OCT. 24/16

5 - Frames and Machines

Structures of interconnected members that do not satisfy the definition of a Truss are designated as Frames if they are designed to remain stationary and support loads, and machines if they are designed to move and apply loads.

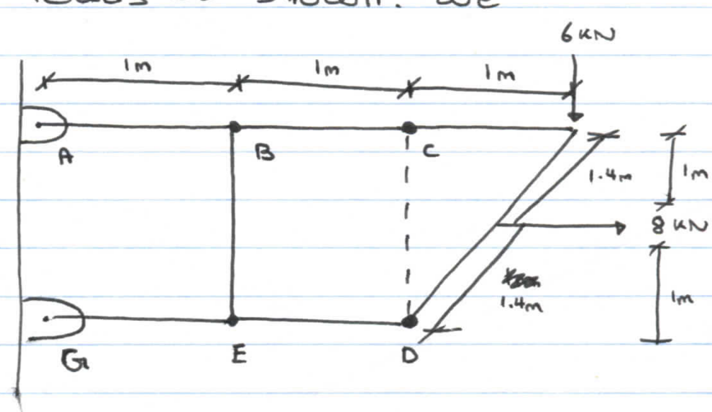
- When analyzing a Frame or a machine, instead of cutting members to obtain Free-body diagrams we isolate entire members, or in some cases Combinations of members, from the structure.

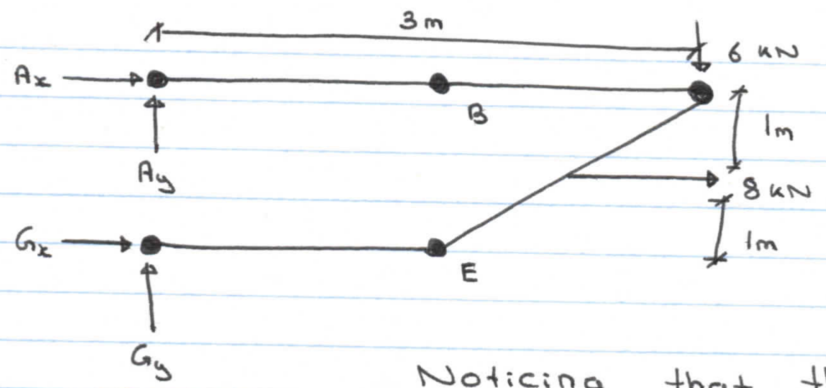
- To begin to analyze a Frame or machine, you draw a F.B.D. of the entire structure and determine as many of the reactions as possible. You then draw F.B.D. of individual members, or selected groups of members, and apply the equilibrium equations to determine the Forces and couples on them.

- If a load acts at a joint, you can place it on any one of the members attached at the joint when you draw the F.B.D. of the individual members, just make sure you don't place it at more than one member.

5.1 - Analyzing the entire structure

Consider the Frame and loads as shown. We draw the F.B.D. of the entire Frame. It is statistically indeterminate. We have Four unknown reactions and can only write three independent equilibrium equations.



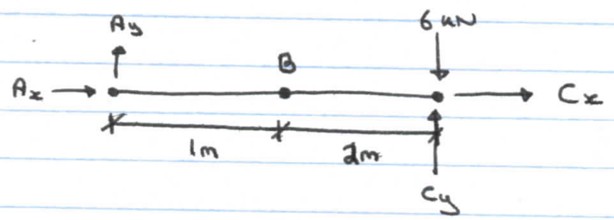


Noticing that the lines of action of three of the unknown forces intersect at A, we take the sum of moments about A.

$\Sigma M_A = 2G_x + (1)(8) - (3)(6) = 0; \quad G_x = 5\text{ kN}$

Then, $\Sigma F_x = A_x + G_x + 8 = 0; \quad A_x = -13\text{ kN}$

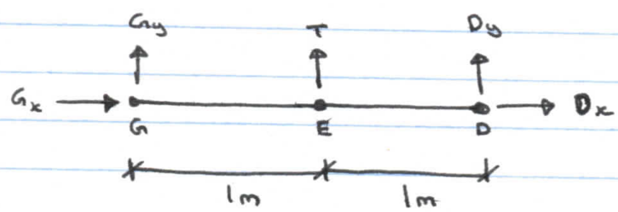
Although we cannot determine A_y or G_y from the F.B.D. we can do so by analyzing the individual members.



Consider member ABC, because we know A_x we can determine C_x from:

$\Sigma F_x = 0 = A_x + C_x;$

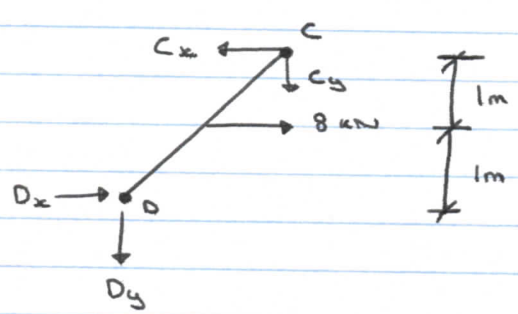
$C_x = A_x = 13\text{ kN}$



Now consider member GED:

$\Sigma F_x = 0 = G_x + D_x = 0;$

$D_x = -G_x \Rightarrow -5\text{ kN}$



Consider the F.B.D. of member CD, we determine C_y by summing moments about D.

$\Sigma M_D = (2)C_x - (1)C_y - (1)(8) = 0; \quad C_y = (2)(13) - 8 = 18\text{ kN}$

$\Sigma F_y = 0 = -C_y - D_y = 0; \quad D_y = -C_y = -18\text{ kN}$

Returning to the F.B.D. of members ABC and GED we determine A_y and G_y.

Summing moments about B of member ABC;

$\Sigma M_B = -(1)(A_y) + 4(C_y) - (2)(6) = 0$; $A_y = 24$ kN

Summing moments about point E of member GED.

$\Sigma M_E = (1)D_y - (1)G_y = 0$; $G_y = D_y = -18$ kN

Finally; From the final F.B.D. of GED:

$\Sigma F_y = D_y + G_y + T = 0$; $T = -D_y - G_y = 36$ kN

Oct. 26 /16

Centroids and Centers of Mass

1 - Centroids

The average position of any set of quantities with which we can associate positions such as $c_i(x_i, y_i)$ can be expressed as:

$$\bar{x} = \frac{\sum_i x_i c_i}{\sum_i c_i} \quad ; \quad \bar{y} = \frac{\sum_i y_i c_i}{\sum_i c_i}$$

The average position obtained from these equations is called a "weighted average position" or centroid.

1-1 Centroids of Area

Consider an arbitrary area A in the x-y plane. Let us divide into parts $A_1, A_2, \ldots A_n$ and denote their positions by $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$.

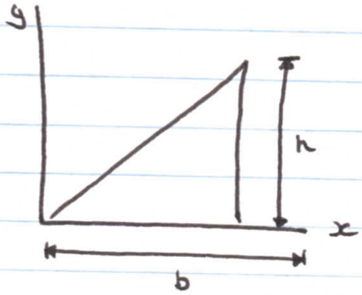
Then to
$$\bar{x} = \frac{\sum_i x_i A_i}{\sum_i A_i} \quad , \quad \bar{y} = \frac{\sum_i y_i A_i}{\sum_i A_i}$$

The question is how do we determine the exact position of these areas? We could reduce the uncertainty in their positions by dividing A into smaller parts, but we would still obtain approximate values for x and y.

To determine the exact location of the centroids we must take the limit as the sizes of the parts approach zero. We do this by using integrals:

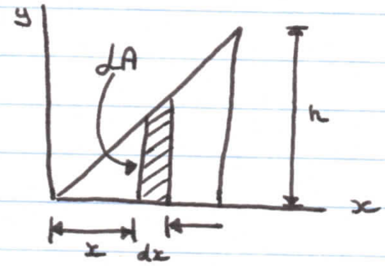
$$\bar{x} = \frac{\int_A x \, dA}{\int_A dA} \quad ; \quad \bar{y} = \frac{\int_A y \, dA}{\int_A dA}$$

Determine the centroid of the triangular area shown.



we will determine the coordinates of the centroid by using element of area dA

In the form of a strip of width dx:



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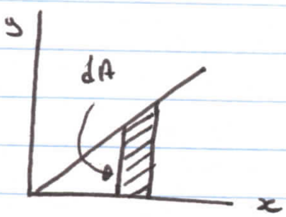
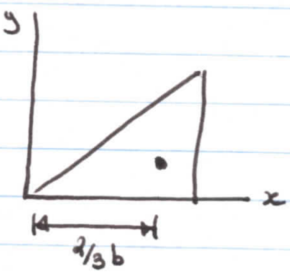
Let dA be the vertical strip.
The height of the strip is $\left(\frac{h}{b}\right)x$ and $dA = \left(\frac{h}{b}x\right)dx$





and $$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^b x\left(\frac{h}{b}x\right)dx}{\int_0^b \left(\frac{h}{b}x\right)dx} = \frac{\frac{h}{b}\left[\frac{x^3}{3}\right]_0^b}{\frac{h}{b}\left[\frac{x^3}{2}\right]_0^b} = \frac{2}{3}b$$

to determine $\bar{y}$ we let y be the coordinate of the midpoint of the strip:

$$\bar{y} = \frac{\int_A y\,dA}{\int_A dA} = \frac{\int_0^b y\left(\frac{h}{b}y\right)dx}{\int_0^b \left(\frac{h}{b}y\right)dx} = \frac{\frac{1}{2}\left(\frac{h}{b}\right)^2\left[\frac{x}{3}\right]_0^b}{\left(\frac{h}{b}\right)\left[x^2\right]_0^b} = \frac{1}{3}h$$

The centroid is located at $C\left(\frac{2}{3}b, \frac{1}{3}h\right)$

1.2 Centroid of Volumes

Consider a volume V, and let dV be a differential element of V with coordinates x, y, and z. The coordinates of the centroid of V are:

$$\bar{x} = \frac{\int_V x\,dV}{\int_V dV} \;;\quad \bar{y} \frac{\int_V y\,dV}{\int_V dV} \;;\quad \bar{z} \frac{\int_V z\,dV}{\int_V dV}$$

If a volume has the form of a plate with uniform thickness and cross-sectional area A, its centroid coincides with the centroid of A, and lies at the midpoint between the two faces.

1.3 Centroids of Lines

The coordinates of the centroid of a line L are:

$$\bar{x} = \frac{\int_L x\,dL}{\int_L dL} \;;\quad \bar{y} = \frac{\int_L y\,dL}{\int_L dL} \;;\quad \bar{z} = \frac{\int_L z\,dL}{\int_L dL}$$

First Moments of Areas and Lines

The integral $\int x\,dA$, in the determination of the centroid of area A, is known as the first moment of the area A with respect to the y-axis and is denoted by Q_y. Similarly, the integral $\int y\,dA$ defines the first moment of A with respect to the x-axis, and is denoted by Q_x.

$$Q_y = \int x\,dA \;;\quad Q_x = \int y\,dA$$

and $Q_y = \bar{x}A \;;\quad Q_x = \bar{y}A$

2 - Centers of Mass

The centre of mass of an object is the centroid of its mass.

$$\bar{x} = \frac{\int_m x\,dm}{\int_m dm} \;;\quad \bar{y} \frac{\int_m y\,dm}{\int_m dm} \;;\quad \bar{z} = \frac{\int_m z\,dm}{\int_m dm}$$

2.1 - Density

The mass density ρ of an object is defined such that the mass of a differential element of its volume is $dm = \rho\, dv$ kg/m³ in SI
slug/ft³ in US unit

and $m = \int_m dm = \int_m \rho\, dV$

An object whose mass density is uniform throughout its volume is said to be homogeneous. In this case, the total mass equals the product of the mass density and the volume.

$$m = \rho \int_v dv = \rho V \quad \text{(homogenous object)}$$

The weight density ($\gamma = g\rho$) in N/m³ in SI
lb/ft³ in US unit

The weight of an element of volume dV of an object is $dw = \gamma\, dV$

its total weight of a homogeneous solution is
$w = \gamma V$

By substituting dm by $\rho\, dv$ in the expressions of the coordinates of the center of mass, we get

$$\bar{x} = \frac{\int_v \rho x\, dv}{\int_v dv} \; ; \; \bar{y} = \frac{\int_v \rho y\, dv}{\int_v dv} \; ; \; \bar{z} = \frac{\int_v \rho z\, dv}{\int_v dv}$$

- The centroid of mass of a homogeneous object coincides with the centroid of its volume
- The center of mass of a homogeneous plate of uniform thickness coincides with the thickness of its cross-sectional area
- The centroid of area of a homogenous slender bar of a uniform cross-sectional area coincides with approximately the centroid of the axis of the bar.