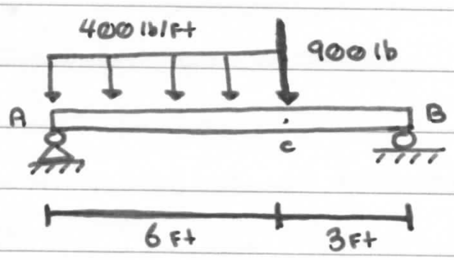
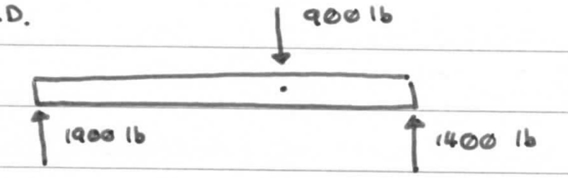
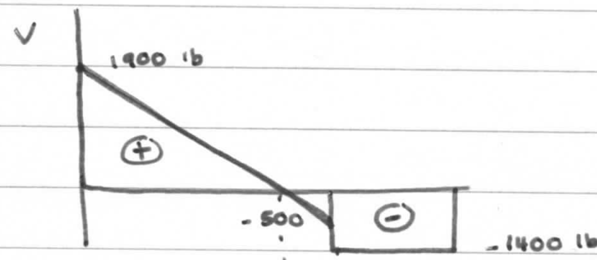
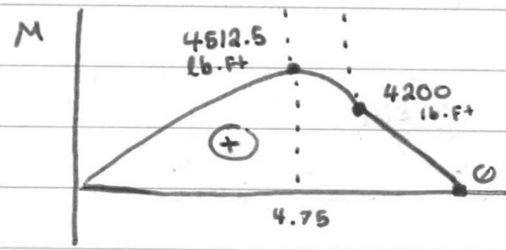



F.B.D.



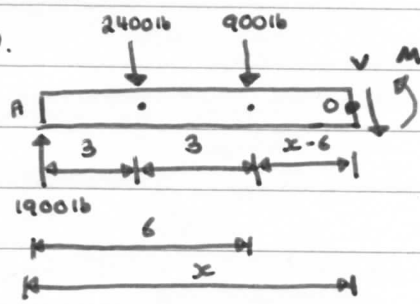




## AC

<u>CB</u> $6 \le x < 9$

F.B.D.




$\uparrow + \sum F_y = 0 \Rightarrow 1900 - 2400 - 900$

$V = -1400$

$\circlearrowleft + \sum M_o = 0$

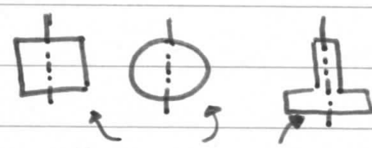
$M - 1900x + 2400(x-3) + 900(x-6) = 0$

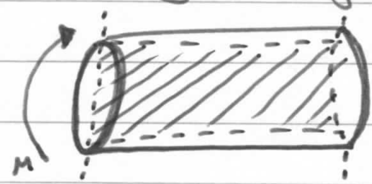
$M = 1400 \cdot (9 - x) = \begin{cases} 4200 \,;\; x = 6 \\ 0 \,;\; x = 9 \end{cases}$

## 6.3 Bending Deformation

Assumptions

- straight beam
- cross-sectional area with a symmetric axis.
- M is applied in the longitudinal plane passing through the symmetric axis.

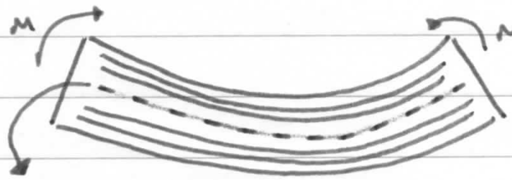




Observations:

- longitudinal (horizontal) lines become curved.
- vertical lines remain straight.
- cross sections $\longrightarrow$ Plane.
- material in the top portion $\rightarrow$ compressed

  bottom portion $\rightarrow$ tension



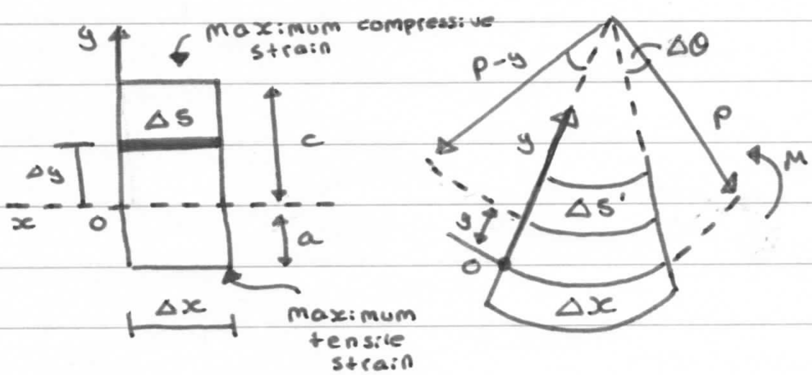

One Fiber must have no change in length.

- neutral plane

  ~ surface in which the longitudinal fibers remain unchanged in length

- neutral axis

  $\rightarrow$ cross sectional surface + neutral surface.




$\Delta S = \Delta x$

$\Delta S' = \Delta\theta \cdot (\rho - y)$

$\Delta x' = \Delta\theta \cdot \rho$

$\rho =$ Curvature radius of the N.S.

Strain: $\quad \varepsilon = \dfrac{\Delta S' - \Delta S}{\Delta S} = \dfrac{\Delta\theta \cdot (\rho - y) - \Delta\theta \cdot \rho}{\Delta\theta \cdot \rho}$

$$\varepsilon = \frac{\rho - y - \rho}{\rho} = -\frac{y}{\rho}$$

longitudinal normal strain varies linearly with y for the N.A.

Max compressive, $\varepsilon^{c}_{max} = -c/\rho$

max tensile, $\varepsilon^{T}_{max} = -\dfrac{-(a)}{\rho} = a/\rho$

$\varepsilon = -y/\rho$

$\varepsilon^{c}_{max} = -c/\rho$

$\dfrac{\varepsilon}{\varepsilon^{c}_{max}} = y/c$

$\varepsilon = y/c \cdot \varepsilon^{c}_{max}$

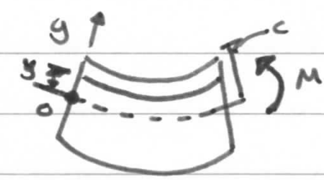
## 6.4 Bending stress

- material has linear elastic properties

  $\sigma = E\varepsilon$

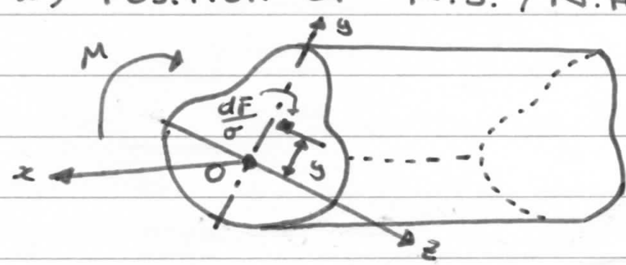
  @ an intermediate $y$

  $\sigma = E\, y/\rho$




$\boxed{\sigma_{max} = -\frac{c}{\rho} E}$ Max $\sigma_{max}$ @ a distance C, Furthest away from N.A.

## 2) Position of N.S. / N.A.




$\overset{+}{\rightarrow} \Sigma F_x = 0$

$dF = \sigma dA$

$\int dF = 0$

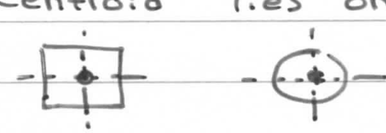
$\int \sigma dA = 0$

$\int y/L\, \sigma_{max}\, dA = 0$

$\dfrac{\sigma_{max}}{L} \cdot \int y dA = 0$

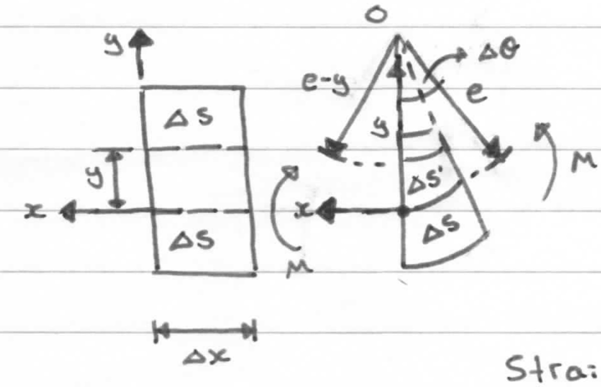
$\int y dA = 0$ $\qquad \bar{y} = \dfrac{\int y dA}{A} = 0$

Centroid lies on the N.A.



## 6.4 Bending Stress

### 1) Bending Deformation




$e$ = radius of curverture of the N.S.

Before deformation:

$\Delta S = \Delta x = \rho \cdot \Delta\theta$

After deformation:

$\Delta S' = (\rho - y) \cdot \Delta\theta$

Strain: $\varepsilon = \dfrac{\Delta S' - \Delta S}{\Delta S} \Rightarrow \dfrac{(\rho-y)\cancel{\Delta\theta} - \rho\Delta\theta}{\rho\Delta\theta}$

because $\Rightarrow \dfrac{\cancel{\Delta\theta}[(\cancel{\rho}-y)-(\cancel{\rho})]}{\rho\cancel{\Delta\theta}}$

then...

$\varepsilon = \dfrac{-y}{\rho}$

Strain varies linearly with y from the N.A. $\varepsilon_{max}$ occurs at a position furthest away from the N.A.

$\varepsilon_{max} = -y/\rho \big|_{y=c} \Rightarrow -c/\rho$

### 2) Bending Stress

$\sigma = E \cdot \varepsilon$

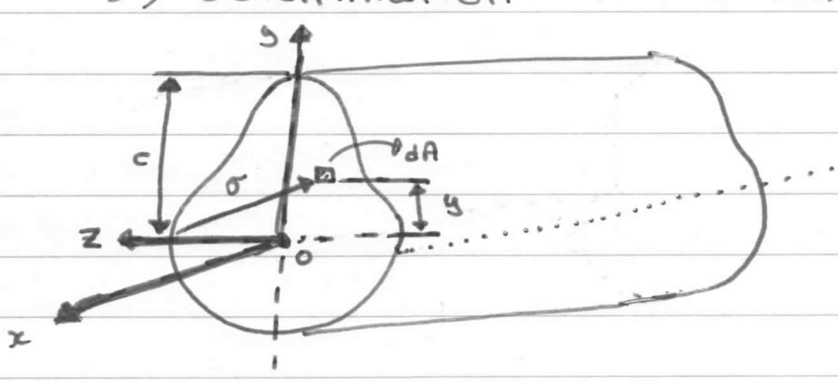
$\sigma = -y/\rho \cdot E$

$\sigma_{max} = E \cdot \varepsilon_{max} = -c/\rho \, E$

$\dfrac{\sigma}{\sigma_{max}} = \dfrac{-y/\rho \cdot E}{-c/\rho \cdot E} = \dfrac{y}{c}$

$\boxed{\sigma = \frac{y}{c}(\sigma_{max})}$

### 3) Determination of the N.A.




$\sigma = -y/\rho \, E$

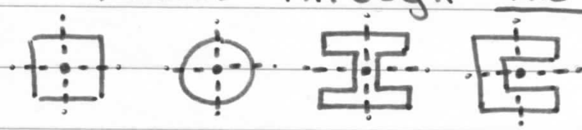
$dF = \sigma \cdot dA$

$\sigma_{max} = -c/\rho \, E$

Centroid

$$\bar{y} = \frac{\int y\,dA^{\Rightarrow 0}}{\int dA \Rightarrow 0} \Rightarrow 0$$

N.A. passes through the centroid.



## 4) Stress Formula

$\circlearrowleft + \sum M_z = 0$

$\int -y\,dy - M = 0$

$-\int y\,\sigma\,dA = M$

$\int y\, y/c\, \sigma_{max}\,dA = -M$

$$\frac{\sigma_{max}}{c} \underbrace{\int y^2\,dA}_{I \text{ (moment of inertia)}} = -M$$

$\sigma_{max}$ = max stress @ position furthest away from the NA ($z$ axis).

$C$ = perpendicular distance from the N.A. to a point farthest away from the N.A.

$I$ = moment of inertia about the N.A.

$$\frac{\sigma_{max}}{c} \cdot I = -M$$

$$\sigma_{max} = \frac{-MC}{I}$$

$$\therefore\ \sigma = -\frac{y}{c} \cdot \frac{MC}{I}$$

$$\sigma = -\frac{My}{I}$$

$\sigma$ = stress @ intermediate position $y$

Note: "−" if $M$ is positive

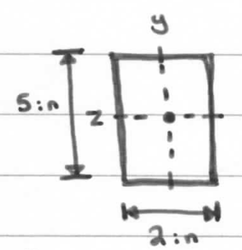
$y > 0$, above the N.A. $\sigma < 0$ (compression)

conversely,

$y < 0$ below the N.A. $\sigma > 0$ (tension)

**EXAMPLE 7-9**

SOLUTION:

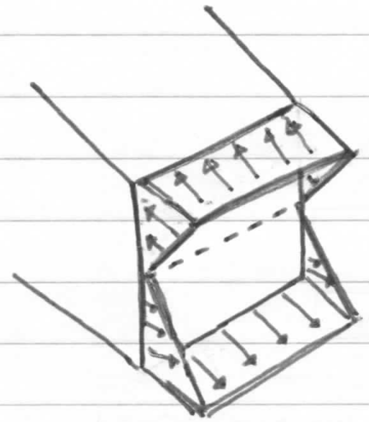



determine $\sigma_{max}$

$$\sigma_{max} = -\frac{MC}{I}$$

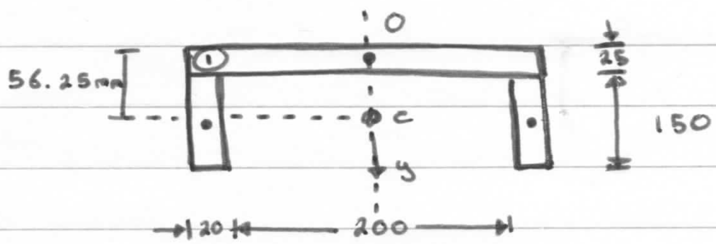
$$= \frac{-4512.5 \cdot 12 \cdot (5/2)}{(1/12)(2)(5)^3}$$

$= 6.498$ psi




| |
|---|
| Assignment Q (due thursday) |
| Q 6 - 50 |
| Q6 - 58 |

Q 6 - 52 (EXAMPLE 6-10)




Solution:

$$\bar{y} = \frac{(12.5)\cdot(25\cdot 240) + \ldots \ldots [(75\cdot 25)\cdot(150\cdot 20)]\cdot 2}{(25\cdot 240)+(2(150\cdot 20))}$$

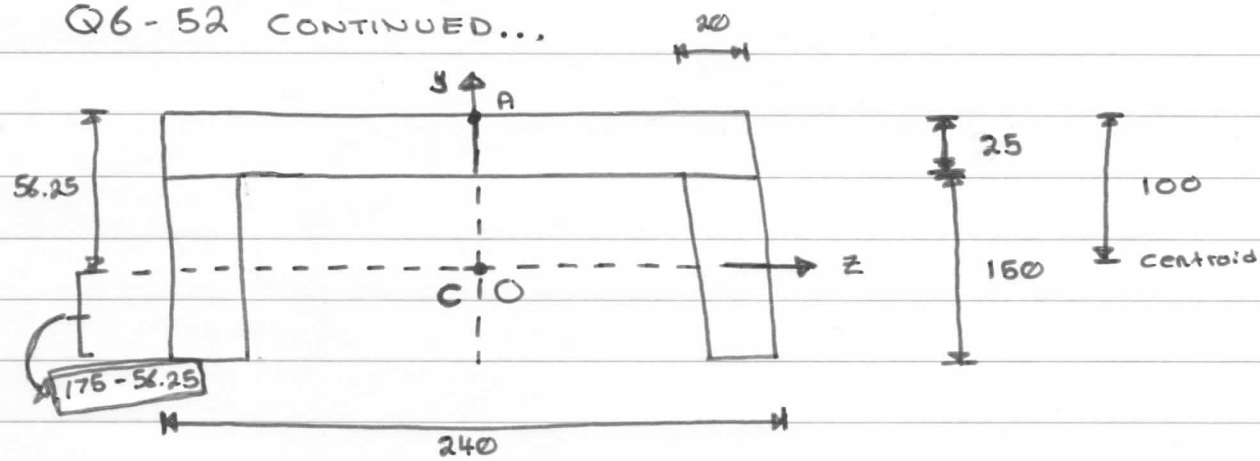
$= 56.25$ mm

$$I = \left[(1/12)\cdot 240 \cdot 25^3\right] + \left[(240\cdot 25)\ldots \ldots (56.25-12.5)^3\right]$$

$$+ \left[(1/12)\cdot(20)(150)^3 + (20\cdot 150)(100-56.25)^2\right]\cdot 2$$

$I = 34.531 \cdot 10^{-6}$ mm$^4$

Q6-52 CONTINUED...



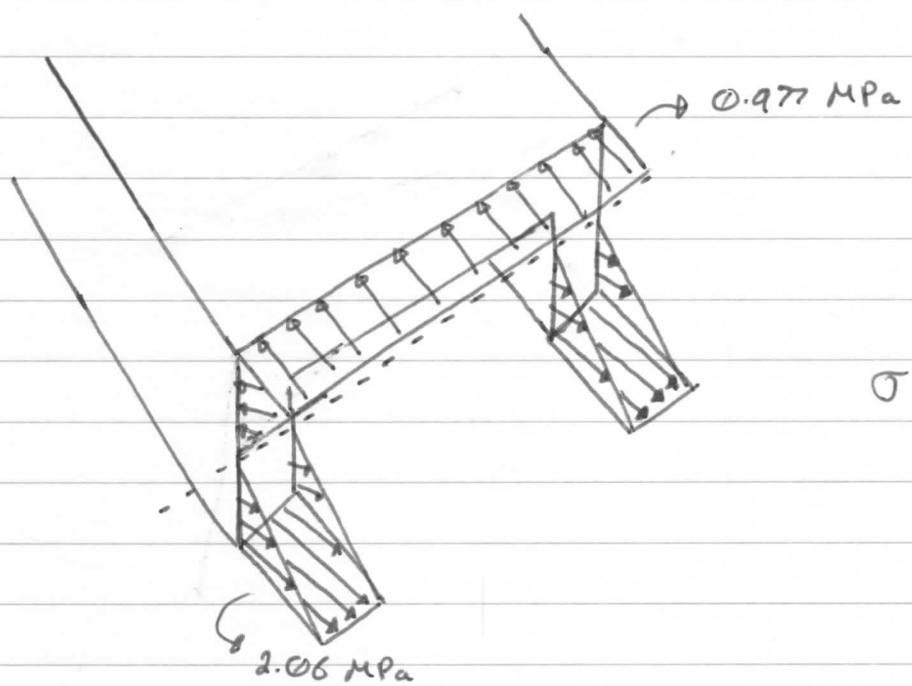

$$I = 34.531 \cdot 10^{6} \text{ mm}^4$$
$$= 34.531 \cdot 10^{-6} \text{ m}^4$$

$$\sigma_A = \frac{-My}{I} = -\frac{600 \cdot (0.05625)}{(34.531 \cdot 10^{-6})}$$

$$\sigma_A = -0.977 \cdot 10^{6} \text{ pa}$$

$$\sigma_D = \frac{-My}{I} = -\frac{600 \cdot (0.175 - 0.05625)}{(34.531 \cdot 10^{-6})}$$

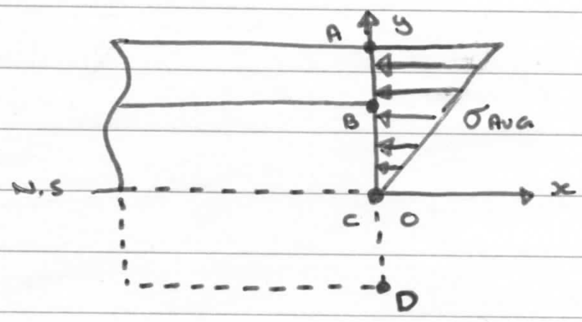
$$\sigma_D = 2.06 \cdot 10^{6} \text{ pa}$$




$$\sigma = -\frac{My}{I}$$

* Resultant Force the bending stress produces on the top board ?

Solution:




$F = \sigma \cdot A$

$\sigma_B = \frac{-My}{I}$

$\Rightarrow \frac{(-600\ N/m)(0.05625 - 0.025)m}{34.531 \cdot 10^{-6}\ m^4}$

$\Rightarrow -0.543 \cdot 10^6\ Pa$

$\sigma_{AVG} = \frac{\sigma_A + \sigma_B}{2}$

$F = \sigma_{AVG} \cdot A = \left( -(0.977 \cdot 10^6) \ldots etc. \right)$

$\Rightarrow 4.56 \cdot 10^3\ N\ (C)$