

Oct. 7/16

Solution:

If $P = 100$ kN

$$\sigma = F/A = \frac{100 \times 10^3 N}{0.05^2_m - 0.04^2_m} = 111.11 \times 10^6 PA = 111.11 \text{ MPa}$$

$$E = \frac{\sigma}{\epsilon} = 250 \text{ MPa} = \frac{250 \times 10^6 \text{ Pa}}{0.00125 \text{ m/m}} = 200 \times 10^9 \text{ Pa} = 200 \text{ GPa}$$

state ~ linear elastic region

→ no permanent ~~elongation~~ deformation = 0
plastic

elastic elongation:

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{111.11 \times 10^6 \text{ Pa}}{200 \times 10^9 \text{ pa}} = 0.000556 \text{ mm/mm}$$

elastic deformation

$$\delta_1 = \varepsilon_1 \cdot l = 0.000556 \text{ mm/mm} \times 600 \text{ mm} = 0.333 \text{ mm}$$

If $P = 360$ kN

$$\sigma_2 = F/A = \frac{360 \times 10^3 N}{0.05^2_m - 0.04^2_m} = 400 \times 10^6 \text{ pa}$$

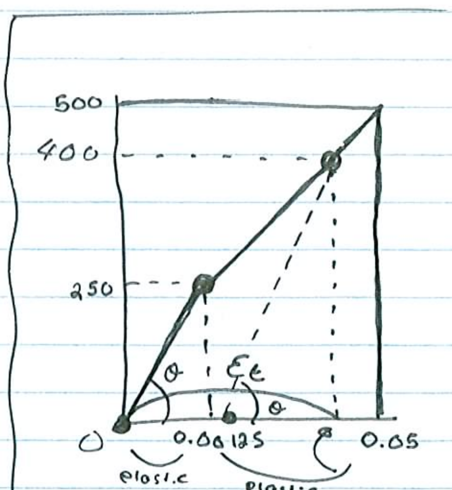
Both elastic and plastic deformation

Total strain $= \varepsilon_T = \varepsilon_e + \varepsilon_p$

$$\frac{500 - 250}{400 - 250} = \frac{0.05 - 0.00125}{\varepsilon_t - 0.00125}$$

(total strain)

↳ $\varepsilon_t = 0.0305$ mm/mm



If P is removed, the strain is recovered linearly along the line parallel to the elastic-linear line.

$$\varepsilon_e = \frac{\sigma}{\tan\theta} = \frac{\sigma_2}{E} = \ldots$$

$$\ldots \frac{400 \times 10^6 \text{ Pa}}{200 \times 10^3 \text{ pa}} = 0.002 \text{ mm/mm}$$

↓

elastic strain

$$\varepsilon_e = \frac{\sigma_2}{\tan\theta} = \frac{\sigma_2}{E} = \frac{400 \times 10^6\ pa}{200 \times 10^3\ pa} = 0.002\ mm/mm$$

plastic strain

0.0305

$$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.0305 - 0.002\ mm/mm = 0.0285\ mm/mm$$

elastic:

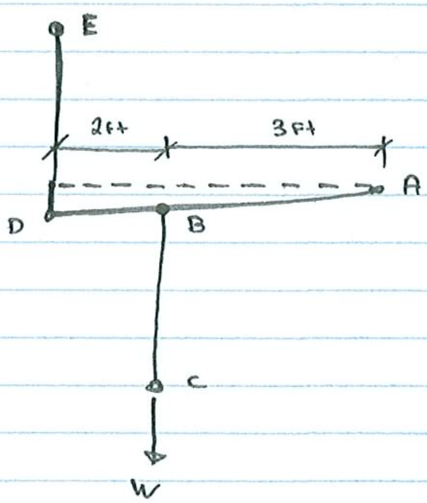
$\delta_e = \varepsilon_e \times 600mm$

plastic:

$\delta_p = \varepsilon_p \times 600mm = 17.1\ mm$

MIDTERM @ 20TH OCT. IN CLASS

Example 3.4 (Question 3-24)



$\varepsilon_E \cdot \varepsilon_{BC}$

$\omega = ?$

$E = 29 \times 10^3\ ksi$

$$\frac{\delta_{DE}}{0.025} = \frac{5\ ft}{3\ ft}$$

$\delta_{DE} = 0.0417\ in$

$$\varepsilon_{DE} = \frac{\delta_{DE}}{L_{DE}} = \frac{0.0417\ in}{3 \times 12\ in} = 0.00116\ in/in$$

Hooke's Law

Stress of DE

$\sigma_{DE} = E\varepsilon_{DE} = 33.56 \times 10^3\ psi$

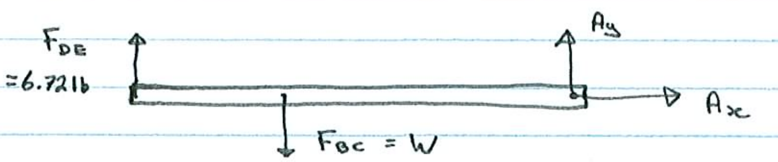
STRESS IN BC

$\sigma_{BC} = \omega / A_{BC} = \frac{112\ lb}{0.002\ in^2} = 55.9 \times 10^3\ psi$

$\varepsilon_{BC} = \sigma_{BC} / E =$

Force @ ED

$F_{DE} = \sigma_{DE} \cdot A_{DE}$



$\circlearrowleft^{+} \sum M_A = 0$

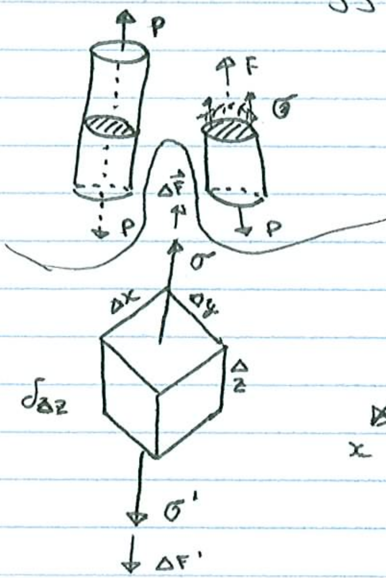
$= -(6.72\ lb)(5) + \omega(3) = 0$

$\omega = 112\ lb$

3.4 Strain Energy

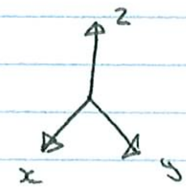
External loading $\rightarrow$ store energy internally

~ strain energy



Stress $\sigma_{AVG} = \sigma$

strain $\varepsilon_{AVG} = \varepsilon$



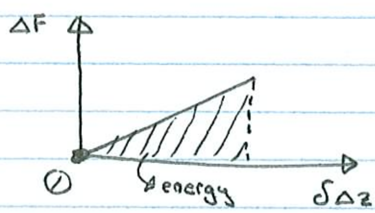
Force :

$\Delta F = \sigma(\Delta x \cdot \Delta y)$

Deformation.

$\delta_{\Delta z} = \varepsilon_{AVG} \cdot \Delta z$

Energy ~ product of Force × distance.



$\longrightarrow (\Delta\delta_z)$

(energy)

$$\Delta W = \frac{1}{2}\Delta F \times \delta_{\Delta z}$$

$$\Delta W = \frac{1}{2}\sigma(\Delta x \cdot \Delta y) \times \varepsilon \times (\Delta z)$$

$$\Delta W = \frac{1}{2}\sigma\varepsilon(\Delta x \cdot \Delta y \cdot \Delta z)$$

$$= \frac{1}{2}\sigma\varepsilon\,\Delta V$$

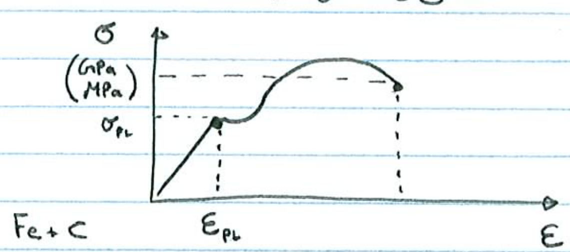
Strain Energy Density

$$u = \frac{\Delta W}{\Delta V} = \frac{\frac{1}{2}\varepsilon\sigma \times \Delta V}{\Delta V} = \tfrac{1}{2}\sigma\varepsilon$$

$$= \tfrac{1}{2}E\varepsilon^2 = \tfrac{1}{2}\sigma^2/E$$

Hookes :

$\therefore \sigma = E\varepsilon$

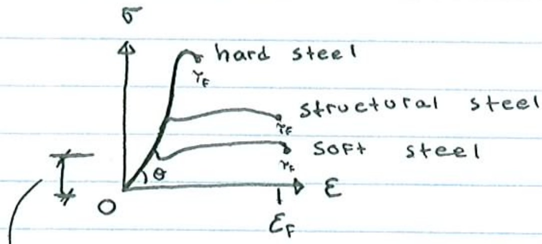


1) Modulus of Resilience

$$U_r = \tfrac{1}{2}\sigma_{PL} \cdot \varepsilon_{PL}$$

$$= \tfrac{1}{2}E\,\varepsilon_{PL}^2$$

$$= \frac{\tfrac{1}{2}\sigma_{PL}^2}{E}$$



hard steel
(0.6% carbon)
highest strength

structural steel
(0.2% carbon)
toughest

Soft steel
(0.1% carbon)
most ductile

These 3 materials have similar E even though they have different proportional limits.

$\tan\theta = E$

U_r ~ ability to absorb energy without permanent deformation (plastic deformation)

- 0.25% or 0.30% most commonly used
- steels can reach 0.8% (maybe 0.9%)

2) Modulus of Toughness

$$U_t = \frac{1}{2}\sigma_F \varepsilon_F$$

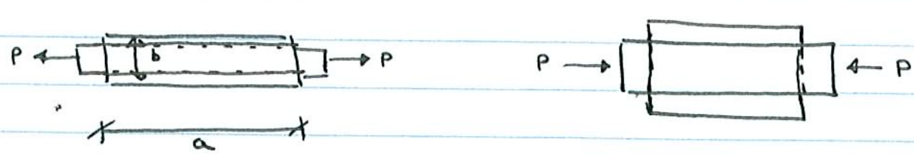
$$U_t = \frac{1}{2}\frac{\sigma_F^2}{E}$$

$$U_t = \frac{1}{2}E\varepsilon_F^2$$

- (represents) ~ capability to absorb energy before failure.

$U_t \uparrow$ ~ overloading $\uparrow$

3.5 Poisson's Ratio



longitudinal $\varepsilon_{long} = \dfrac{\delta_{long}}{a} = \dfrac{a' - a}{a}$ (+)

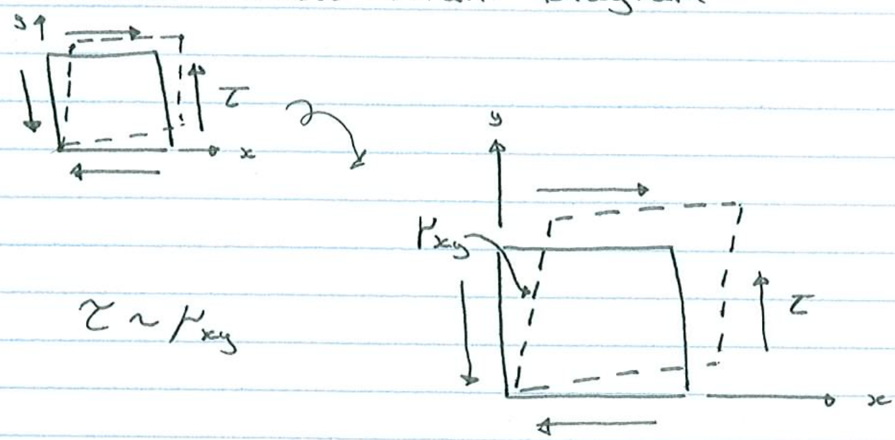
lateral $\varepsilon_{lat} = \dfrac{\delta_{lat}}{b} = \dfrac{b' - b}{b}$ (−)

$0 < \nu < 0.5$

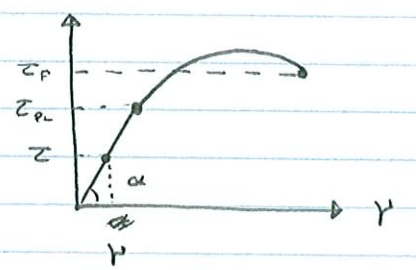
Poisson's ratio $\nu = -\dfrac{\varepsilon_{lat}}{\varepsilon_{long}}$

Note: - negative sign
- suitable for both tension + comp
- ν is a constant for a given mat.

3.6 Shear Stress - Strain Diagram



$\tau \sim \gamma_{xy}$



$\tan \alpha = \dfrac{\tau}{\gamma} \rightarrow G$

$\tau = G\gamma$

G = shear modulus of elasticity
pa, KPa, Mpa, Gpa, Ksi, psi (units)

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

Stainless steel (304)
$E = 28 \times 10^3$ Ksi
$G = 11 \times 10^3$ Ksi
$\nu = 0.27$

* ASSIGNMENT 2
- 3 - 5
- 3 - 6
- 3 - 22
- 3 - 25

Due on Tuesday after reading week.

Solution:
$\sigma_u = 76 \times 10^6$ psi
$F_u = \sigma_u \times A = 76 \times 10^6 \text{psi} \times \frac{\pi(0.5)^2}{4}$

Solution: (question 1)

$E = \dfrac{\sigma}{\varepsilon} = \dfrac{\sigma_{PL}}{\varepsilon_{PL}} = \dfrac{40 \times 10^3 \text{ psi}}{0.001 \text{ in/in}} = 40 \times 10^6 \text{ psi} = 40 \cdot 10^3 \text{ kpsi}$

Solution : (question 2)

$\sigma_y = 40 \times 10^6$ psi $\rightarrow \sigma_y = \dfrac{F_y}{A} \Rightarrow F_y = \sigma_y \times A$
$\Rightarrow 40 \times 10^6 \text{ psi} \cdot \dfrac{\pi(0.5)^2}{4}$