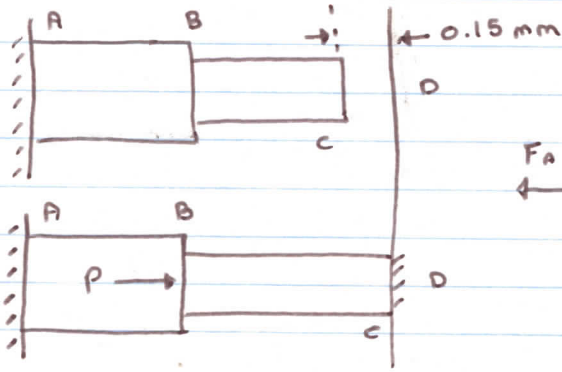
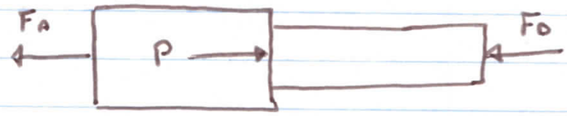
Nov. 8/16

EXAMPLE 4-46 (From textbook, maybe)

Solution

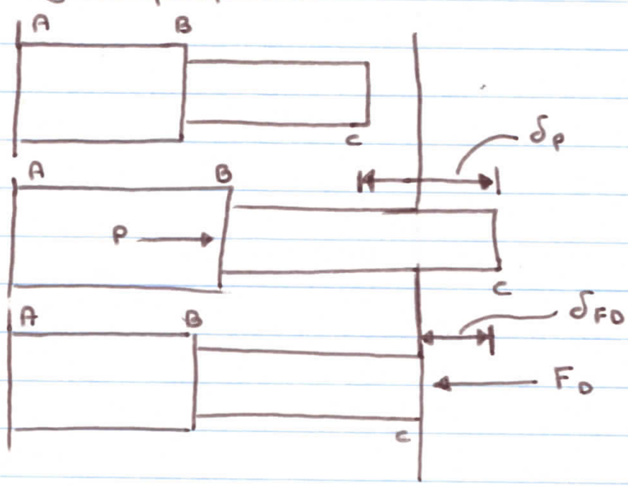




$\xrightarrow{+} \Sigma F_x = 0$

$-F_A - F_D + 200\ kN = 0$ (Eq. 1)

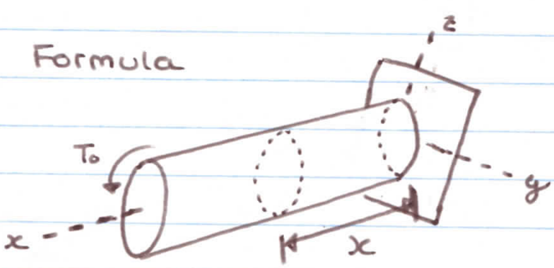
By superposition:

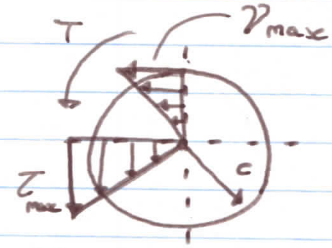


$\delta_P - \delta_{FD} = 0.15\ mm\ (\cdot 10^{-3}\ m)$

By: $\frac{\Sigma F l}{EA}$

5.2 Torsion Formula





$\tau = G\gamma$

$\tau = \frac{T \cdot \rho}{J}$

Torsional strain

Strain γ

$\gamma \sim$ linearly along any radial direction

T = internal resultant torque

ρ = intermediate distance from O

J = Polar moment of inertia

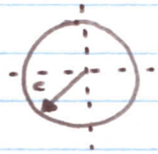
$\tau_{max} = \frac{TC}{J}$, when $\rho = C$ at the outer surface

C = radius

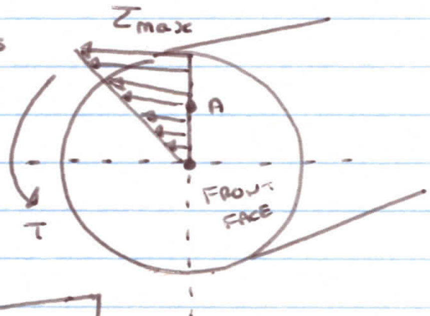
1) Solid shaft

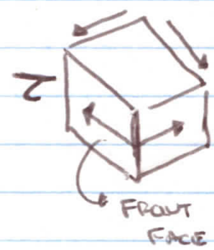
$J_o = \frac{1}{2}\pi c^4$

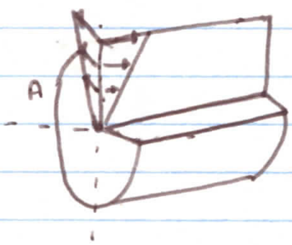
(m^4)



2) Shear stress

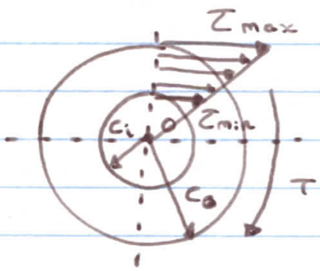






T → a radial shear stress, + shear stress along the adjacent axial plane.

3) Hollow shafts



$J = \frac{\pi}{2}(c_o^4 - c_i^4)$

$\tau_{max} = \frac{T c_o}{J}$

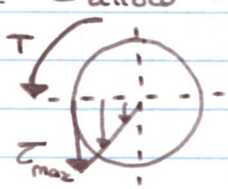
$\tau_{min} = \frac{T c_i}{J}$

Example 5-3 (from textbook)

Solution:

$\tau_{max} = \frac{T \cdot 0.05\,m}{\frac{\pi}{2} \cdot 0.05\,m^4} \leq \tau_{allow} = 100 \cdot 10^6\,pa$

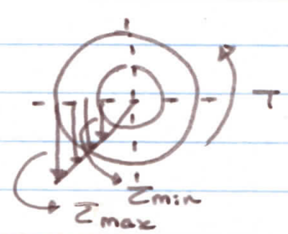
$T \leq 19.6 \times 10^3\ N \cdot m$



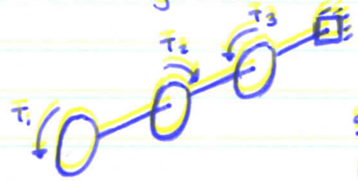
$\tau_{max} = \frac{T' \cdot 0.05\,m}{\pi/2\left(0.05^4 - \left(\frac{0.075}{2}\right)^4\right)} \leq \tau_{allow} = 100 \cdot 10^6\,pa$

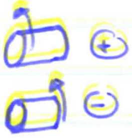
$T' \leq 13.4 \cdot 10^3\ N \cdot m$

$\tau_{min} = \frac{T' \cdot \left(\frac{0.075}{2}\right)}{J}$



4) Torque Diagram





Sign convention
R.H.R.
Fingers curl → T
if the thumb is directed outward = (+)
if the thumb is directed inwards = (-)

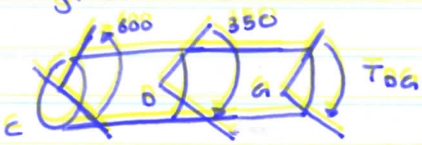
Example 5-5

Solution:

Segment CD

$T_{CD} = 600$ lb·ft

Segment DG

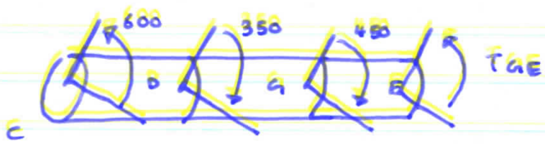


$+\circlearrowleft \Sigma M_x = 0$

$600 - 350 - T_{DG} = 0$

$T_{DG} = 250$ lb·ft

Segment GE



$+\circlearrowleft \Sigma M_x = 0$

$600 - 350 - 450 + T_{GE}$

$T_{GE} = 200$ lb·ft

$\tau_A = \dfrac{T_A \cdot c}{J}$

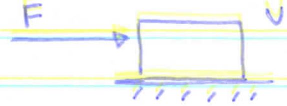
T_B ...

Assignment Question

5-12

5.3 - Power Transmission

Shaft - power, motion (Power = work/s)



work: N·m/s

Rotation

$P = T\omega$

T is torque

ω is angular speed (rad/sec)

If rotation $n \rightarrow$ r.p.m ≃

$\omega = \frac{2\pi n}{60\,s}$ rad/sec

If rotation $f \rightarrow$ Hz (cycle/sec)

$\omega = 2\pi f$ (rad/sec) r.p.s. (revolutions per second)

Units

P: SI: 1 W = 1 Nm/s

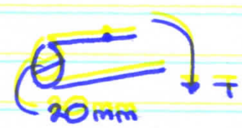
1 kW = 10^3 W

US: lb·ft/sec

1 hp = 550 lb·ft/sec 1 hp = 750 W

Solution: (5-32 on board)

$\omega = \frac{2\pi n}{60} = \frac{2\pi \cdot 150\ rpm}{60}$



$P = T\omega$

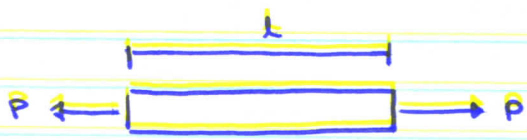
$85\,W = T\omega$

$T = 5.41\ Nm$

$\tau_A = \frac{T\rho}{J} \Rightarrow \frac{(5.41\ N\cdot m)\cdot\left(\frac{0.02}{2}\right)m}{\frac{\pi}{2}\left(\frac{0.02}{2}\right)^4}$

τ_{max}

$\Rightarrow 3.44 \cdot 10^6$



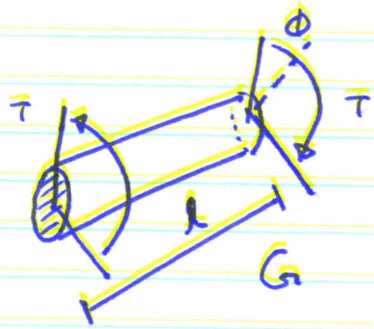
r: E

$\delta = \frac{F \cdot l}{EA}$

5.4 Angle of Twist

1) Constant T, J, G

$$\phi = \frac{TL}{GJ}$$



T = internal torque (RHR)
J = polar moment of inertia

2) Shaft with variable T, J, G

$$\phi_i = \frac{T_i l_i}{G_i J_i}$$

can be applied to each segement of the shaft with constant T, J, G.

The angle of twist:

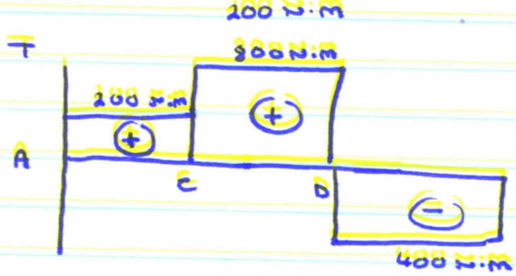
$$\phi = \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} \cdots$$

RHR: T⊕ → φ⊕
T⊖ → φ⊖

SOLUTION (5-49)

AC

T_{AC} = 200 N·m



CD

T_{CD} = 800 N·m

DB

$T_{DB} = 400\ N \cdot m$

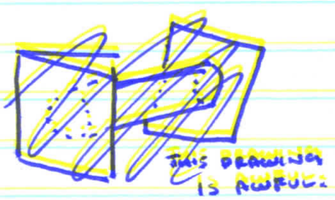
By RHR = ⊖ (Fingers along direction, thumb points inwards)

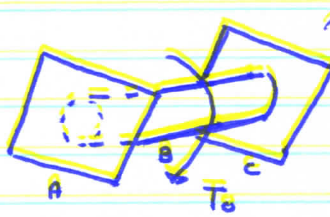
$$\phi_{A/B} = \phi_{A/E} + \phi_{E/D} + \phi_{D/B}$$

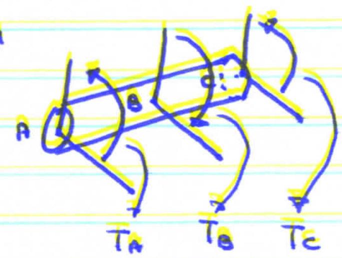
$$\phi_{A/B} = \frac{200\ Nm \cdot 0.4m}{75 \cdot 10^9\ Pa \cdot \pi/2 \cdot (0.02)^4} + \frac{800\ Nm \cdot 0.5m}{GJ} + \frac{(-400\ Nm)\cdot(0.6m)}{GJ}$$

$\Rightarrow 0.01273$ rad

$\Rightarrow 0.01273 \text{ rad} \cdot \frac{180°}{\pi} = 0.730°$







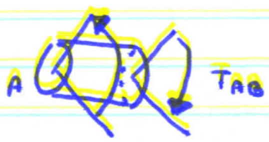
5.5 Statically Indeterminate Torque-Loaded Members

$\phi_{A/C} = 0$

$T_A - T_B + T_C = 0$

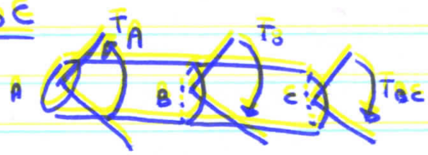
Deformation Compatability

AB



$T_{AB} = T_A$ ⊕

BC



$+\circlearrowleft \sum M_x = 0$

$T_A - T_D - T_{BC} = 0$

$T_{BC} = T_A - T_D$ ⊕

$\phi_{A/C} = 0$

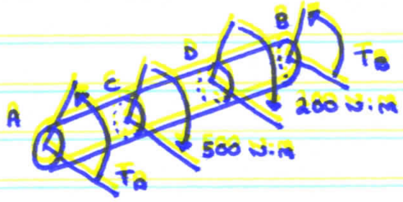
$\phi_{A/B} + \phi_{B/C} = 0$

$$\frac{T_A l_{AB}}{GJ} + \frac{(T_A - T_D)\, l_{BC}}{GJ} = 0$$

Ass. Ques.
5-12
5-34
5-53
5-82

Example 5-78

Solution:



$+\circlearrowleft \sum M_x = 0$

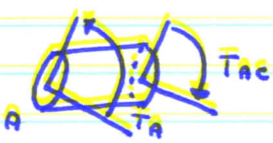
$T_A - 500 - 200 + T_B = 0$

$T_A + T_B = 700 \text{ N·m}$

Need 2nd equation (via deformation compatability)

$\phi_{A/B} = 0$

Segment AC



$T_{AC} = T_A$ (+)

Segment CD

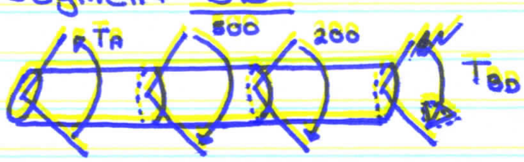
$+\circlearrowleft \sum M_x = 0$

$T_A - 500 - T_{CD}$

$T_{CD} = T_A - 500$ (+)

$\hookrightarrow = -85.71 \text{ Nm}$

Segment DB



$+\circlearrowleft \sum M_x = 0$

$T_A - 500 - 200 + T_{BD}$

~~$T_{BD} = 700 - T_A$ (+)~~

$T_{BD} = T_A - 700$ (+)

$\hookrightarrow = 285.71 \text{ Nm}$

$\phi_{A/B} = 0$

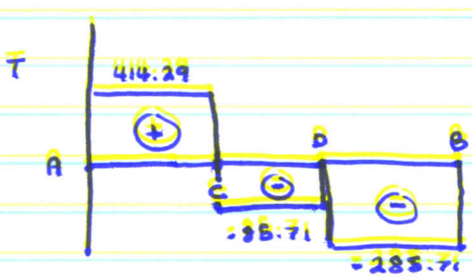
$\phi_{A/C} + \phi_{C/D} + \phi_{D/B} = 0$

$$\frac{T_A \cdot 1}{\pi/2 \cdot \left(\frac{0.06}{2}\right)^4 \cdot G} + \frac{(T_A - 500)\, 1.5}{JG} + \frac{(T_A - 700)\cdot 1}{JG} = 0$$

$\hookrightarrow T_A = 414.29 \text{ Nm}$

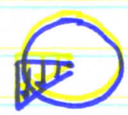
$(414.29 \text{ Nm}) + T_B = 700 \text{ Nm}$

$T_B = 285.71 \text{ Nm}$



$\tau_{max} \rightarrow AC$

absolute max

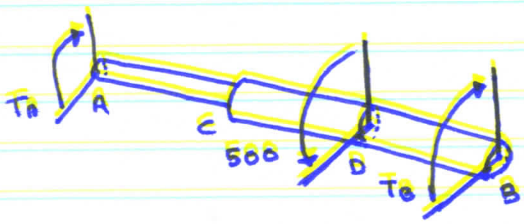


$$\tau_{max} = \frac{(414.29)\left(\frac{0.06}{2}\right)}{\left(\pi/2\right)\left(\frac{0.06}{2}\right)^4}$$

$\tau_{max} = 9.77 \cdot 10^6 \text{ Pa}$

5-79 (EXAMPLE)

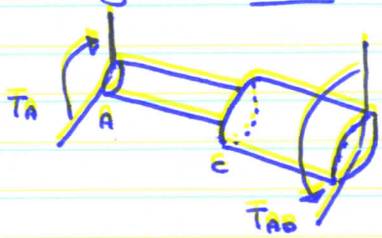
SOLUTION:



$\circlearrowleft + \Sigma M_x = 0$

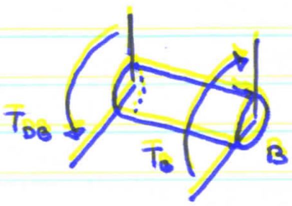
$T_A = 500 + T_B$ [Eq. 1]

Segment AD



$T_{AD} = T_A$ (+)

Segment DB



$T_{DB} = T_B$ (−)

Eq'n of D.C

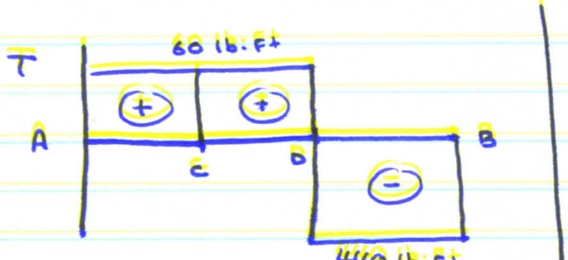
$\phi_{A/B} = 0$

$\phi_{A/C} + \phi_{C/D} + \phi_{D/B} = 0$

$$\frac{T_A\,(5\,in)}{\left(\frac{\pi}{2}\cdot\left(\frac{0.05}{2}\right)^4\cdot G\right)} + \frac{T_A\,(8\,in)}{\left(\frac{\pi}{2}\cdot\left(\frac{1}{2}\right)^4\cdot G\right)} + \frac{-T_B\,(12\,in)}{\left(\frac{\pi}{2}\cdot\left(\frac{1}{2}\right)^4\cdot G\right)} = 0$$

$\Rightarrow$ $T_A = 60$ lb ft

$T_B = 440$ lb ft



AC

$$\tau_{max}^{AC} = \frac{60\cdot 12\,lb\cdot in\ \cdot\ 0.5/2\ in}{\frac{\pi}{2}\left(0.5/2\right)^4}$$

$\Rightarrow 29.3\cdot 10^3$ psi → max

$$\tau_{max}^{DB} = \frac{440\cdot 12\cdot(1/2)}{}$$

$\Rightarrow 26.9\times 10^3$ psi