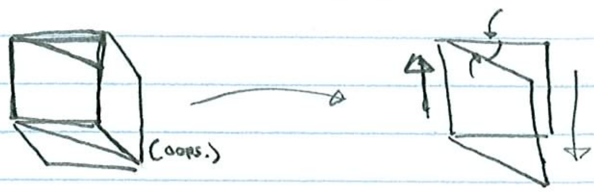
Sept. 27/16

Chapter 2. - Strain

2.1 Deformation

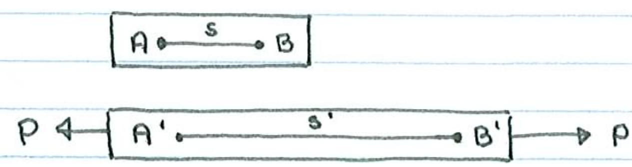
Most engineering materials ~ slight deformation



2.2 Strain

1) Normal strain

~ the elongation or connection by unit of length



Change in length : $\Delta S = S' - S$

S' = the new dimension

S = the original ——

average normal strain

$$\varepsilon_{AVG} = \frac{S' - S}{S} = \frac{\Delta S}{S}$$

Strain at point A

$$\varepsilon = \lim_{\substack{S \to \emptyset \\ (B \to A)}} \frac{S' - S}{S}$$

if ε is known,

$$S' = (1 + \varepsilon) S$$

Notes

- if $\varepsilon > \emptyset$, the member is subjected to tension
- if $\varepsilon < \emptyset$, the member is subjected to compression
- Segment could be a straight line, curve

2) Units

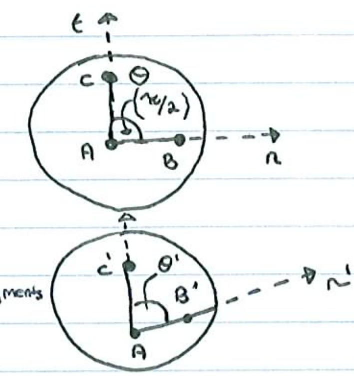
$\varepsilon = \dfrac{s' - s}{s}$ (no units)
(mm/mm) or (in/in) or (mm/m) or (in/ft) etc.

Strength	Stiffness
(Force)	(deformation)

3) Shear strain

line AB + AC

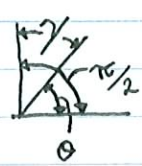
$\theta = \pi/2$



The change in angle, between two line segments is defined as:

$\gamma = \theta - \theta'$

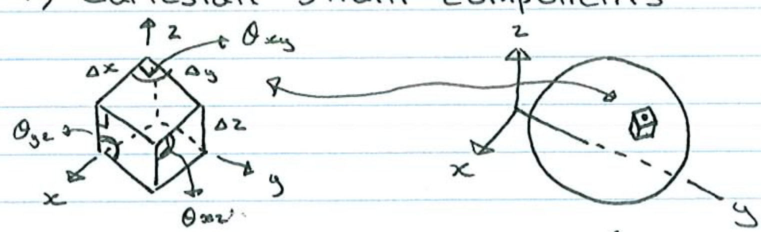
(gamma) = (theta) - (theta prime)



θ = original angle ($\pi/2$ rad) = 90°

$\gamma = 90° - \theta°$

4) Cartesian strain components



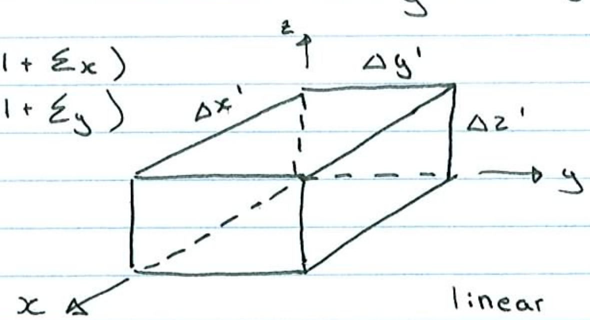
Three deformations

$\varepsilon_x, \varepsilon_y, \varepsilon_z$

$\gamma_{xy}, \gamma_{yz}, \gamma_{xz}$

$\Delta x' = \Delta x(1 + \varepsilon_x)$

$\Delta y' = \Delta y(1 + \varepsilon_y)$



linear strains change the volume of the element.

$\theta'_{xy} = \pi/2(1 + \gamma_{xy})$ $[\Rightarrow \pi/2 - \gamma_{xy}]$

$\theta'_{xz} = \pi/2(1 + \gamma_{xz})$ $[\Rightarrow \pi/2 - \gamma_{xz}]$

$\theta'_{zx} = \pi/2(1 + \gamma_{zx})$ $[\Rightarrow \pi/2 - \gamma_{zx}]$

- Shear strains change the shape of the element
- These two types of deformation occur simultaneously

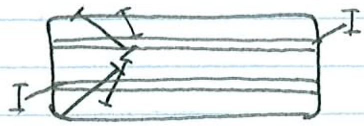
5) Small strain analysis

only small deformations are allowed

$\varepsilon \ll 1$ (much smaller than 1)

Assume $\varepsilon^2 \approx 0$

High order components ≈ 0



angular deformation

$\gamma \sim$ very small

$\sin\gamma \approx \gamma$

$\cos\gamma \approx 1$

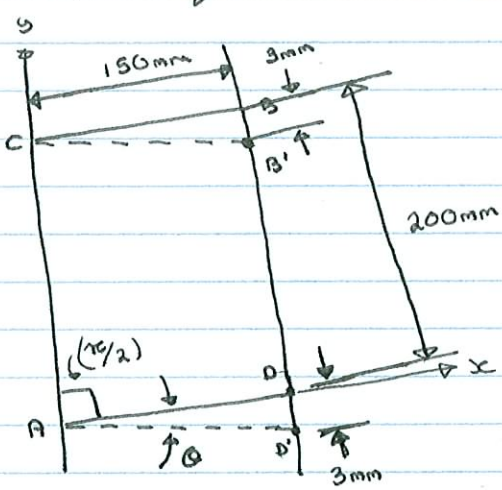
$\tan\gamma = \dfrac{\sin\gamma}{\cos\gamma} \approx \gamma$

Assignment Questions

→ 2-3

→ 2-29

Example question (2-5)



$\tan\theta = \dfrac{3mm}{150mm}$

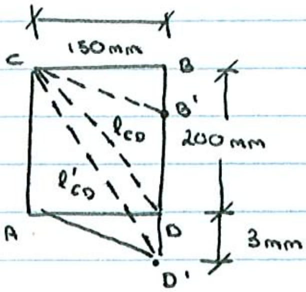
$\therefore \theta = 3/150 = 0.02$ rad

Solution

$\gamma = 90° - (90° + \theta) = -\theta$

$\gamma_{xy} = -\theta = -0.02$ rad

$= -0.02 \times \dfrac{180°}{\pi} =$ deg



$l_{CD} = \sqrt{(150)^2 + (200)^2}$

$l_{CD} =$

$l'_{CD} = \sqrt{(150)^2 + (200+3)^2}$

$l'_{CD} =$

$\varepsilon_{CD} = \dfrac{l'_{CD} - l_{CD}}{l_{CD}}$

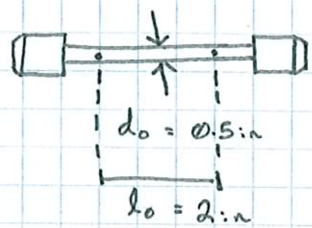
$\triangle CBD'$

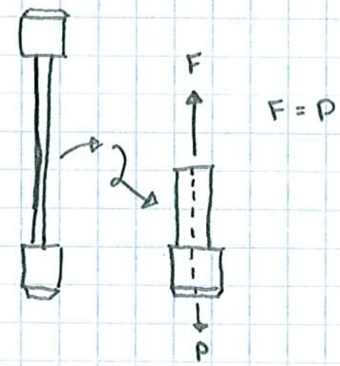
Sept. 29/18

Chapter 3: Mechanical Properties of Materials

3.1 Tension and compression Test

- used for metals, polymer, ceramics





Force P (measured by load cell)

Deformation $= \delta = l - l_0$

(measured by ~~strain~~ gauges)
strain

$\varepsilon = \delta / L_0$ $\qquad \sigma = F/A = P / \frac{\pi d^2}{4}$

area of cylinder face

3.2 Stress-Strain Diagram

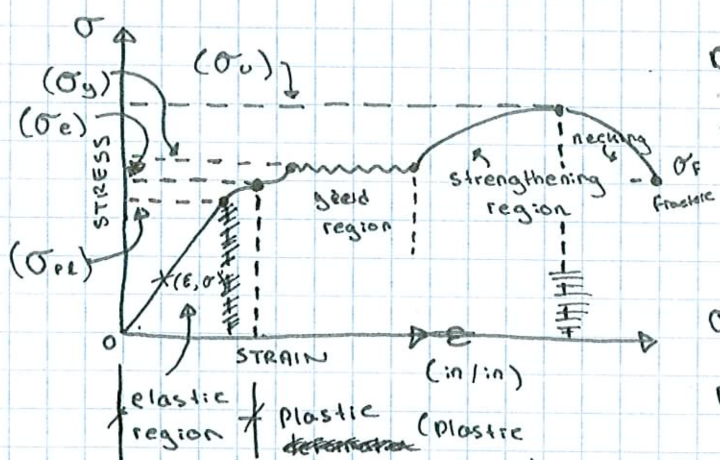
Average stress

uniformly distributed

$\sigma = F/A = (\cancel{\ldots}) \rightarrow \left(P / \frac{\pi d_0}{4}\right)$ write as

Average strain

constant between gauge marks $\qquad \varepsilon = \delta / l_0$



σ_{PL} = Proportional limit

elastic region - if the ~~force~~ (load) is removed the specimen will return to normal to its original shape.

σ_e = elastic limit

From σ_{PL} to σ_e is no longer linear.
(region)

Plastic behavior: permanent deformation occurs.

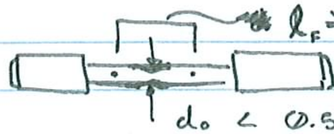
- engineering design occurs in elastic region (mostly)
- necking is when material becomes smaller (in terms of d_0) and can't support original stress.

σ_y = yielding

σ_u = ultimate limit

σ_F = fracture stress

* Assume σ_{pl}, σ_e, σ_y coincide unless specified otherwise



(Necking)

C content < 0.35% (low carbon) ~ low carbon steel C + Fe (alloy)
(carbon) steel - alloy with C, Fe, ...

ductile { C content < 0.35% ~ low C steel
C content 0.35 ~ 0.6% ~ medium C. steel
brittle { C content > 0.6% ~ (~1.2/1.9) ~ high C. steel

3.3 Stress-strain ($\sigma - \varepsilon$) behavior of ductile and brittle materials

Materials can be classified as either ductile or brittle

< ductile
brittle

1) Ductile materials
~ be subjected to large strains before failure
~ can absorb impacts. overloading

- percent elongation

$$PE = \frac{L_F - L_o}{L_o} \times 100\%$$

L_F = length at failure
L_o = original length

- Percent reduction in area

$$PR = \frac{A_o - A_F}{A_o} \times 100\%$$

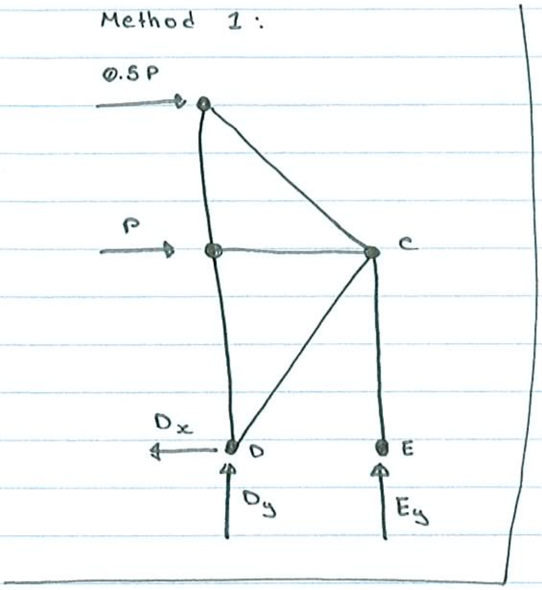
A_o = original area
A_F = Area in Fracture

- PE > 5% ~ ductile material
PE < 5% ~ brittle material

2) Brittle materials:
P.E < 5%
~ exhibits little or/no yielding before failure

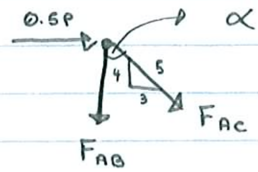
(Question 1-64 From Chapter 1)

Method 1:



Method 2:

Point A



$\overset{\rightarrow}{+} \Sigma F_x = 0$

$0.5P + F_{AC} \sin\alpha = 0$

$0.5P + F_{AC} \sin\alpha$

$(=) \; 6 \; F_{AC} (3/5) = 0$

$\dfrac{3.00 \text{ kip}}{(3/5)} = -F_{AC}$

$-F_{AC} = -5$ (change direction)

Member is in tension.

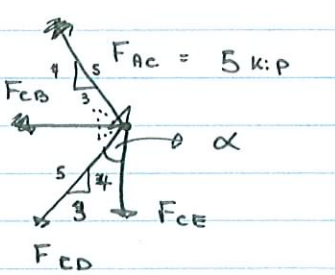
$+\uparrow \Sigma F_y = 0$

$-F_{AB} + F_{AC} (\cos\alpha)$

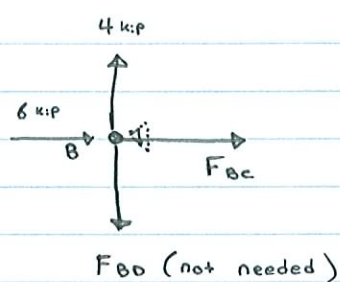
$F_{AC} (4/5) = F_{AB}$

$F_{AB} = (5)(4/5) = 4$

Point C



Point B



$\Sigma F_x = 0$

$\therefore F_{BC} = -6 \text{ kip } (c)$

~~$\overset{\rightarrow}{+} \Sigma F_x = 0$ (cos α)~~

~~$F_{BC} - F_{CD} = 0$~~

~~$F_{BC} = F_{CD}$~~

~~$3/5$~~

~~$F_{CD} = 8$ kip~~

oops..

$\overset{\rightarrow}{+} \Sigma F_x = 0$

$6 \text{ kip} + F_{AC_x} - F_{CD_x}$

$6 \text{ kip} + (3/5) F_{AC} - (3/5) F_{CD}$

$\Rightarrow F_{CD} = 15$ (Tension)

$+\uparrow \Sigma F_y = 0$

$15 \cdot \cos\alpha - F_{CD} \cos\alpha - F_{CE} = 0$

$F_{CE} = -16$ kip (comp.)

$\sigma_{CE} = \dfrac{-16 \times 10^3 \text{ lb}}{1.25}$

... no answer.

Example 2-14 (From textbook.)

$$l_{AB} = 1000 \text{ mm}$$

$$l_{AB'} = \sqrt{\overline{AC}^2 + \overline{CB'}^2 + 2\overline{AC}\cdot\overline{CB'}\cos(90^\circ + 0.5^\circ)}$$

$$= \cancel{}\ 1004.18 \text{ mm}$$

$$\varepsilon_{AB} = \frac{l_{AB'} - l_{AB}}{l_{AB}} = 0.00418 \text{ mm/mm}$$