Chapter 12
Kinematics of Particles
Newton's second Law
3 - Sections in total

Introduction (pp. 719-720)
§12.1 Newton's Second Law + Linear Momentum
12.1A Newton's Second Law
12.1C System of Units
12.1D Equations of motion
- Rectangular
- Tangential - Normal
- Radial - Transverse

§12.1 Newton's Second Law and Linear Momentum
(12.1 B not covered)

12.1A Newton's 2nd Law

When a particle is subject to a single Force $\vec{F}$ then

$$\vec{F} = m\vec{a}$$

When there are a number of Forces applied upon the particle simultaneously, then

$$\Sigma \vec{F} = m\vec{a}$$

12.1C Systems of Units

SI : mass is a base quantity
m: kg, or g
weight is a derived quantity
$\omega = m \cdot g$ $\omega$: N

US Customary:

Weight is a base quantity

$w$ : lb

Mass is a derived quantity

$$m = \frac{w}{g}$$

m: slug or blob, depending on unit used with length

$$1 \text{ slug} = \frac{1 \text{ lb}}{1 \text{ ft/s}^2} = 1 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$1 \text{ blob} = \frac{1 \text{ lb}}{1 \text{ in/s}^2} = 1 \text{ lb}\cdot\text{s}^2/\text{in}$$

e.g. the mass associated with 1-lb weight is:

$$m = \frac{1 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.031056 \text{ slug}$$

$$m = \frac{1 \text{ lb}}{386. \text{ in/s}^2} = 2.5907 \times 10^{-3} \text{ blob}$$

12.1D Equations of Motion

Problem Solving

Kinematics (Ch. 11) + FBD (Statics)

kinematics : rectilinear motion

curvilinear motion, planar

Forces : can be 3-dimensional

∴ need 3 unit vectors

rectangular : $\vec{i}, \vec{j}, \vec{k}$

tangential - normal : $\vec{e_t}, \vec{e_n}$

and $\vec{e_b} = \vec{e_t} \times \vec{e_n}$ (binormal)

radial - transverse : $\vec{e_r}, \vec{e_\theta}$

and $\vec{k} = \vec{e_r} \times \vec{e_\theta}$

Fig 12.9, pp. 725 ~ 726

$$\sum \vec{F} = m\vec{a}$$

$\sum \vec{F} \downarrow$ FBD

$m\vec{a} \downarrow$ Kinematic diagram (KD)

What are equations of motion?

$$\sum \vec{F} = m\vec{a}$$

When cast/written in components form, the resulting equations are known as EOM (equations of motion)

1) Rectangular Components

From statics $\sum \vec{F} = (\sum F_x)\vec{i} + (\sum F_y)\vec{j} + (\sum F_z)\vec{k}$

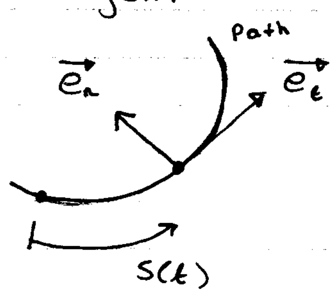
From chapter 11 $\vec{a} = a_x\vec{i} + a_y\vec{j} + \emptyset k$

$$\therefore (\sum F_x)\vec{i} + (\sum F_y)\vec{j} + (\sum F_z)\vec{k} = (ma_x)\vec{i} + (ma_y)\vec{j} + \emptyset\vec{k}$$

$$\begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = \emptyset \end{cases}$$

Applicable to rectilinear motions

2. Tangential - Normal Components




$$\vec{a} = a_t\vec{e_t} + a_n\vec{e_n} + \emptyset\vec{e_b}$$

$$\sum \vec{F} = (\sum F_t)\vec{e_t} + (\sum F_n)\vec{e_n} + (\sum F_b)\vec{e_b}$$

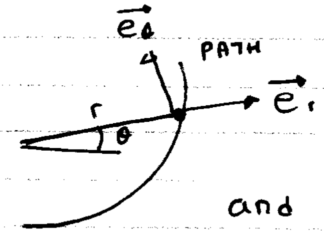
$$\therefore (\sum F)\vec{e_t} + (\sum F_n)\vec{e_n} + (\sum F_b)\vec{e_b}$$

$$= (ma_t)\vec{e_t} + (ma_n)\vec{e_n}$$

$$= (m\dot{v})\vec{e_t} + \left(m\frac{v^2}{\rho}\right)\vec{e_n}$$

$$\therefore \begin{cases} \sum F_t = ma_t = m\dot{v} \\ \sum F_n = ma_n = m\dfrac{v^2}{\rho} \\ \sum F_b = \emptyset \end{cases}$$

3. Radial and Transverse Components




and $\vec{k} = \vec{e_r} \times \vec{e_\theta}$

$$\begin{cases} \sum F_r = ma_r \\ \quad\;\; = m(\ddot{r} - r\dot{\theta}^2) \\ \sum F_\theta = m a_\theta \\ \quad\;\; = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \\ \sum F_z = 0 \end{cases}$$

Sample Problems

12.1 - in-Class

12.2 - Force in terms of $x$

12.6 tangential - normal

7 " "

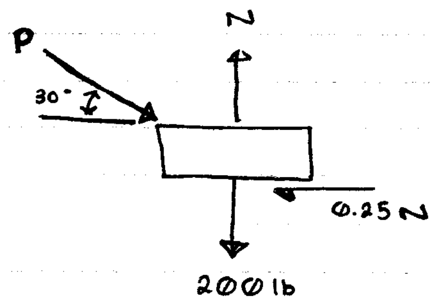
8 " "

9 " "

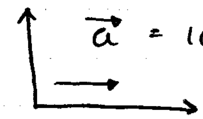
12.10 radial - transverse

SAMPLE PROBLEM 12.1 (slug - mass, lb - force)

(1)




$P_x = (\cos 30°)P$
$P_y = (\sin 30°)P$




(2) $\Sigma F_x = m a_x$

$$P\cos(30°) - 0.25N = \left(\frac{200}{32.2}\right)(10)$$

$\Sigma F_y = m a_y$

$$N - 200 - P\sin(30°) = 0$$

$\Sigma F_z = 0$

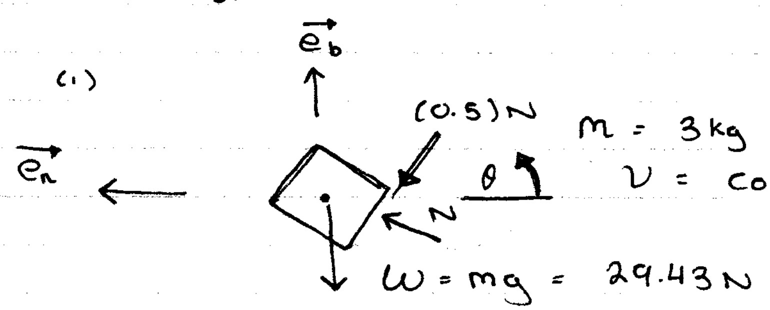
$0 = 0$

Solving ⓐ + ⓑ

$P = 151.3$ lb

PROBLEM 12.55

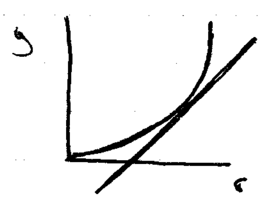
(1)




$m = 3$ kg , $\rho = T = 2$m

$v =$ constant

$\theta$ : $y = \frac{r^2}{4}$




$$\left.\frac{dy}{dr}\right|_{r=2} = 1$$

$$\theta = \tan^{-1}\left(\frac{dy}{dr}\right)\Big|_{r=2} = 45°$$

(2) $\Sigma F_b = 0$

$$(N\sin 45°) - (0.5)N\sin 45° - 29.43 = 0$$

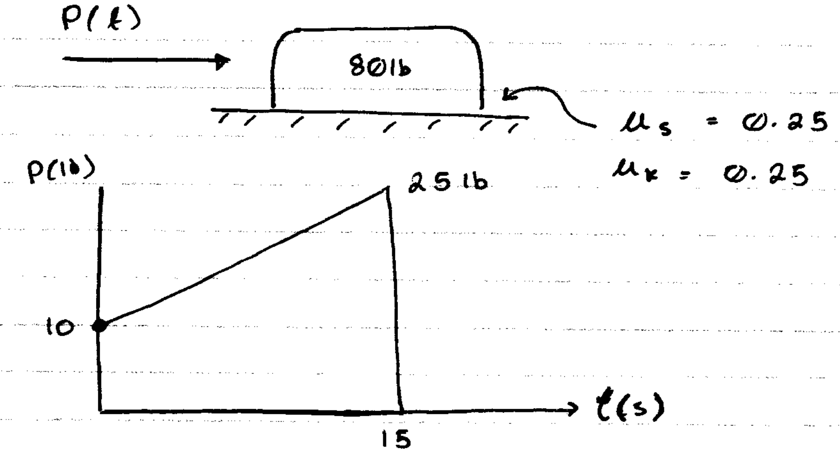
$\therefore N = 83.24$ N

$\Sigma F_n = m a_n$

$$N\cos 45° + (0.5)N\cos 45° = 3\frac{v^2}{2}$$

$\therefore v = 7.672$ m/s

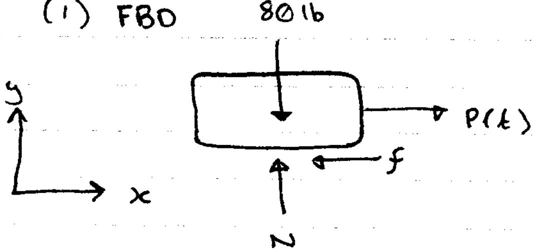
Example

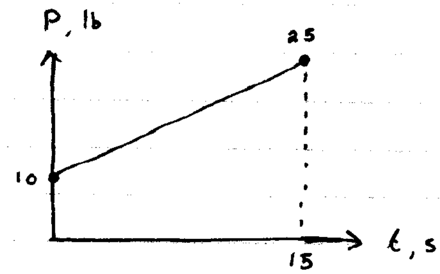



Find : (1) time $t_1$ when block starts moving
(2) distance traveled by the block when it stops

Example

(1) FBD




$$P(t) = \begin{cases} 10 + t & 0 \le t \le 15\text{s} \\ 0 & t > 15\text{s} \end{cases}$$

$\Sigma F_x = 0 \qquad P(t) - f = 0$

at time instant $t_1$, $f = f_{max} = \mu_s \cdot N$

$\Sigma F_y = 0 \qquad N - 80 = 0$

$\therefore N = 80 \text{ lb}$

$\therefore f_{max} = (0.25)(80) = 20 \text{ lb}$

$P(t_1) - f_{max} = 0$

$\therefore 10 + t_1 - 20 = 0$

$\therefore t_1 = 10\text{s}$

(2) $a \rightarrow v \rightarrow x$

$a = ?$

$t_1 \le t \le 15\text{s}$

$\Sigma F_x = m a_x = ma$

$P(t) - f = \left(\frac{80}{32.2}\right) a$

$10 + t - (0.25)(80) = \left(\frac{80}{32.2}\right) a$

$\therefore a(t) = 0.4025\,(t - 10) \quad \text{ft/s}^2$

Initial conditions : $t_0 = 10\text{s}$, $v_0 = 0$, $x_0 = 0$

$\therefore v(t) = 0.20125\, t^2 - 4.025\, t + 20.125 \quad \text{ft/s}$

$x(t) = \frac{0.20125\, t^3}{3} - 2.0125\, t^2 + 20.125\, t$

$= -67.083 \quad \text{ft}$

and $v|_{t=15} = 5.03125$ ft/s
$x|_{t=15} = 8.38575$ ft

$t > 15$ s $\qquad P(t) = 0$

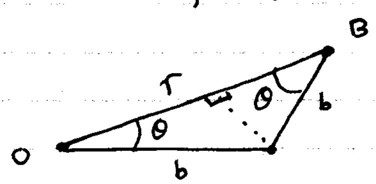
$$\sum F_x = ma$$
$$\therefore \; -f = \left(\frac{80}{32.2}\right)a$$
$$-20 = \left(\frac{80}{32.2}\right)a$$
$$a = -8.05 \text{ ft/s}^2$$
$$\therefore \; v^2 - v_0^2 = 2a(x - x_0)$$

but $v_0 = v|_{t=15} = 5.03125$ ft/s
$x_0 = x|_{t=15} = 8.38575$ ft
and $v = 0$

$$\therefore \; x = 9.958 \text{ ft}$$

Problem 12.70
Solution:

(a) $a_r$, $a_\theta$ $\longrightarrow$ need to know $r, \dot{r}, \ddot{r}$
$\theta, \dot{\theta}, \ddot{\theta}$ given.



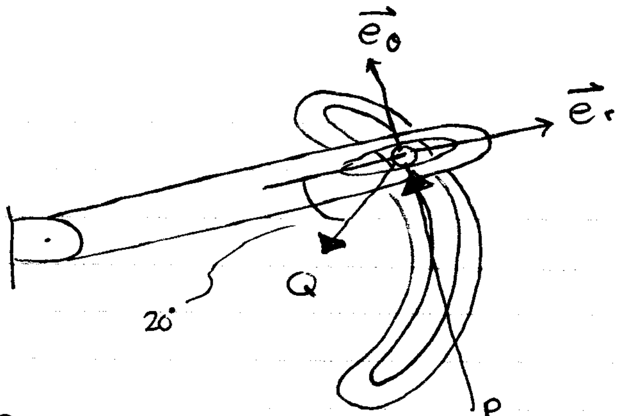

$$r = 2b\cos\theta$$
$$\dot{r} = -2b\sin\theta \cdot \dot{\theta}$$
$$\ddot{r} = -2b\cos\theta \cdot \dot{\theta}^2 + -2b\sin\theta \cdot \ddot{\theta}$$
$$\ddot{r} = -2b(\cos\theta \cdot \dot{\theta}^2 + \sin\theta \cdot \ddot{\theta})$$

$$\therefore \; a_r = \ddot{r} - r\dot{\theta}^2 = -20{,}334 \text{ in/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3{,}240.7 \text{ in/s}^2$$

$$\therefore \; \sum F_r = ma_r = \left(\frac{0.25 \text{ lb}}{386 \text{ in/s}^2}\right)(-20{,}334)$$
$$= -13.176 \text{ lb}$$

$$\sum F_r = ma_r = \left(\frac{0.25 \text{ lb}}{386 \text{ in/s}^2}\right)(-20{,}334) = -13.176 \text{ lb}$$

$$\sum F_\theta = ma_\theta = 2.0989$$




P → Pin on rod
Q → Pin against track

$\Sigma F_r = -13.176 - Q\cos 20°$
$\therefore\ Q = 14.022$ lb

$\Sigma F_\theta = 2.0989 = P - Q\sin 20°$
$\therefore\ P = 6.8947$ lb