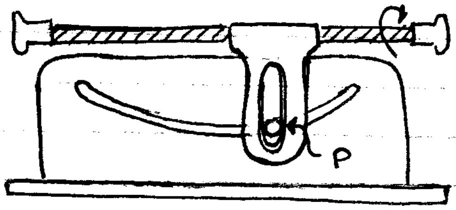
JAN. 30/17



$y = x^2/160$

Given:

$x = 100m$

$v_x = 20\ mm/s = \dot{x} = dx/dt$

$a_x = -3\ mm/s^2 = \ddot{x} = d^2x/dt^2$

$$v_y = \dot{y} = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = y'\dot{x} = \frac{2x}{160} \cdot \dot{x}$$

$$= \frac{x}{80} \cdot \dot{x}$$

$$= \frac{x\dot{x}}{80}$$

$$a_y = \ddot{y} = \frac{d}{dt}\left(\frac{x\dot{x}}{80}\right) = \frac{1}{80}\left[\frac{dx}{dt} \cdot \dot{x} + x\frac{d\dot{x}}{dt}\right]$$

$$= \frac{1}{80}\left[(\dot{x})^2 + x\ddot{x}\right]$$

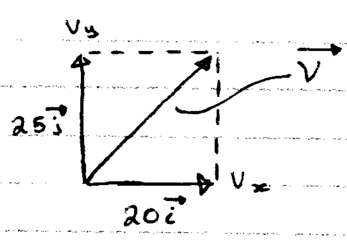
Substituting values.

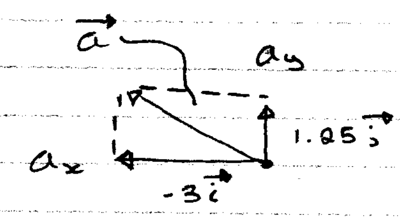
$v_y = 25\ mm/s$, $a_y = 1.25\ mm/s^2$

$$\therefore |\vec{v}| = \sqrt{v_x^2 + v_y^2} = 32.02\ mm/s$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = 3.25\ mm/s^2$$

Visualization:





§11.4 Curvilinear Motion of Particles

11.4A $\vec{r}$, $\vec{v}$, and $\vec{a}$

11.4B over-dot notation

11.4C rectangular components of $\vec{v}$ and $\vec{a}$

11.4D

Sample Prob. 11.10 ~ 11.12 (Projectile motion)

§ 11.5 Non-rectangular components

11.5A Tangential and Normal Components (motion in space not req'd)

11.5B Radial and transverse components (motion in space not req'd)

Sample Prob.: 11.16 ~ 11.20

11.5A Tangential and Normal Components

A) Geometric properties of a planar curve

1) Curve: $y = f(x)$

2) slope of tangent to curve at a given point:

$$\frac{dy}{dx}\ (= \tan\theta), \quad \text{or} \quad y'\ (= \tan\theta)$$

3) Curvature

every point on the curve possesses a curvature which measures the curved-ness of the curve;

e.g. a straight line has zero curvature at every point

↳ a circle is a curve with identical curvature.

4) radius of curvature at a given point x in the radius of a circle which touches the curve at a given point, has the same tangent and curvature at that point

$$\underset{(\text{length})}{\rho} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\sqrt{\left[1 + (y')^2\right]^3}}{|y''|}$$

($\frac{1}{\text{length}}$) Curvature is the reciprocal of ρ

5) Center of curvature at a given point x

11.5B Radial and Transverse Components

A) Polar Coordinates

Geometric properties of a planar curve

$x - y$

Curve $y = f(x)$

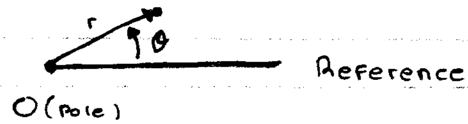
slope y' or $\dfrac{dy}{dx}$

Curvature, radius of curvature

Polar Coordinates (r, θ)

r : radial coordinate

θ : angular coordinate



Units r: m, in, ft, ...

θ : radian

radian as a unit: (to differentiate from degrees, gradients, ...)

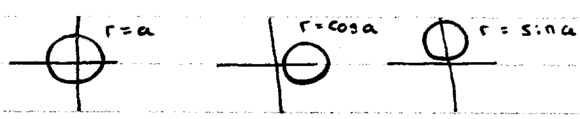
rad / s = 1/s

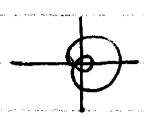
m · rad/s = m/s

$r \in (-\infty, \infty)$ limited (For this class) $r \in [0, \infty)$

$\theta \in (-\infty, \infty)$ $\theta \in (-\infty, \infty)$

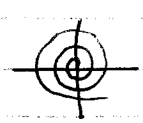
Polar curves $r = f(\theta)$ $y = f(x)$





$r = a + b\cos\theta$

Can be used to graph functions that are difficult to graph with other systems. (x and y)



$r = a\theta > 0$

$\vec{a_t}$: tangential (component of) acceleration
measures the time-rate of change in speed

$$\dot{v} \begin{cases} > \emptyset & \text{increasing} \\ = \emptyset & \text{constant} \\ < \emptyset & \text{decreasing} \end{cases}$$

$\vec{a_n}$: normal (component of) acceleration
measures the time-rate of change in the direction of velocity

$$\frac{v^2}{\rho} \begin{cases} > \emptyset \\ = \emptyset \end{cases} \qquad \rho \to \infty \text{ , or } y'' = \emptyset$$

Midterm: Feb. 13th

1:00 → 2:30

↪ Covers Chapter 11

Assignments 1 ~ 5

Assignment 5: not required

Tomorrow's Tutorial: File is available

Lecture on Feb. 16th/17, 1:30 → 2:30, UC 2011

Vector Form:

$\vec{v} = v\vec{e}_t$

$\vec{a} = \vec{a}_t + \vec{a}_n = a_t\vec{e}_t + a_n\vec{e}_n$

Scalar Form:

$v = \dot{s}$

$a_t = \dot{v} = \ddot{s}$

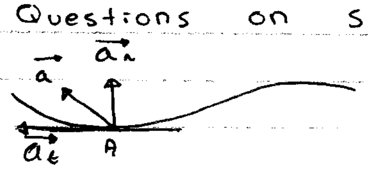
$a_n = v^2/\rho$

Problem Solving

	Rectilinear Motion	Tangential Component of Curvilinear motion
Position	$x(t)$	$s(t)$ — arc length
Velocity	$v(t) = \dot{x}$	$v(t) = \dot{s}$
Acceleration	$a(t) = \dot{v} = \ddot{x}$	$a_t(t) = \dot{v} = \ddot{s}$

From Questions on screen:

#1

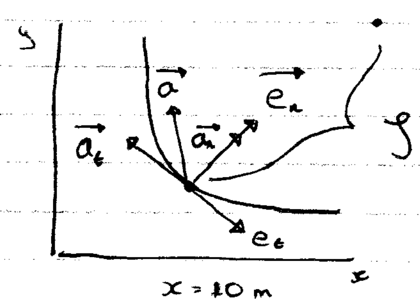


#2 Curve $xy = 20$

$x = 10\text{ m}$, Speed $= 5\text{ m/s}$

decreasing at 1 m/s^2

$a_n, a_t, \vec{a}$



Solution:

$a_n = \dfrac{v^2}{\rho} \Rightarrow \rho$

y', y''

$\therefore y = 20/x$

$$\therefore\ y = \frac{20}{x}$$

y', y''

at $x = 10\,m$, $y' = -0.2$, $y'' = 0.04$

$$\rho = \frac{\left[1 + (y')^2\right]^{3/2}}{|y''|} = 26.52\ m$$

$$\therefore\ a_n = \frac{v^2}{\rho} = \frac{5^2}{26.52} = 0.9429\ \ m/s^2$$

$$a_t = -1\,m/s^2$$

$$\vec{a} = (-1)\vec{e_t} + (0.9429)\vec{e_n} \quad (m/s^2)$$

Sample Problem 11.16

1 mile = 5280 ft, and given $\rho = 2500\ ft$, $v_A = 60$ mi/hr

$v_B = 45$ mi/hr

$$60\ \frac{mile}{hour} \times \frac{1\ hour}{3600\ s} \times \frac{5280\ ft}{1\ mile}$$

60 mile/hour = 88 ft/s

45 mile/hour = 66 ft/s

$$a_n = \frac{v_A^2}{\rho} = \frac{(88)^2}{2500} = 3.098\ \ ft/s^2$$

a_t : $v_A \rightarrow v_B$ at a constant rate

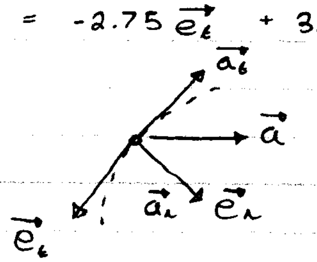
Constant acceleration (as in rectilinear motion)

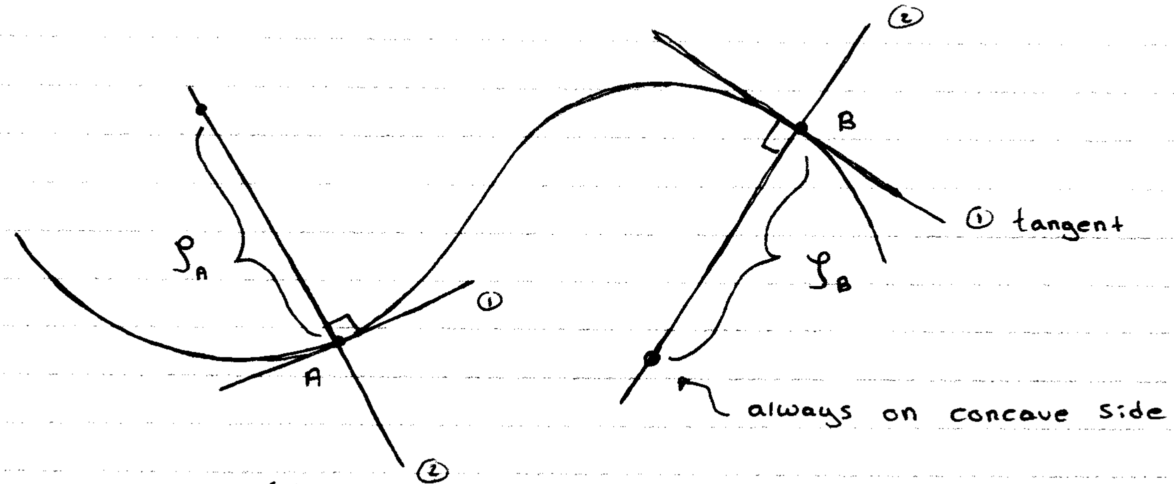
$$v = v_0 + a(t - t_0)$$

$$\therefore\ v_B = v_A + a_t\,(8\,sec)$$

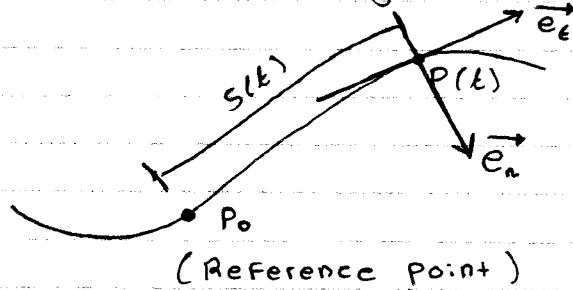
$$\therefore\ a_t = \frac{v_B - v_A}{8\ sec} = -2.75\ ft/s^2$$

$$\therefore\ \vec{a} = -2.75\,\vec{e_t} + 3.098\,\vec{e_n} \quad (ft/s^2)$$





Planar motion (tangential - normal components)



PATH OF MOTION OF PARTICLE

$y = f(x)$

$s(t)$: Position arc-length from $P_o \rightarrow P$

$$\dot{\vec{e}_t} \neq \vec{0}, \quad \dot{\vec{e}_n} \neq \vec{0}$$

$\vec{e}_t$: tangential direction $\left\{ \begin{array}{l} \text{tangent to curve,} \\ \text{increasing } s(t) \end{array} \right.$

$\vec{e}_n$: normal direction $\left\{ \begin{array}{l} \text{normal to tangent} \\ \text{directed towards} \\ \text{centre of curvature} \end{array} \right.$

both are rotating unit vectors

Position : $s(t)$

Velocity : $\vec{v} = v\,\vec{e}_t = \dot{s}\,\vec{e}_t$

acceleration : $\vec{a} = \dot{\vec{v}} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}\left(\dot{s}\,\vec{e}_t\right)$

$$= \frac{d\dot{s}}{dt}\,\vec{e}_t + \dot{s}\,\frac{d\vec{e}_t}{dt}$$

$$= \ddot{s}\,\vec{e}_t + \dot{s}\left(\frac{\dot{s}}{\rho}\,\vec{e}_n\right)$$

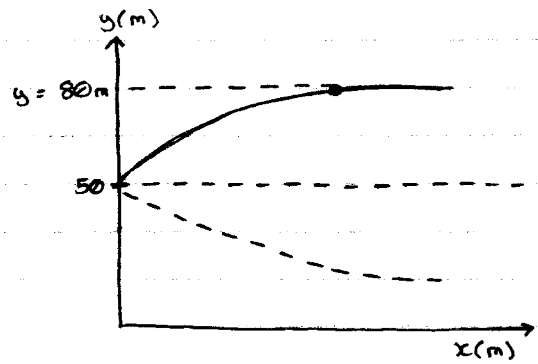
$$= \ddot{s}\,\vec{e}_t + \frac{(\dot{s})^2}{\rho}\,\vec{e}_n$$

$$= \underbrace{\dot{v}\,\vec{e}_t}_{=\vec{a}_t} + \underbrace{\frac{v^2}{\rho}\,\vec{e}_n}_{=\vec{a}_n}$$

FEB. 2/17

1. When a rocket reaches the altitude of 50 m, it begins to travel along the path given by $(y-50)^2 = 140x$, where x and y are in meters. Given that $V_y = 180$ m/s and is constant. Determine the velocity and acceleration of the rocket when it reaches an altitude of $y = 80$ m.

(Both Questions are posted on D2L)



$$(y-50)^2 = 140x$$
$$x = \tfrac{1}{140}(y-50)^2$$
$$\dot{x} = v_x = \frac{d}{dv}\left[\frac{(y-50)^2}{140}\right] = \frac{y-50}{70} \cdot v_y$$

$$a_x = \dot{v}_x = \frac{v_y}{70} \cdot \frac{d}{dy}(y-50) \cdot \frac{dy}{dt} = \frac{v_y^2}{70}$$

At $y = 80$ m, $v_x = \dfrac{80-50}{70}(180) = 77.14$ (m/s)

and $a_x = \dfrac{(180)^2}{70} = 462.9$ (m/s²)

$\therefore \vec{v} = 77.14\,\vec{i} + 180\,\vec{j}$ (m/s)

$\vec{a} = 462.9\,\vec{i}$ (m/s²)

2. $y^2 = 4Hx$ $\quad H \neq 0$

Given: $v_y = ct$ $\quad c \neq 0$

$$a_y = \dot{v}_y = c$$
$$x = \frac{y^2}{4H}$$
$$v_x = \dot{x} = \frac{y}{2H}\dot{y}$$
$$a_x = \dot{v}_x = \frac{(\dot{y})^2}{2H} + \frac{y\ddot{y}}{2H}$$

a_x, a_y in terms of H, c, t, and x

$\dot{y} = v_y = c \cdot t$

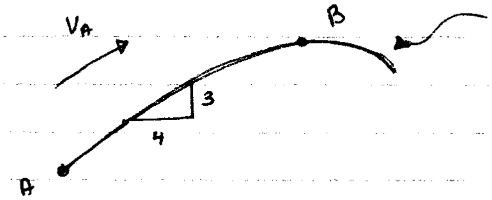
$\ddot{y} = \dot{v}_y = a_y = c$

and: $y = \sqrt{4Hx} = 2\sqrt{Kx}$

$\therefore a_x = \dfrac{c}{2H}(ct^2 + 2\sqrt{Kx})$

$a_y = c$

From the textbook: Problem 11.148

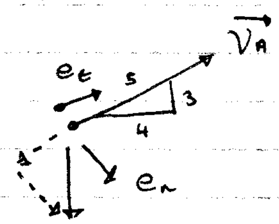


Given: $\rho_A = 25\,m$

Find: v_A , ρ_B

Solution

Pt. A



$\vec{a} = -9.81\,\vec{j}$

$\vec{v} = v\,\vec{e}_t$

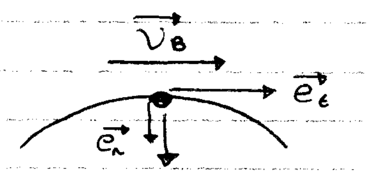
$(a_n)_A = 7.848\ m/s^2$

but $(a_n)_A = \dfrac{v_A^{\,2}}{\rho_A}$

$\therefore\ v_A = 14.01\ m/s$

Pt. B

$(a_n)_B = \dfrac{v_B^{\,2}}{\rho_B}$



$\vec{a} = -9.81\,\vec{j}$

$\therefore\ (a_n)_B = 9.81$

$v_B = (v_A)_x = 11.21\ m/s$

$\therefore\ \rho_B = v_B^{\,2}/(a_n)_B = 12.80\ m$