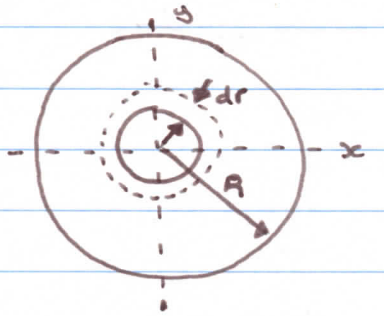
Nov. 21/16

Q1: Determine the moments of inertia and radii of gyration of the circular area.



By letting r change by an amount or dr, we obtain an annular element of area $dA = 2\pi r\,dr$

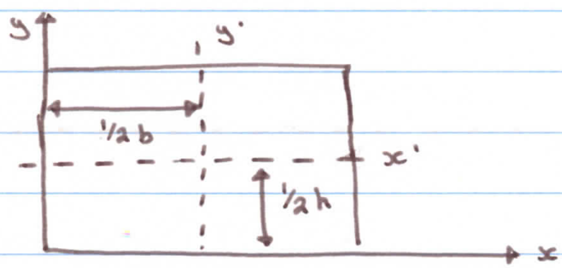
$$J_o = I_x + I_y$$
$$= \int_A r^2 dA = \int_0^R 2\pi r^3 dr$$
$$= 2\pi \left[r^4/4 \right]_0^R = \tfrac{1}{2}\pi R^4$$

$$k_o = \sqrt{\frac{J_o}{A}} = \sqrt{\frac{\tfrac{1}{2}\pi R^4}{\pi R^2}} = \frac{1}{\sqrt{2}} R$$

The moment of inertia $I_x = I_y = \tfrac{1}{2} J_o = \tfrac{1}{4}\pi R^4$

$$k_x = k_y = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\tfrac{1}{4}\pi R^4}{\pi R^2}} = \frac{1}{2} R$$

Q2:



Determine the moment of inertia in terms of the xy coordinate system.

$$I_{x'} = \tfrac{1}{12} bh^3 \quad ; \quad I_{y'} = \tfrac{1}{12} hb^3$$

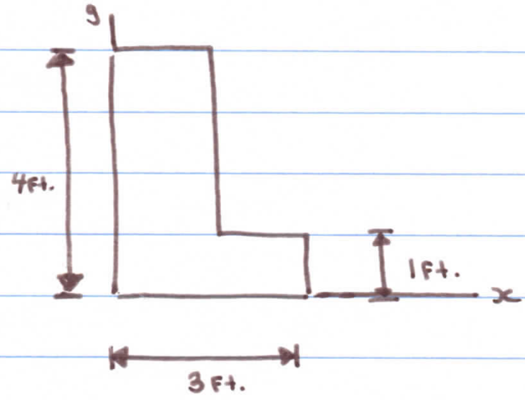
$$I_x = I_{x'} + d_y^2 A \Rightarrow \tfrac{1}{12} bh^3 + (\tfrac{1}{2}h)^2 (bh)$$
$$\Rightarrow \tfrac{1}{3} bh^3$$

$$I_y = I_{y'} + d_x^2 A \Rightarrow \tfrac{1}{12} hb^3 + (\tfrac{1}{2}b)^2 (hb)$$
$$\Rightarrow \tfrac{1}{3} hb^3$$

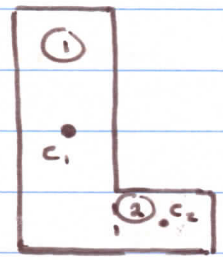
$$J_o = J_o' + d^2 A = \tfrac{1}{12}(bh^3 + hb^3) + \left[(\tfrac{1}{2}b)^2 + (\tfrac{1}{2}h)^2 \right](bh)$$
$$= \tfrac{1}{3}(bh^3 + hb^3)$$

<u>OR</u> $J_o = I_x + I_y$

Q3:



Determine I_x, k_x, and I_{xy}.



PART #	d_y (ft)	A (ft²)	$I_{x'}$ (ft⁴)	$I_x = I_{x'} + d_y^2 A$ (ft⁴)
1	2	(1)(4)	$(1/12)(1)(4)^3$	21.33
2	0.5	(2)(1)	$(1/12)(2)(1)^3$	0.67

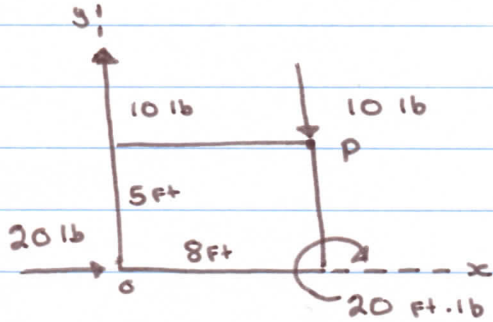
$$I_x = (I_x)_1 + (I_x)_2$$

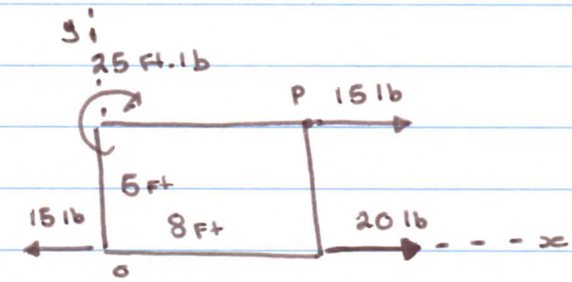
$$I_x = 21.33 + 0.67 = 22.00 \text{ ft}^4$$

$$k_x = \sqrt{\frac{I_x}{A}} \Rightarrow \sqrt{\frac{22}{6}} \Rightarrow 1.91 \text{ ft}$$

	d_x / d_y (ft)	A (ft²)	$I_{x'y'}$ (ft⁴)	$I_{xy} = I_{x'y'} + (d_x d_y)A$ (ft⁴)
1	0.5 / 2	(1)(4)	0	4
2	2 / 0.5	(2)(1)	0	2

Q4:





$(\Sigma F)_1 = 20i + 0j$

$(\Sigma M_o)_1 = -8\text{ ft}(10\text{ lb}) - 20\text{ ft.lb}$

$= -100\text{ ft.lb}$

$(\Sigma F)_1 = 20i + 0j$

$(\Sigma M_o)_2 = -5\text{ ft}(15\text{ lb}) - 25\text{ ft.lb}$

$= -100\text{ ft.lb}$

Are the two systems equivalent?

– YES.

Q5 :

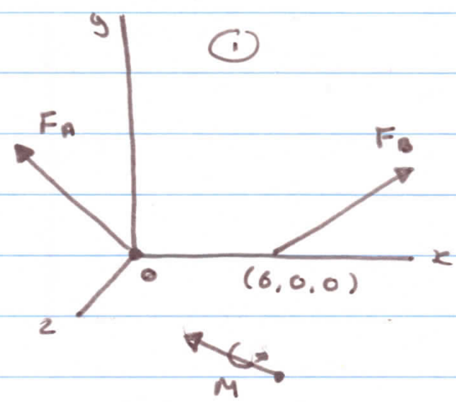
$F_A = -10i + 10j - 15k$

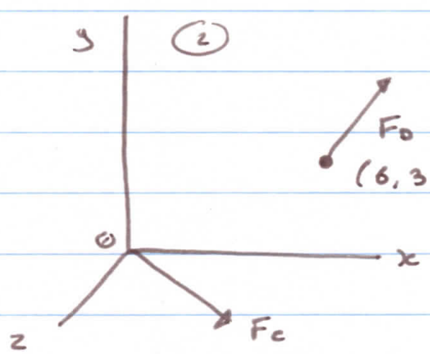
$F_B = 30i + 5j + 10k$

$M = -90i + 150j + 60k$

$F_C = 10i - 5j + 5k$

$F_D = 10i + 20j - 10k$





$(\Sigma F)_1 = F_A + F_B$

$= 20i + 15j - 5k$

$(\Sigma F)_2 = F_C + F_D$

$= 20i + 15j - 5k$

$$(\Sigma M_o)_1 = \begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ 30 & 5 & 10 \end{vmatrix} + (-90i + 150j + 60k)$$

$(\Sigma M_o)_1 = 90i + 90j + 90k$

$$(\Sigma M_o)_2 = \begin{vmatrix} i & j & k \\ 6 & 3 & 3 \\ 10 & 20 & -10 \end{vmatrix}$$

$$\Rightarrow -90i + 90j + 90k$$

$\therefore$ the two systems are equivalent