Radian as a unit: (to differentiate from degrees, gradients, ...)

$rad/s = 1/s$

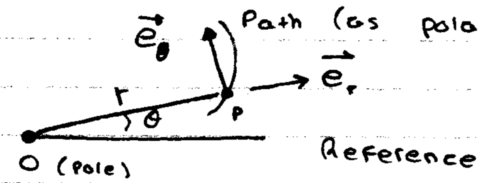
$m \cdot rad/s = m/s$

$r \in (-\infty, \infty)$ limited $r \in [0, \infty)$

$\theta \in (-\infty, \infty)$ $\theta \in (-\infty, \infty)$

Polar curves $r = f(\theta)$ $y = f(x)$

B) Radial and Transverse Components



$\vec{e}_r, \vec{e}_\theta$

$\vec{e}_r = O \to P$

$\vec{e}_\theta =$ rotate $\vec{e}_r$ by 90°, ccw

$\dot{\vec{e}}_r \neq \vec{0}$

$\dot{\vec{e}}_\theta \neq \vec{0}$

$\dot{\vec{e}}_r = \dot{\theta}\vec{e}_\theta$

$\dot{\vec{e}}_\theta = -\dot{\theta}\vec{e}_r$

Position: $\vec{r} = r\vec{e}_r$

Velocity: $\vec{v} = \dot{\vec{r}} = \dfrac{d}{dt}(r\vec{e}_r)$

$$\vec{v} = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt}$$

$$= \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$= \vec{v}_r + \vec{v}_\theta$$

Acceleration:

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt}(\dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta)$$

$$= \ddot{r}\vec{e}_r + \dot{r}\dot{\theta}\vec{e}_\theta + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta$$

$$= \ddot{r}\vec{e}_r + \dot{r}\dot{\theta}\vec{e}_\theta + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}(-\dot{\theta}\vec{e}_r)$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$= \vec{a}_r + \vec{a}_\theta$$

Vector Form:

$$\vec{r} = r\vec{e}_r$$

$$\vec{v} = \vec{v}_r + \vec{v}_\theta$$

$$= v_r\vec{e}_r + v_\theta\vec{e}_\theta$$

$$\vec{a} = \vec{a}_r + \vec{a}_\theta$$

$$= a_r\vec{e}_r + a_\theta\vec{e}_\theta$$

Scalar Form :

$v_r = \dot{r}$, $v_\theta = r\dot{\theta}$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\vec{e}_r$, $\vec{e}_\theta$: 90° apart

Problem Solving :

1) $\theta = \theta(t)$, $r = r(t)$

2) $r = r(\theta)$, θ in terms of t

3) $\vec{e}_r - \vec{e}_\theta \Leftrightarrow \vec{e}_t - \vec{e}_n$

Sample Problem 11.18

$\theta = 0.15t^2$ rad $\qquad r = 0.9 - 0.12t^2$

$\dot{\theta} = 0.3t$ rad/s $\qquad \dot{r} = 0 - 0.24t$

$\ddot{\theta} = 0.3$ rad/s² $\qquad \ddot{r} = -0.24$

$\theta = 30° = \frac{\pi}{6}$

$\therefore \pi/6 = 0.15t^2$

$\therefore t = 1.868$ sec

$\therefore \dot{\theta} = 0.5605$ rad/s

$\ddot{\theta} = 0.3$ rad/s²

$r = 0.4811$ m

$\dot{r} = -0.4484$ m/s

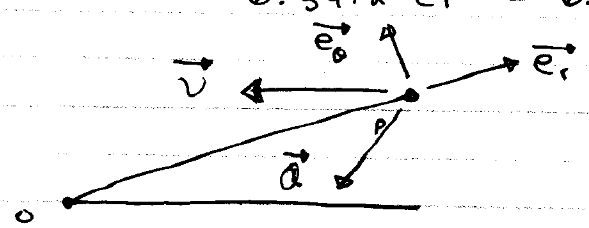
$\ddot{r} = -0.24$ m/s²

(a) $\therefore \vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$

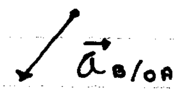
$= -0.4484\,\vec{e}_r + 0.2697\,\vec{e}_\theta$ (m/s)

(b) $\therefore \vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$ (m/s²)

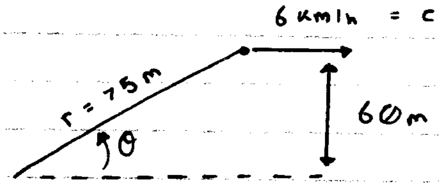
$= -0.3912\,\vec{e}_r - 0.3583\,\vec{e}_\theta$



(c) $\vec{a}_{B/OA} = \ddot{r}\,\vec{e}_r = -0.24\,\vec{e}_r$



SAMPLE PROBLEM 11.19

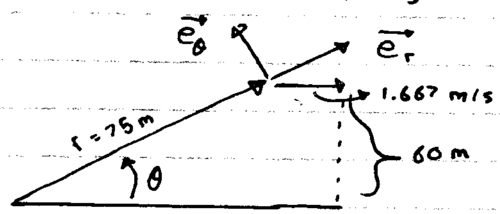


6 km/h = constant ; $\vec{v}$ = constant

60 m = constant

SOLUTION:

$$v = \frac{6\text{ km}}{\text{h}} \cdot \frac{1\text{ h}}{3600\text{ s}} \cdot \frac{1000\text{ m}}{1\text{ km}} = 1.667\text{ m/s}$$



$$\therefore\ 60 = r\sin\theta$$

$$\therefore\ r = \frac{60}{\sin\theta}$$

$$\dot{r} = \frac{dr}{d\theta}\cdot\dot{\theta}$$

$$= -60\,\frac{\cos\theta}{\sin^2\theta}\,\dot{\theta}$$

$$\ddot{r} = -60\left[\frac{d}{d\theta}\left(\frac{\cos\theta}{\sin^2\theta}\right)\cdot\dot{\theta}\cdot\dot{\theta}\right.$$

$$\left.\cdots + \left(\frac{\cos\theta}{\sin^2\theta}\right)\ddot{\theta}\right]$$

Where $\dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\sin^2\theta}\right) = \dfrac{-\sin^2\theta - 2\cos^2\theta}{\sin^3\theta}$

$$= -\frac{1+\cos^2\theta}{\sin^3\theta}$$

$$\therefore\ \ddot{r} = 60\left[\frac{1+\cos^2\theta}{\sin^3\theta}\,\dot{\theta}^2 - \frac{\cos\theta}{\sin^2\theta}\,\ddot{\theta}\right]$$

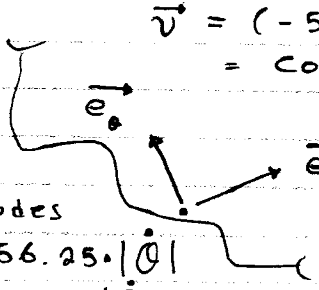
At the given time instant:

$r = 75$ m, $\theta = 0.9273$ rad

$\dot{r} = -56.25\,\dot{\theta}$, $\ddot{r} = 102.2\,\dot{\theta}^2 - 56.25\,\ddot{\theta}$

$\vec{v} = (-56.25\,\dot{\theta})\,\vec{e}_r + (75\cdot\dot{\theta})\,\vec{e}_\theta$

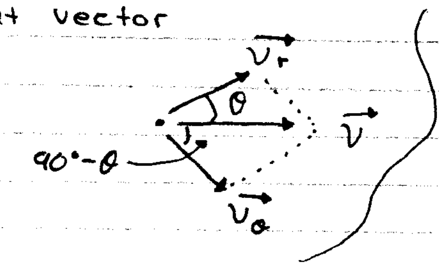
$=$ Constant vector



Magnitudes

$v_r = 56.25\cdot|\dot{\theta}|$

$v_\theta = 75\cdot|\dot{\theta}|$



$\therefore\ ((56.25)|\dot{\theta}|)\cos\theta$

$+\ ((75|\dot{\theta}|)\cos(\pi/2-\theta))$

$= 1.667$

$|\dot{\theta}| = 0.01778$ rad/s

or $\dot{\theta} = -0.01778$ rad/s

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... continued from previous lecture.

then $\dot{r} = -56.25\dot{\theta} = 1\ m/s$

$\ddot{r} = ?$

$\therefore \vec{v}$ = constant vector

$\therefore \vec{a} = \frac{d\vec{v}}{dt} = \vec{0}$

On the other hand, $\vec{a} = a_r\vec{e_r} + a_\theta\vec{e_\theta} = \vec{0}$

$\therefore a_r = 0,\ a_\theta = 0$

$\therefore a_r = \ddot{r} - r\dot{\theta}^2$

$\therefore \ddot{r} = r\dot{\theta}^2 = 0.02371\ m/s^2$

$\ddot{\theta}$? $a_\theta = 0$

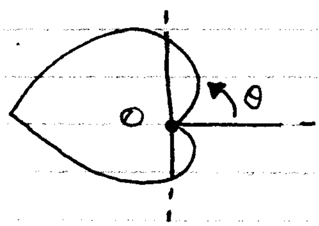
Next Example:

Given $\theta = \pi t$ (radians), $r = 2(1-\cos\pi t)$ (inches)

Determine, at $t = 0.5\ s$

(1) magnitude of $\vec{v}$ and $\vec{a}$; and

(2) radius of curvature



(1) $\theta, \dot{\theta}, \ddot{\theta}$

$r, \dot{r}, \ddot{r}$

$v_r, v_\theta \Rightarrow v$

$a_r, a_\theta \Rightarrow a$

$t = 0.5\ s$

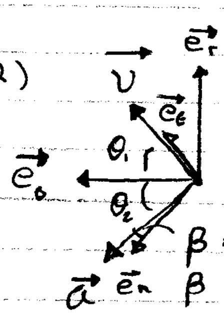
$v_r = 2\pi, \quad v_\theta = 2\pi$

$\therefore v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{2}\ 2\pi$

$a_r = -2\pi^2, \quad a_\theta = 4\pi^2$

$\therefore a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{5}\ 2\pi^2$

(2)



$\theta_1 = 45°$

$\theta_2 = 26.57°$

$\beta = 90° - 45° - 26.57°$

$\beta = 18.43°$

$\therefore a_n = a\cos\beta$

$a_n = 4.243\pi^2$

but $a_n = v^2/\rho$

$\therefore \rho = \frac{v^2}{a_n} = 1.885\ (in)$

Ch. 11 - Kinematics of Particles

Rectilinear motion

x v a

Position $\longleftrightarrow$ velocity $\longleftrightarrow$ acceleration

$a(t)$
$a(x)$
$a(v)$

integration / differentiation
graphical solution (limits, when not to use)

accelerating vs. decelerating
irreversible motion vs. reversible motion
max / min velocity / position
distance traveled between t_1 and t_2

Uniformly accelerated rectilinear motion
(a = const)
Uniform rectilinear motion

Curvilinear motion

$$\left\{\begin{array}{l}\text{rectangular components } (x, y),\ \vec{i} - \vec{j} \\ \text{tangential - normal components } \vec{e_t} - \vec{e_n} \\ \text{radial - transverse components } (r, \theta),\ \vec{e_r} - \vec{e_\theta}\end{array}\right.$$

$\left\{\begin{array}{l}(x, y) \\ (r, \theta)\end{array}\right.$

Rectangular Components

$x = x(t)$, $y = y(t)$
$\Rightarrow v_x, v_y, \vec{v}$
$\Rightarrow a_x, a_y, \vec{a}$

$y = y(x)$, $x, \dot{x}, \ddot{x}$
$\Rightarrow \dot{y}, \ddot{y}$
$\Rightarrow \vec{v}, \vec{a}$

Radial Transverse Components

$\theta = \theta(t)$, $r = r(t)$

$\Rightarrow \dot{\theta}, \ddot{\theta}, \dot{r}, \ddot{r}$

$\Rightarrow v_r, v_\theta, a_r, a_\theta \Rightarrow \vec{v}, \vec{a}$

$r = r(\theta)$, $\theta, \dot{\theta}, \ddot{\theta}$

$\Rightarrow \dot{r}, \ddot{r}$

$\Rightarrow v_r, v_\theta, a_r, a_\theta \Rightarrow \vec{v}, \vec{a}$

Tangential - normal components

Path coordinate $s(t)$

$s(t)$ = arc-length P_0 to P

line integral $\begin{cases} \text{rectangular based} \\ \text{polar based} \end{cases}$

$\vec{e_t} - \vec{e_n}$ combination has the most physical meaning

a_t measures . . . . , range of value

a_n measures . . . , range of value

$s(t) \longrightarrow v(t) \longrightarrow a_t(t)$

Solving the tangential component is equivalent to solving a rectilinear motion

$$a_n = \frac{v^2}{\rho}$$

$\begin{cases} \text{Rectangular Components} \\ \text{Tangential - normal Components} \\ \text{Radial - transverse components} \end{cases}$

1) They are inter-related

$$\vec{v} = v_x \vec{i} + v_y \vec{j} = v \vec{e_t} = v_r \vec{e_r} + v_\theta \vec{e_\theta}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} = a_t \vec{e_t} + a_n \vec{e_n} = a_r \vec{e_r} + a_\theta \vec{e_\theta}$$

Tangential-normal components have the most physical meanings.

$$\vec{v} \begin{cases} \text{mag} & v \\ \text{direction} & \vec{e_t} \end{cases}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \therefore \text{ it's the time rate of change in } \vec{v}$$

time rate of change in mag of $\vec{v}$: $\dot{v}\,\vec{e_t}$

time rate of change in direction of $\vec{v}$: $\dfrac{v^2}{\rho}\,\vec{e_n}$

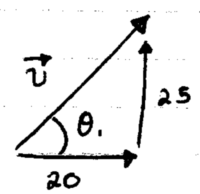
From Question:

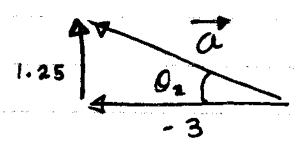
Given $y = x^2/160$ (y is in mm) $\quad x = 100\text{ mm} \quad \dot{x} = 20\text{ mm/s} \quad \ddot{x} = -3\text{ mm/s}^2$

Find: a_t, a_n and ρ at $x = 100\text{ mm}$

Solution: $\dot{x} = 20\text{ mm/s} \quad v_y = \dot{y} = 25\text{ mm/s}$

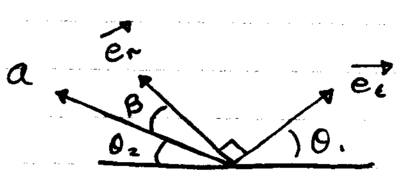
$\ddot{x} = -3\text{ mm/s}^2 \quad a_y = \ddot{y} = 1.25\text{ mm/s}^2$





$v = 32.02\text{ mm/s} \qquad a = 3.25\text{ mm/s}^2$

$\theta_1 = 51.34° \qquad \theta_2 = 22.62°$



$\beta = 16.04°$

$\therefore a_t = -a\sin\beta$

$= -0.8980\text{ mm/s}^2$

$a_n = a\cos\beta = 3.123\text{ mm/s}^2$

$\therefore a_n = \dfrac{v^2}{\rho} \quad \therefore \rho = 328.3\text{ mm}$

11.137, 11.138, 11.143 :

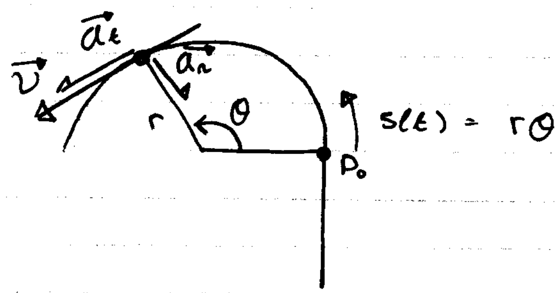
tangential component $\longleftrightarrow$ rectilinear motion

a_t = constant

137: $t = 8.560$ s

138: (b) $t = 25.20$ s

143: $\vec{e_t} - \vec{e_n}$



$x - y$

$s(t)$

$v_x = -47.55$ m/s

$a_y = -33.73$ m/s²

11.161 : $\theta = \theta(t)$ $\quad$ b) $\vec{a} = -49.94\vec{e_r} - 9.744\vec{e_\theta}$ in/s²

$r = r(t)$

(*) 11.165 : $\theta = \theta(t)$, $r = r(\theta)$

$$r = \frac{2b}{1+\cos\theta}$$

a) $\theta = 0°$; $\vec{v} = bk\vec{e_\theta}$, $\vec{a} = -\frac{k^2 b}{2}\vec{e_r}$

b) $\theta = 90°$

$\vec{v} = 2kb\vec{e_r} + 2kb\vec{e_\theta}$

$\vec{a} = 2k^2b\vec{e_r} + 4k^2b\vec{e_\theta}$

Notes on Assignment 4 P1

↳ rectangular all the way $\quad x, \dot{x}, \ddot{x}$ given

$y = y(x)$, $x, \dot{x}, \ddot{x}$

$\rightarrow \dot{y}, \ddot{y}$ (To find)

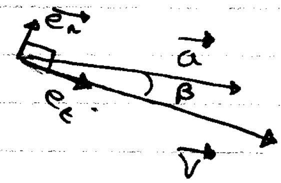
$\rightarrow$ values at $x = 12$ m

$\vec{v} = 6\vec{i} - 0.6646\vec{j}$ m/s

$\vec{a} = -0.07273\vec{i} - 0.5679\vec{j}$ m/s²

Notes on P1 (assignment 5)
x-y components $\rightarrow$ $\vec{e_t} - \vec{e_n}$

$\dot{y} = -0.1472$ m/s
$\ddot{y} = -0.2649$ m/s



(Note: $\vec{e_t}$ and V share direction)

($\vec{e_n}$ must be positive)

$a_t = 4.009$ m/s²
$a_n = 0.03009$ m/s²