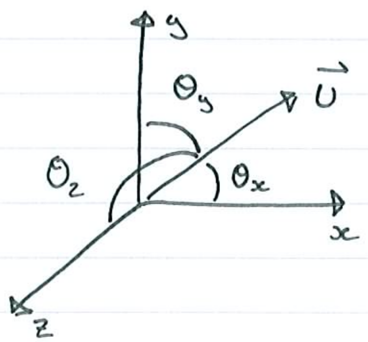
Sept. 14/16

Any vector $\vec{U}$ can be expressed in terms of its Cartesian Coordinate as:

$$\vec{U} = U_x \vec{i} + U_y \vec{j} + U_z \vec{k}$$

$$|\vec{U}| = U = \sqrt{U_x^2 + U_y^2 + U_z^2}$$

2.9 - Direction cosines



The components of the Vector $\vec{U}$ are given in terms of the angles θ_x, θ_y, θ_z by:

$$U_x = U\cos\theta_x$$
$$U_y = U\cos\theta_y$$
$$U_z = U\cos\theta_z$$

but $\vec{U} = U\vec{e}$

where $\vec{e}$ is a unit vector in the direction of $\vec{U}$.

In terms of components this can be written as:

$$U_x \vec{i} + U_y \vec{j} + U_z \vec{k} = U(e_x \vec{i} + e_y \vec{j} + e_z \vec{k})$$

or:

$$U_x = Ue_x$$
$$U_y = Ue_y$$
$$U_z = Ue_z$$

and therefore,

$$\cos\theta_x = e_x$$
$$\cos\theta_y = e_y$$
$$\cos\theta_z = e_z$$

The direction cosines of any Vector U are the components of a unit vector with the same direction as vector $\vec{U}$.

and $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = e_x^2 + e_y^2 + e_z^2 = 1$

Sept. 14/16

2.10 - Position Vector in terms of Components, 3D

$$\vec{r}_{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

2.11 - Dot Products

The dot product of two vectors is $\vec{u}$ and $\vec{v}$ denoted by $\vec{u}\cdot\vec{v}$ is defined as the product of their magnitudes and the cosine of the angle θ between them.

$$\vec{u}\cdot\vec{v} = UV\cos\theta$$

The dot product is commutative (order does not matter)

$$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$$

The dot product is associative with respect to scalar multiplication.

$$a(\vec{u}\cdot\vec{v}) = (a\vec{u})\cdot\vec{v} = \vec{u}\cdot(a\vec{v})$$

The dot product is distributive with respect to vector addition.

$$\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$$

2.12 - Dot product in terms of Components

$$\vec{i}\cdot\vec{i} = ii\cos(0) = ii(1) = 1$$

$$\vec{i}\cdot\vec{j} = ij\cos 90° = ij(0) = 0 \quad \text{(perpendicular to each other)}$$

Therefore,

$$\vec{i}\vec{i} = 1, \quad \vec{i}\cdot\vec{j} = 0, \quad \vec{i}\cdot\vec{k} = 0,$$

$$\vec{j}\vec{j} = 1, \quad \vec{j}\vec{i} = 0, \quad \vec{j}\cdot\vec{k} = 0,$$

$$\vec{k}\cdot\vec{k} = 1, \quad \vec{k}\cdot\vec{i} = 0, \quad \vec{k}\cdot\vec{j} = 0,$$

Sept. 14/16

$\vec{u} \cdot \vec{v} = U_x V_x + U_y V_y + U_z V_z$

but $\vec{U} \cdot \vec{V} = UV\cos\theta$

$$\cos\theta = \frac{\vec{U} \cdot \vec{V}}{UV} = \frac{U_x V_x + U_y V_y + U_z V_z}{UV}$$

2.13 - Cross Products (vector quantity)

The cross product of two vectors $\vec{u}$ and $\vec{v}$ denoted $\vec{U} \times \vec{V}$ is defined as

$\vec{U} \times \vec{V} = UV\sin\theta\,\vec{e}$

The vector $\vec{e}$ is a unit vector defined to be perpendicular to both $\vec{U}$ and $\vec{V}$ and its direction is given by the right-hand rule.

- Moment of two Forces.

The cross product is not commutative

$\vec{U} \times \vec{V} = -\vec{V} \times \vec{U}$

The cross product is associative with respect to scalar multiplication.

$a(\vec{U} \times \vec{V}) = (a\vec{U}) \times \vec{V} = \vec{U} \times (a\vec{U})$

The cross product is distributive with respect to vector addition.

$\vec{U} \times (\vec{V} + \vec{W}) = \vec{U} \times \vec{V} + \vec{U} \times \vec{W}$

Sept. 14/16

2-14 - Cross Products in Terms of Components

$\vec{i} \times \vec{i} = (1)(1)\sin(0)\,\vec{e} = \emptyset$

$\vec{i} \times \vec{j} = (1)(1)\sin(90)\,\vec{e} = \vec{e} = \vec{k}$

Continuing we obtain

$\vec{i} \times \vec{i} = \emptyset \quad ; \quad \vec{i} \times \vec{j} = \vec{k} \quad ; \quad \vec{i} \cdot \vec{k} = -\vec{j}$

$\vec{j} \times \vec{i} = -\vec{k} \quad ; \quad \vec{j} \times \vec{j} = \emptyset \quad ; \quad \vec{j} \cdot \vec{k} = \vec{i}$

$\vec{k} \times \vec{i} = \vec{j} \quad ; \quad \vec{k} \times \vec{j} = -\vec{i} \quad ; \quad \vec{k} \cdot \vec{k} = \emptyset$

Which leads to

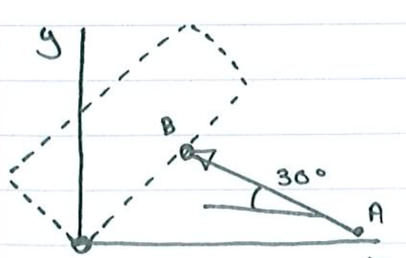
$$\vec{U} \times \vec{V} = (U_y V_z - U_z V_y)\vec{i} - (U_x V_z - U_z V_x)\vec{j} + (U_x V_y - U_y V_x)\vec{k}$$

The result can be expressed as the determinant

$$\vec{U} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix} = \vec{i}\begin{vmatrix} U_y & U_z \\ V_y & V_z \end{vmatrix} - \vec{j}\begin{vmatrix} U_x & U_z \\ V_x & V_z \end{vmatrix} + \vec{k}\begin{vmatrix} U_x & U_y \\ V_x & V_y \end{vmatrix}$$

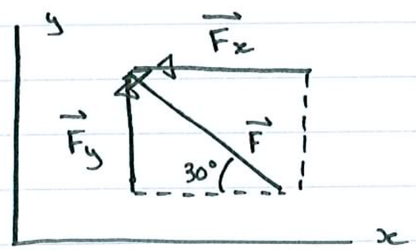
Example:

Hydrolic Cylinders are used to exert Forces in many mechanical devices. The force is exerted by pressurized liquid pushing against a piston within a cylinder. The hydrolic cylinder in the AB in the figure exerts 4000 lb force $\vec{F}$ on the bed of the dump truck at B. Express $\vec{F}$ in terms of scalar components using the coordinate system shown.



When the direction of a vector is specified by an angle formed by a vector and its component, we draw the vector $\vec{F}$ and its vector components.

Sept. 14/16



$\vec{F}_x$ points in the negative x-direction

so, $F_x = -3464\vec{i}$ lb

$F_x = |\vec{F}| \cos 30° = 4000 \cos 30° = 3464$ lb

$F_y = |\vec{F}| \sin 30° = 4000 \sin 30° = 2000$ lb

(consider SOH CAH TOA for right-handed triangles)

$\vec{F}_y$ points in the positive y-direction

so, $F_y = 2000\vec{j}$ lb

The vector $\vec{F}$ in terms of its components is:

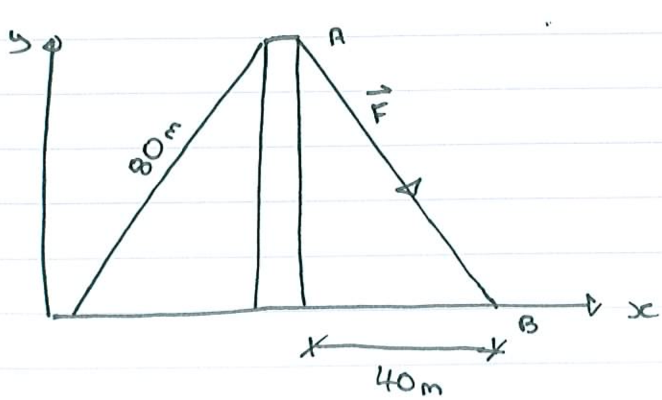
$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F} = -3464\vec{i} + 2000\vec{j} \text{ (lb)}$$

$$|F| = \sqrt{(-3464)^2 + (2000)^2} = 4000$$

Example:

The cable from point A to point B exerts an 800 N force $\vec{F}$ on the top of the television transmission tower. Resolve $\vec{F}$ into components using the coordinate system shown.



Sept. 14/16

Assignment # 1

2.5, 2.6, 2.9, 2.24, 2.34, 2.43

↳ Solution given within 2 weeks.

Sept. 16 /11

ASSIGNMENT #1 (not due to be handed in)

2.5, 2.6, 2.9, 2.24, 2.34, 2.43

#1 Suppose that Einstein's equation

$$E = mc^2$$

The mass m is in kg and the velocity of light c in m/s.

a - what are the SI units of E?

b - if the value of E in SI units is 20, what is its value in U.S. Customary base units.

#2 The force in the Figure lies in the plane defined by the intersecting lines L_A and L_B. It's magnitude is 400 lb. Suppose that you want to resolve $\vec{F}$ into vector components parallel to L_A and L_B. Determine the magnitudes of the vector components

a - graphically and

b - using trigonometry