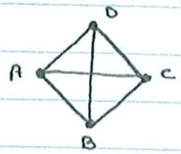
Oct. 17/16

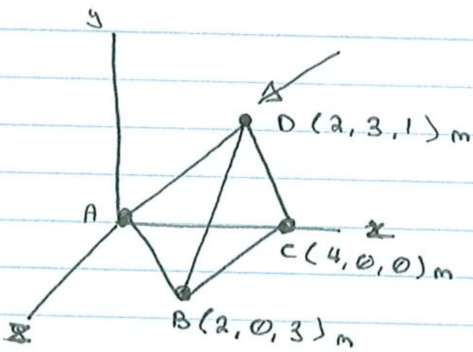
4 - Space Trusses

We can form a simple three-dimensional structure by connecting six bars at their ends to obtain a tetrahedron.



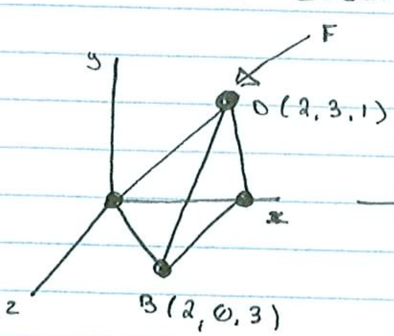
By adding members, we can obtain more elaborate structures. Three-dimensional structures such as these are called space trusses if they have joints that do not exert couples on the members and they are loaded and supported at the joints.

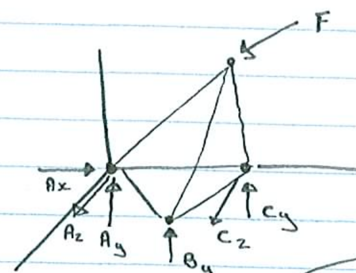
→ Let's consider the space Truss ABCD. Suppose that the load is $\vec{F} = 2\vec{i} - 6\vec{j} - \vec{k}\ (kN)$



Joints ABC rest on the smooth floor. Joint A is supported by the corner where the smooth walls meet. We can apply the method of joints to this truss.

First we determine the reactions exerted by the supports.





$$\therefore \xrightarrow{+} \Sigma F_x = A_x - 2 = 0$$

$$+\uparrow \Sigma F_y = A_y + B_y + C_y - 6 = 0$$

$$\swarrow^{+} \Sigma F_z = A_z + C_z - 1 = 0$$

(negative)

$$\Sigma M_{(\text{point } A)} = \bar{r}_{AB} \times B_y\vec{j} + \bar{r}_{AC} \times (\bar{C}_y\vec{j} + \bar{C}_z\vec{k})$$

$$\ldots + \bar{r}_{AD} \times F$$

$$\bar{r}_{AB} = 2\vec{i} + 3\vec{k}$$

$$r_{AC} = 4\vec{i}$$

$$r_{AD} = 2\vec{i} + 3\vec{j} + \vec{k}$$

$$\Sigma M_A = \begin{vmatrix} i & j & k \\ 2 & 0 & 3 \\ 0 & B_y & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 0 & 0 \\ 0 & C_y & C_z \end{vmatrix} + \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ -2 & -6 & -1 \end{vmatrix}$$

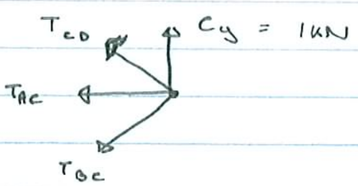
$= (-(3B_y + 3)\vec{i} + (-4C_y)\vec{j} + (2B_y + 4C_y - 6)\vec{k} = 0$

Using these equations to obtain:

$A_x = 2\,kN$; $A_y = 4\,kN$; $A_z = 1\,kN$; $B_y = 1\,kN$

$C_y = 1\,kN$; $C_z = 0$

→ In this example, we determine the axial forces in members AC, BC, and CD from the free-body diagram of joint C.



To write the equilibrium equations for the joint, we must express the three ~~forces~~ (axial forces) in terms of their components. The force exerted by the axial force T_{AC} on joint C is $-T_{AC}\vec{i}$ (T_{AC} is along the x-axis)

The position vector from C to B is:

$\vec{r}_{CB} = (2-4)i + (0-0)j + (3-0)\vec{k} = -2i + 3k \ (m)$

Dividing the vector by it's magnitude to obtain a unit vector that points from C to B.

$\vec{e}_{CB} = \dfrac{\vec{r}_{CB}}{|r_{CB}|} = -0.555i + 0.832k$

The force exerted by the axial force T_{BC} on joint C expressed as a vector is $T_{BC}\vec{e}_{CB} = T_{BC}(-0.555\vec{i} + 0.832\vec{k})$

The position vector from C to D is $\vec{r}_{CD} = (2-4)\vec{i} + (3-0)\vec{j} + \ldots$

rewritten ... $(1-0)\vec{k}$

$= -2\vec{i} + 3\vec{j} + \vec{k}$

$\vec{r}_{CD} = (2-4)\vec{i} + (3-0)\vec{j} + (1-0)\vec{k} = -2i + 3j + k$

$\vec{e}_{CD} = \dfrac{\vec{r}_{CD}}{|r_{CD}|} = 0.535\vec{i} + 0.807\vec{j} + 0.267\vec{k}$

Setting the sum of the forces on the joint to zero:

$-T_{AC}\vec{i} + T_{BC}(-0.555\vec{i} + 0.832\vec{k}) + T_{CD}(-0.535\vec{i} + 0.807\vec{j} + 0.267\vec{k}) = 0$

which gives us the following 3 equilibrium equations.

$$\Sigma F_x = -T_{AC} - 0.55\, T_{BC} - 0.535\, T_{CD} = 0$$
$$\Sigma F_y = 0.802\, T_{CD} + 1 = 0$$
$$\Sigma F_z = 0.832\, T_{BC} + 0.267\, T_{CD} = 0$$

Solving these equations to find:

$T_{AC} = 0.444$ kN (Tension) ; $T_{BC} = 0.401$ kN (Tension) ; $T_{CD} = -1.247$ kN (comp.)

As we have seen, we obtain three equilibrium equations from the FBD of a joint, in three dimensions, so we should choose joints to analyze that are subjected to known forces and are no more than three unknown forces.

* Go through last tutorial, understand those two problems.
↳ Re: Midterm.

- Equivilent system example on board.

Are the sums of the forces equal?

$$(\Sigma F)_1 = 20\vec{i} + 10\vec{j} - 10j = 20i \text{ lb}$$
$$(\Sigma F)_2 = 20\vec{i} + 15\vec{i} - 15\vec{i} = 20i \text{ lb}$$

Are the sums of the moments about one arbitrary point equal?

$$(\Sigma M_o)_1 = (-8 \text{ ft})(10 \text{ lb}) - (20 \text{ ft} \cdot \text{lb}) = -100 \text{ ft} \cdot \text{lb}$$
$$(\Sigma M_o)_2 = (5 \text{ ft})(15 \text{ lb}) - (25 \text{ ft} \cdot \text{lb}) = -100 \text{ ft} \cdot \text{lb}.$$