Sept. 26/16

2 - The moment Vector

The moment of Force "$\vec{F}$" about a point "O" is the vector $\vec{M}_2 = \vec{r} \times \vec{F}$.

Where $\vec{r}$ is a position vector from "O" to any point on the line of action of "$\vec{F}$"

2-1. Magnitude of the Moment

The magnitude of the $\vec{M}_o$ is:

$|\vec{M}_o| = |\vec{r}||\vec{F}| \sin\theta$

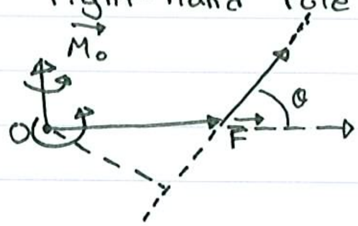
Where θ is the angle between vectors $\vec{r}$ and $\vec{F}$ when they are placed tail to tail.

Note that $|\vec{r}| \sin\theta$ is the perpendicular distance from "O" to the line of action of $\vec{F}$. There fore,

$|M_o| = D|\vec{F}|$

2.2 - Sense of the Moment

$\vec{M}_o$ is perpendicular to the plane containing "O" and "$\vec{F}$." It's direction is given by the right-hand rule



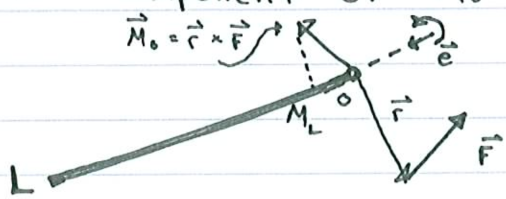
3 - Moment of a Force about a Line

The measure of the tendency of a force to cause rotation about a line, or axis, is called the moment of the force about the line.

3.1 - Definition of a Line

Consider a line L and a force $\vec{F}$. Let $\vec{M_o}$ be the moment of $\vec{F}$ about an arbitrary point "O" on L.

The moment of $\vec{F}$ about L is $\vec{M_L}$ which is the component of $\vec{M_o}$ parallel to L.

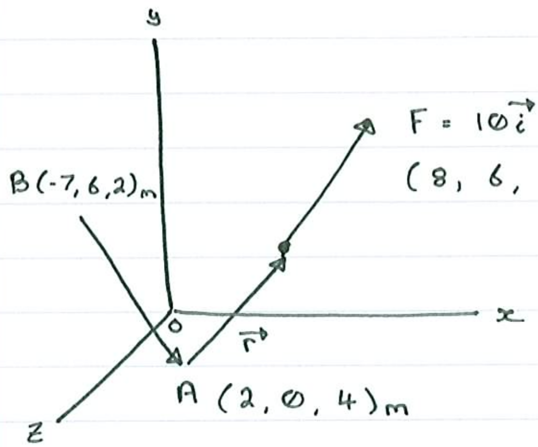


The magnitude of the moment of $\vec{F}$ about L is $|\vec{M_L}|$ and its direction is given by the right-hand rule. In terms of a unit vector $\vec{e}$ along L, $\vec{M_L}$ is given by

$$\vec{M_L} = (\vec{e} \cdot \vec{M_o})\,\vec{e}$$
$$\vec{M_L} = [\vec{e} \cdot (\vec{r} \times \vec{F})]\,\vec{e}$$

Note: the unit vector $\vec{e}$ can point in either direction. The value of the scalar $\vec{e} \cdot \vec{M_o} = \vec{e} \cdot (\vec{r} \times \vec{F})$ tells you both the magnitude and direction of $\vec{M_L}$.
The absolute value of $\vec{e} \cdot \vec{M_o}$ is the magnitude of $\vec{M_L}$. If $\vec{e} \cdot \vec{M_o}$ is positive, $\vec{M_L}$ points in the direction of $\vec{e}$ and if it is negative, $\vec{M_L}$ points in the opposite direction.

Let's determine the moment of a Force about an arbitrary line L. The First step is to choose a point on the line. If we chose the point A, the vector $\vec{r}$ from A to the point of application of $\vec{F}$ is $\vec{r}$.



$$\vec{r} = (8-2)\vec{i} + (6-0)\vec{j} + (4-4)\vec{k} = 6\vec{i} + 6\vec{j} \text{ (m)}$$

The moment of $\vec{F}$ about A is:

$$M_A = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 6 & 0 \\ 10 & 60 & -20 \end{vmatrix} = 120\vec{i} + 120\vec{j} + 300\vec{k} \text{ N·m}$$

The next step is to determine a unit vector along L. The vector from A to B is $(-7-2)\vec{i} + (6-0)\vec{j} + (2-4)\vec{k}$

$= -9\vec{i} + 6\vec{j} - 2\vec{k}$ (m)

The unit vector $\vec{e}_{AB}$ that points from point A to B is

$$\vec{e}_{AB} = \frac{\vec{L}_{AB}}{|\vec{L}_{AB}|} = \frac{-9}{11}\vec{i} + \frac{6}{11}\vec{j} - \frac{2}{11}\vec{k}$$

root square of Magnitude comp.

The moment of $\vec{F}$ about L is:

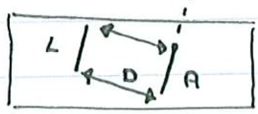
$\vec{M_L} = (\vec{e}_{AB} \cdot \vec{M_A})\, \vec{e}_{AB}$

$[(-9/11)(-120)\ \ (6/11)(120)\ \ (-2/11)(300)]$ ... and he erased everything.

3.2- Special Cases

1- When the line of action of $\vec{F}$ is perpendicular to a plane containing L, the magnitude of the moment of $\vec{F}$ about L is equal to the product of the magnitude of $\vec{F}$ and the perpendicular distance D from L to the point where the line of action intersects the plane.

$\vec{F}$ $|M_L| = |\vec{F}|D$



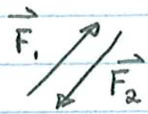
2 - When the line of action of $\vec{F}$ is parallel to L, the moment of $\vec{F}$ about L is zero.

$\vec{M_L} = 0$ since $\vec{M_o} = \vec{r} \times \vec{F}$ is perpendicular to F. $\vec{M_o}$ is perpendicular to L and the vector component of M_o parallel to L is zero.

3 - When the line of action of $\vec{F}$ intersects L, the moment of $\vec{F}$ about L is zero. Since we can choose any point on L to evaluate $\vec{M_o}$, we can use the point where the line of action of $\vec{F}$ intersects L. The moment M_o about that point is zero, so it's vector component parallel to L is zero.

4. Couples

1 Two forces that have equal magnitudes, opposite directions and different lines of action are called a couple. A couple tends to cause rotation and it has ~~a~~ the remarkable property that the moment it exists is the same about any point.



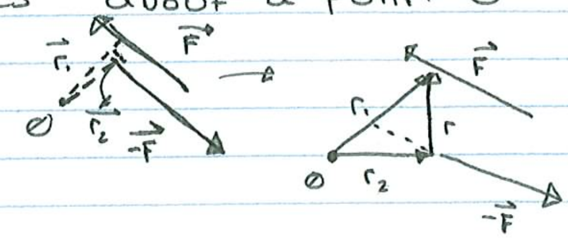
The moment of a couple is simply the sum of the moments of the forces about a point O

$\vec{M} = \vec{r}_1 + \vec{r}_2 \times (-\vec{F})$

$= (\vec{r}_1 - \vec{r}_2) \times (\vec{F})$

The vector $\vec{r}_1 - \vec{r}_2 = \vec{r}$

and $\vec{M} = \vec{r} \times \vec{F}$



Since $\vec{r}$ does not depend on the position of "O", the moment $\vec{M}$ is the same for any point "O".

$\vec{M} = \vec{r} \times \vec{F}$ is the moment of $\vec{F}$ about a point on the line of action of the force $-\vec{F}$. The magnitude of this moment is $|\vec{M}| = D|\vec{F}|$, where D is the perpendicular distance between the lines of action of the two forces. It's direction is given by the right-hand rule.

5. Equivalent Systems

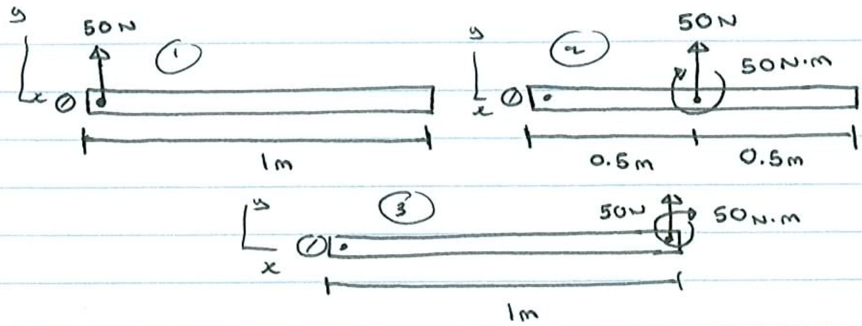
We define two systems 1 and 2 to be equal if the sums of the forces are equal,

$(\Sigma(\vec{F}_1) = \Sigma(\vec{F})_2)$

and the sums of the moments about a point "O" are equal.

$(\Sigma_1(\vec{M}_0)_1 = \Sigma_2(\vec{M}_0)_2)$

Example: Three systems of forces and moments act on the beam. Are they equivalent?



1st + 3rd = yes

2nd = No.

Are the sums of the forces equal?

$$(\Sigma \vec{F})_1 = 50\vec{j}\,N$$
$$(\Sigma \vec{F})_2 = 50\vec{j}\,N$$
$$(\Sigma \vec{F})_3 = 50\vec{j}\,N$$

Are the sums of moments equal?

$$(\Sigma M_o)_1 = \emptyset$$
$$(\Sigma M_o)_2 = (50N)(\emptyset.5\,m) - (50\,N.m) = -25\,N.m$$
$$(\Sigma M_o)_3 = (50N)(1m) - (50N.m) = \emptyset$$

$\therefore$ System 1 and 3 are equivalent.

6 - Representing Systems by Equivalent Systems

Let's consider an arbitrary system of forces and moments and a point "∅", system 1. We can represent this system by one consisting of a simple force acting at "∅" and a simple couple such that $\vec{F} = (\Sigma \vec{F})_1$; $\vec{M} = (\Sigma M_o)_1$

6.1 - Representing a Force by a Force and a couple:

You can represent a Force $\vec{F}_p$ acting at point "P" a Force $\vec{F}$ acting at a different point "O" and a couple $\vec{M}$ such that: $\vec{F} = \vec{F}_p$ and $M = \vec{M}_o = \vec{r} \times \vec{F}$ where $\vec{M}_o$ is the moment of $\vec{F}_p$ about "O."

6.2 - Concurrent Forces Represented by a Force:
A system of Forces whose lines of action intersect at a point "O" can be represented by a single Force whose line of action intersects "O" such that

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots$$

6.3 - Parallel Forces Represented by a Force
A system of parallel forces $\vec{F_i}$ can be represented by a single Force $\vec{F}$ such that

$$\vec{F} := \sum_i \vec{F_i}$$

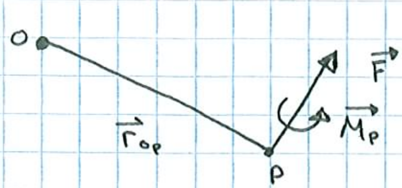
and $\vec{M_o} = (\sum \vec{M_o})_i$

6.4 - Representing a System by a Wrench
A Force and a couple M_P that is parallel to $\vec{F}$ is called a wrench; it is the simplest system that can be equivalent to an arbitrary system of Forces and moments.

If the system is more complicated than a single Force and a single couple; begin by choosing a convenient point "O" and representing the system by a Force $\vec{F}$ acting at "O" and a couple "M."

representing this system by a wrench requires two steps.

1. Determine the components of $\vec{M}$ parallel and normal to $\vec{F}$
2. The wrench consists of the Force $\vec{F}$ acting at point "P" and the parallel component $\vec{M_P}$ such that the moment of $\vec{F}$ about O equals the normal component $\vec{M_a}$



$$\vec{M}_a = \vec{r}_{op} \times \vec{F}$$

Objects in Equilibrium

1 - The equilibrium equations

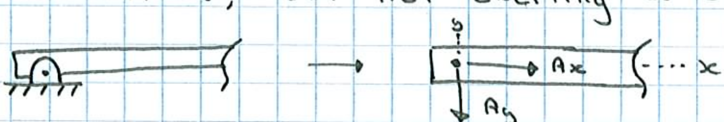
An object acted upon by a system of forces and moments is in equilibrium if and only if.

1. The sum of the forces is zero.
   $\Sigma \vec{F} = 0$
2. The sum of the moments is zero (about any point "O")
   $\Sigma \vec{M}_{(\text{any point})} = 0$

2 - Two - dimensional applications

2.1 - The pin support

The pin support is used to represent any real ~~system~~ support capable of exerting a force in any direction, but not exerting a couple.



← typical bridge support.