Sept. 20/16

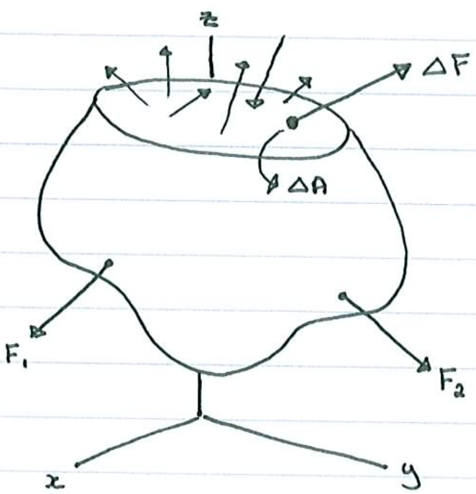
1.3 Stress

Stress is referred to as the internal Force over a specific area.

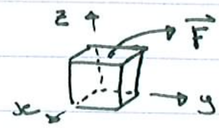
(P/A) (force / area) - where P = internal loading

Two basic assumptions : material is homogeneous, and material is isotropic.

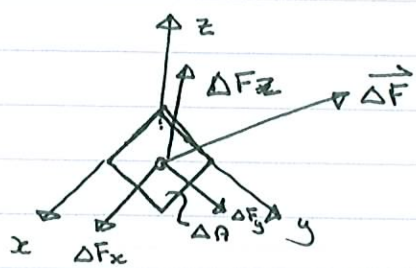
Point ~



Small cubic volume element.



$\Delta V \rightarrow 0$



If a part is in equilibrium, each element of a part is in equilibrium.

$\Delta A \rightarrow 0$

1) Normal Force (ΔF_z)

σ_z (normal stress)

$$\sigma_z = \lim_{\Delta a \to 0} \frac{\Delta F_z}{\Delta A}$$

- tensile stress (tensile Force)
- compressive stress (compressive Force)
- Subscript (z) ~ the direction line of the normal ~~Force~~ (stress)

2) Shear stress

$$\tau_{xx} = \lim_{\Delta a \to 0} \frac{\Delta F_x}{\Delta A}$$

→ 1st subscript specifies normal direction

→ 2nd subscipt specifies stress line direction

→ => $\tau_{zx} = \lim_{\Delta a \to 0} \frac{\Delta F_x}{\Delta A}$

as well as:

$$\tau_{zy} = \lim_{\Delta a \to 0} \frac{\Delta F_y}{\Delta A}$$

3) Units of Stress

(S.I.) F = newtons (N) pa = Pascal

A = (metres)

1 pa = $1 N/m^2$

(U.S. Unit) F = Pounds psi = pounds per square inch

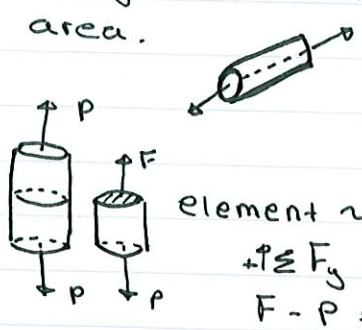
A = inch

psi = lb/in^2

1.4 Average Normal Stress in an Axially loaded Bar

- long and slender
- weight neglected
- material: homogeneous + isotropic

- loading applied to the ends through the centroid of the area.



element ~ uniform deformation

$+\uparrow \Sigma F_y = 0$

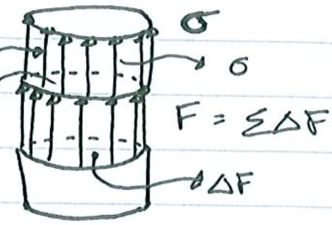
$F - P = 0 \quad \therefore F = P$

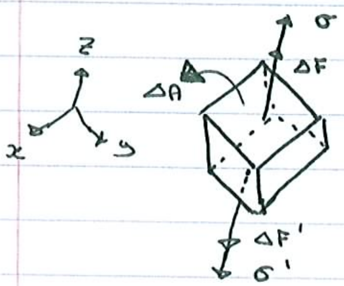
Average Stress

$\sigma = F/A$

A = Area

F = internal loading





$\sigma = \dfrac{\Delta F}{\Delta A}$; $\Delta F = \sigma \cdot \Delta A$

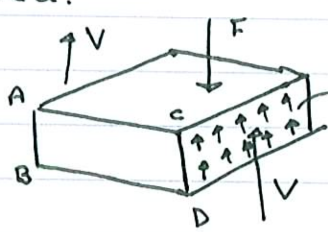
$\sigma' = \dfrac{\Delta F'}{\Delta A}$; $\Delta F' = \sigma' \cdot \Delta A$

$+\uparrow \Sigma F_z = 0$ $\quad \Delta F - \Delta F' = 0$

$\sigma \cdot \Delta A - \sigma' \cdot \Delta A = 0$

$\sigma = \sigma'$

Stress is uniformly distributed over the sectioned area.



$+\uparrow \Sigma F_z = 0$

$2V = F$

$V = F/2$

Average shear stress

$\tau = V/A' = \dfrac{F/2}{A'}$

V = resultant shear Force

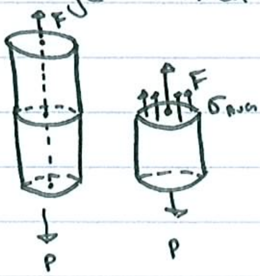
A' = area

Next Tuesday (1-32, 1-38, 1-66) Due?

Sept. 22nd

Tutorial @ Friday Sept. 22ND

Average normal stress



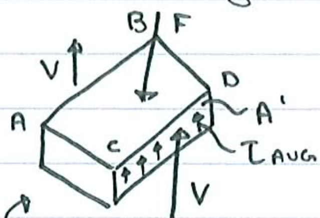
$F = P$

$\sigma_{AVG} = F/A$ (σ_{AVG} ~ uniformly distributed over the sectioned area)

F = internal resultant

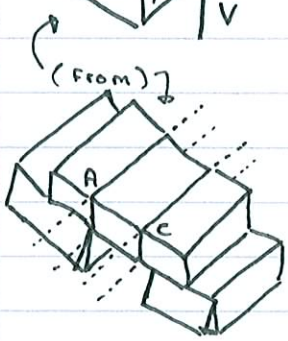
A = area

1.5 Average Shear Stress



$+\uparrow \Sigma F_y = 0$

$2V - F = 0 \Rightarrow V = F/2$

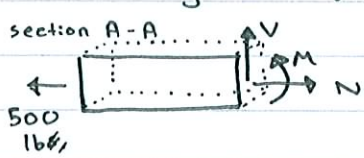


$\tau_{AVG} = V/A'$

where V = internal shear force

A' = area

For Figure 1-2 (in textbook)



$A = 2in \times 3in = 6 in^2$

$V = 0$

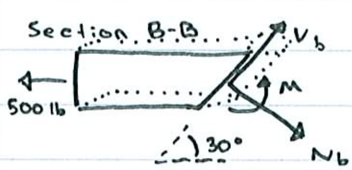
$M = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$N = 500 lb$

$\sigma_{AVG} = N/A = 500 lb / 6 in^2$

$=$ psi



$M = 0$

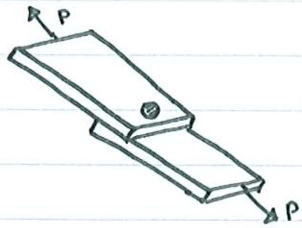
$V_b = 250 lb$

$N_b = 433 lb$

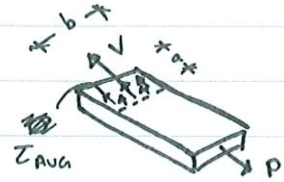
$\sigma_{AVG} = N/A = 433 lb / 2 \times (3/\cos 30°)$

$= -$ psi

2) Single Shear



(fastened w/ glue)

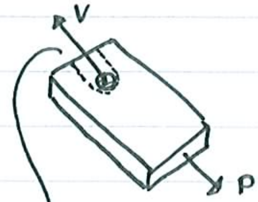


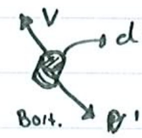
$V = P$

$\tau_{AVG} = \dfrac{V}{A}$

$= P/(a \cdot b)$

(with pin) →





Pin (bolt):

$$\tau_b = \frac{V_b}{A_b} = \frac{P}{\left(\frac{\pi d^2}{4}\right)} \le [\tau_{allow}]_b$$

$V = P$

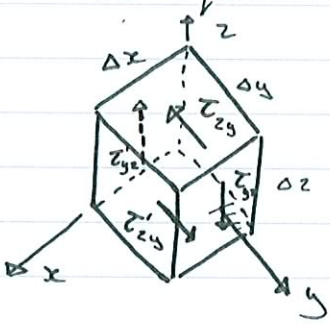
This peice could be removed from the shear force (if it's too large)

Plate (2 shear surface) $V = 2V_p$

$V_p = V/2 = P/2$

$$\tau_p = \frac{V_p}{A_p} = \frac{(P/2)}{(a \cdot b)} \le [\tau_{allow}]_p$$

3) Stress equilibrium



$\Delta V \to 0$

$\tau_{zy} = \dfrac{\Delta F}{\Delta A'}$

$\nearrow^{+} \sum F_y = 0$

$0 = -\tau_{zy} \cdot (\cancel{\Delta y \cdot \Delta x}) + \tau_{zy}'(\cancel{\Delta y \cdot \Delta x})$

$\tau_{zy} = \tau_{zy}'$

Same in magnitude, opp. in directions

$+\uparrow \sum F_z = 0$

$0 = \tau_{yz} \cdot (\cancel{\Delta x \cdot \Delta y}) - \tau_{yz}' \cdot (\cancel{\Delta x \cdot \Delta y})$

$\tau_{yz} = \tau_{yz}'$

Same in magnitude, but opposite in directions.

$\circlearrowleft^{+} \sum M_x = 0$

$(\tau_{zy} \cdot \cancel{(\Delta x \cdot \Delta y)) \cdot \Delta z} - (\tau_{yz} \cdot \cancel{(\Delta x \cdot \Delta z)) \cdot \Delta y}) = 0$

$\tau_{zy} = \tau_{yz}$

All shear stresses have some magnitude.
They are diverted toward an edge / away
From an edge.

1.6 Allowable Stress
1.7 Limited state design

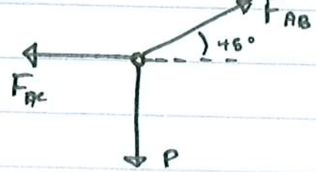
$\}$ Read sections.

(Example 1-87) in textbook

Solution $d_{AB} = 10\,mm$ $d_{AC} = 8\,mm$ $\sigma_{allow} = 150\,MPa$

Consider point A

F.B.D.



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_{AB}\cos(45°) - F_{AC} = 0$

$\uparrow + \Sigma F_y = 0$

$-P + F_{AB}\sin(45°) = 0$

$F_{AB} = P/(\sin 45°)$

$\therefore F_{AC} = \frac{P}{(\sin 45°)} \cdot \cos(45°)$

$\Rightarrow F_{AC} = \frac{P\cos(45°)}{(\sin 45°)}$

AB fails:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{(P/\sin 45°)}{\left(\frac{(10 \times 10^{-3}\,m)^2 \pi}{4}\right)}$$

$P \leq 8.33 \times 10^3\,N$

AC Fails

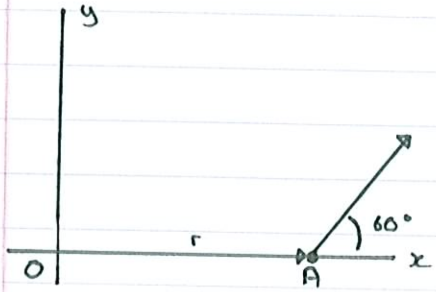
$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{\left(\frac{P\cos(45°)}{\sin 45°}\right)}{\left(\frac{\pi\,(8 \times 10^{-3}\,m)^2}{4}\right)} \leq 150 \times 10^6\,Pa$$

$P \leq 7.54 \times 10^3\,N$

The magnitude of force F in Fig 2.39 is 100 lb
The magnitude of the vector r from point
O to point A is 8 ft.

(a) Use the definition of the cross product
to determine r × F.

(b) Use Eq. (2.34) to determine r × F.

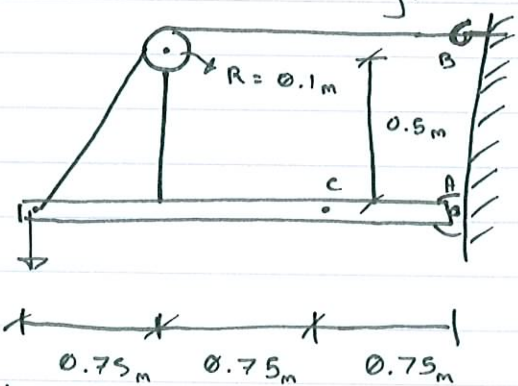


$\Sigma F_x = 0$

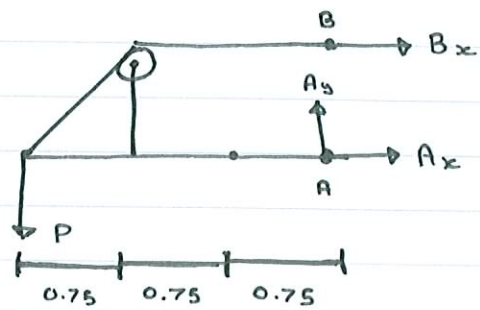
Consider the straight

Sept. 23/16

The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at the cross section through point C for this loading.



$\Rightarrow$



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$\Sigma F_x = B_x + A_x \Rightarrow A_x = -2\text{ kN}$

but $B_x = 2\text{ kN}$

$+\uparrow \Sigma F_y = 0$

$\Sigma F_y = A_y - P = 0$

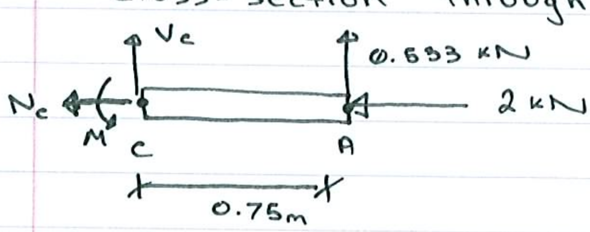
$\therefore A_y = P = 0.533\text{ kN}$

$\circlearrowleft + \Sigma M_A = 0$

$\Sigma M_A = -2\text{ kN}(0.6\text{m}) + P(2.25\text{m})$

$P = \dfrac{2\text{kN}(0.6\text{m})}{(2.25\text{m})} \Rightarrow P \leq 0.53\text{kN}$

Cross-section through point C.



$+\uparrow \Sigma F_y = 0$

$V_c + 0.533\text{ kN} = 0$

$V_c = -0.533\text{ kN}$

$\circlearrowleft + \Sigma M_c = 0$

$M + (+0.533\text{ kN})(0.75\text{ m}) = 0$

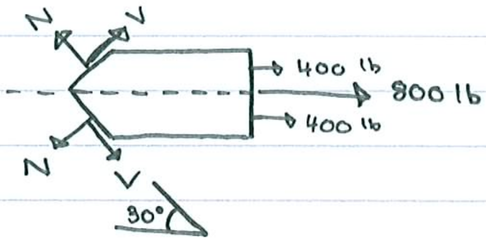
$M = -0.4\text{ kN}\cdot\text{m}$ ($\therefore + \circlearrowleft$ ccw)

$\overset{+}{\leftarrow} \Sigma F_x = 0$

$N_c + 2\text{ kN} = 0$

$N_c = -2\text{ kN}$

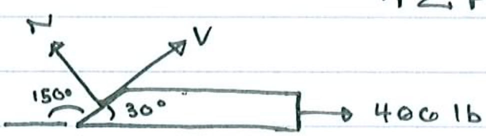
Problem 1-54 (From textbook)



$\overset{+}{\rightarrow} \sum F_x = 0$

$400 \cdot 2 \text{ lb} + 2V\cos 30° = 0$

$+\uparrow \sum F_y = 0$



$+\uparrow \sum F_y = 0$

$V\sin 30° + N\cos 30° = 0$

$\overset{+}{\rightarrow} \sum F_x = 0$

$400 \text{ lb} + V\cos 30° - N\sin 30° = 0$

(eventually..) ?

$N = (200 \text{ lb})$ (↖)

$V = (346.41 \text{ lb})$ (↙)

$\sigma_{AVG} = N/A = 200 \text{ lb} / [(1.5 \text{ in} \cdot 1 \text{ in})/\sin 30°] = 66.7 \text{ psi}$

$\tau_{AVG} = V/A = 346.41 \text{ lb} / [(1.5 \text{ in} \cdot 1 \text{ in})/\cos 30°] = 115 \text{ psi}$

Problem 1-64/65 (From textbook)

(to be clarified next lecture)