(From previous example):

(2) Acceleration Analysis

P on AP

$$\vec{a}_P = \vec{\alpha}_{AP} \times \vec{r}_{P/A} - \omega_{AP}^2 \vec{r}_{P/A}$$
$$= -4.322\vec{i} - 5.074\vec{j} \ (m/s^2)$$

P as slider w.r.t. BD

$$\vec{a}_P = \cancel{\vec{a}_o}^{\,0} + \vec{\dot{\Omega}} \times \vec{r} - \Omega^2 \vec{r} + 2\vec{\Omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

$\dot{\Omega}$ ↺ $\vec{\dot{\Omega}} = \dot{\Omega}\vec{k}$,

$\vec{a}_{rel} = a_{rel}\vec{i}$ ⟶

$$\vec{a}_P = \vec{0} + (\dot{\Omega}\vec{k}) \times (0.1195\vec{i}) - (7.864)^2(0.1195\vec{i}) + 2(7.864\vec{k}) \times (-0.9397\vec{i}) + a_{rel}\vec{i}$$

Solving: $\dot{\Omega} = 81.22 \ rad/s^2$ ↺

$a_{rel} = 3.068 \ m/s^2$ ⟶

Problem 15.176 (*)

Chapter 9 - Distributed Forces; Moments of Inertia

Mass moments of inertia:

by §9.11 ~ §9.18 :

Moment of Inertia of a mass (or rigid body)

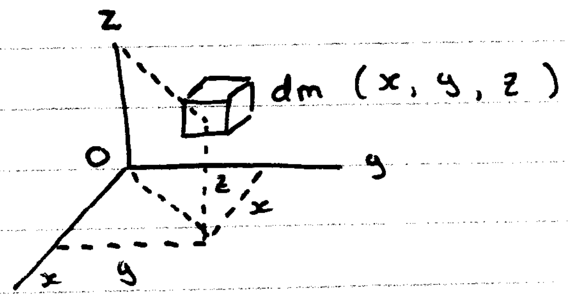
1) Mass : the resistance to being accelerated by forces $\Sigma\vec{F} = m\vec{a}$

moment of inertia of mass, or mass moment of inertia: the resistance to rotational acceleration about an axis;

the higher the mass moment of inertia, the higher the resistance;

rotary mass;

2) Definition :



$$I_x = \int_m (y^2 + z^2)\, dm \longrightarrow (\text{distance to } x)^2$$

$$I_y = \int_m (x^2 + z^2)\, dm \longrightarrow (\text{distance to } y)^2$$

$$I_z = \int_m (y^2 + x^2)\, dm \longrightarrow (\text{distance to } z)^2$$

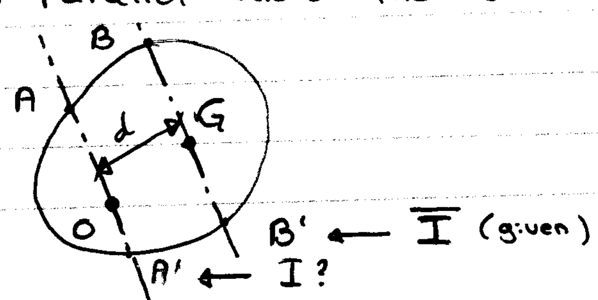
I_x, I_y, I_z : moment of inertia about (w.r.t.) x, y, z-axis respectively.

Units : $\begin{cases} kg \cdot m^2 \\ slug \cdot ft^2, \ blob \cdot in^2 \end{cases}$

(*) moments of inertia such as I_x, I_y, I_z are positive.

(*) moments of inertia depend on the orientations of the axes about which moments are taken

3) Parallel-axis theorem



G : centre of gravity

O : arbitrary point

→ BB' passes through G

moment of inertia about BB' is $\bar{I}$

→ AA' passes through O

moment of inertia about AA' is I

→ AA' // BB', distance between the two lines is d

then,

$$I = \bar{I} + md^2$$

(*) moments of inertia depend on the location of axes about which moments are taken.

4) Radius of Gyration k

Given that I is the moment of Inertia about a certain axis that passes through a certain point, then:

$$k = \sqrt{I/m} \qquad (m\text{: total mass})$$

is the radius of gyration about the same axis.

units: $\begin{cases} \text{meter} \\ \text{ft, in} \end{cases}$

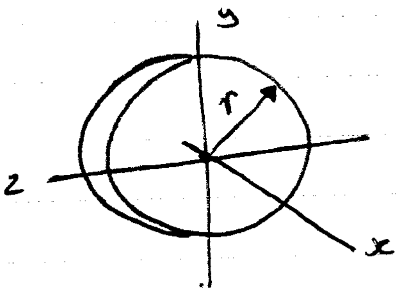
→ For complex shapes, k will be given such that I can be easily determined.

Back to parallel axis theorem $I = \bar{I} + md^2$

$$k^2 = \bar{k}^2 + d^2$$

k : radius of gyration about AA'

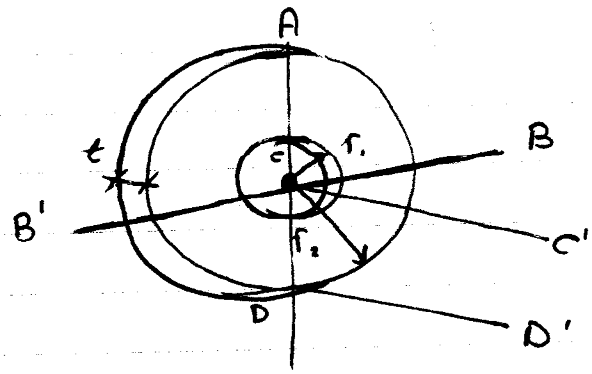
$\bar{k}$: radius of gyration about BB'



$$I_x = \tfrac{1}{2} m r^2$$
$$I_y = I_z = \tfrac{1}{4} m r^2$$

$0.0078 \ g/mm^3$

$\rho = 7800 \ kg/m^3$
$t = 10 \ mm$
$r_1 = 250 \ mm$
$r_2 = 500 \ mm$



P9.112

(1) masses :

$$m_1 = 0.0078(\pi (250)^2 \cdot 10)$$
$$= 15.315 \ kg$$
$$m_2 = 0.078(\pi (500)^2 \cdot 10)$$
$$= 61.261 \ kg$$

(2) about AA'

$$I_{AA'} = \tfrac{1}{4}(61.261)(0.5)^2 - \tfrac{1}{4}(15.315)(0.25)^2$$
$$= 3.829 - 0.2393 = 3.590 \ kg \cdot m^2$$

(3) About CC'

$$I_{cc'} = \tfrac{1}{2}(61.261)(0.5)^2 - \tfrac{1}{2}(15.315)(0.25)^2$$
$$= 7.658 - 0.4786 = 7.179 \ kg \cdot m^2$$

(4) About DD'

CC' is centroidal axis

$$\therefore I_{DD'} = I_{cc'} + md^2$$
$$I_{DD'} = 7.179 + (m_2 - m_1) r_2^2$$
$$= 7.179 + 11.487$$
$$= 18.67 \ kg \cdot m^2$$

Chapter 16 - Plane Motions of Rigid Bodies : Forces and Accelerations

Introduction

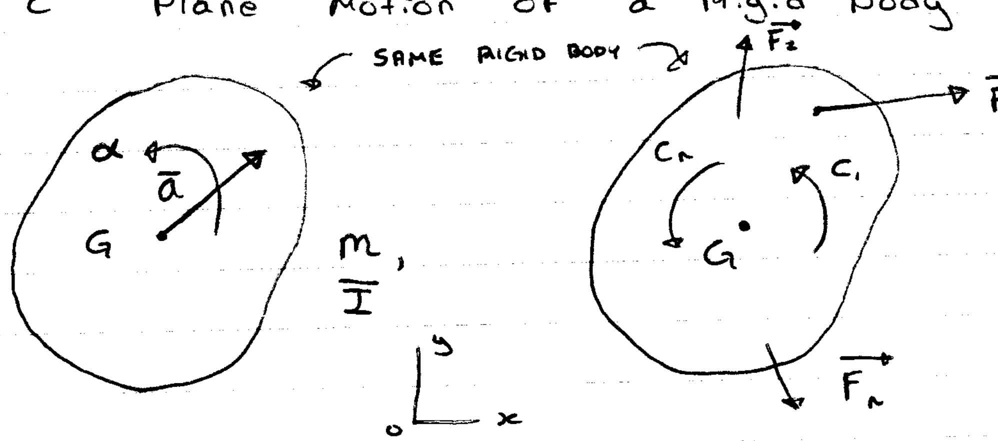
§16.1 Kinetics of a Rigid Body

SAMPLE PROBS

§16.1 : 16.1, 16.3 - 16.5

§16.2 : 16.8 - 10, 16.12, 16.13

16.1 C Plane Motion of a Rigid Body



$\bar{a}$: acceleration vector at G, the center of mass

α : angular acceleration of the rigid body

m : mass of the rigid body

$\bar{I}$: mass moment of inertia of the rigid body about the axis perpendicular to the plane of motion, and passing through G

then the equations are

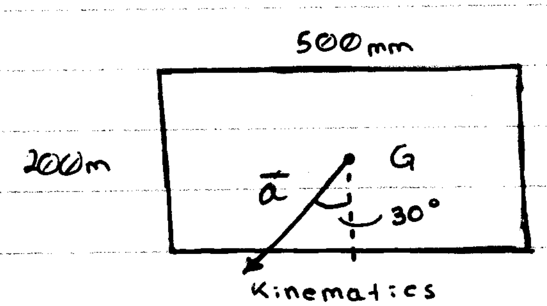
$$\begin{cases} \Sigma F_x = m\bar{a}_x \\ \Sigma F_y = m\bar{a}_y \\ \Sigma M_G = \bar{I}\alpha \end{cases}$$

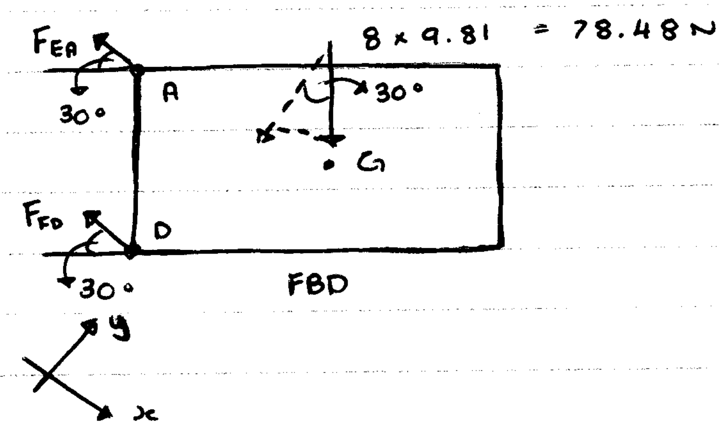
where ΣM_G is sum of moments about G, due to $\vec{F_i}$, and of applied couples.

§16.2 Constrained Plane Motion

1) Most engineering applications involve rigid bodies connected in some manners to achieve desired motion.
2) Friction needs to be dealt with;
3) rolling without slip requires specific conditions; the case of rolling with slip is a kinetics problem.

SAMPLE PROBLEM 16.3 (curvilinear translation)





$\alpha = 0$

$\bar{I}$ not needed

$$\Sigma F_x = m\bar{a}_x$$

$$-F_{EA} - F_{FD} + 78.48 \sin 30° = 80(0) = 0 \qquad (1)$$

$$\Sigma F_y = m\bar{a}_y$$

$$-78.48 \cos 30° = (80)(-\bar{a})$$

$$\therefore \ \bar{a} = 8.496 \ (m/s^2)$$

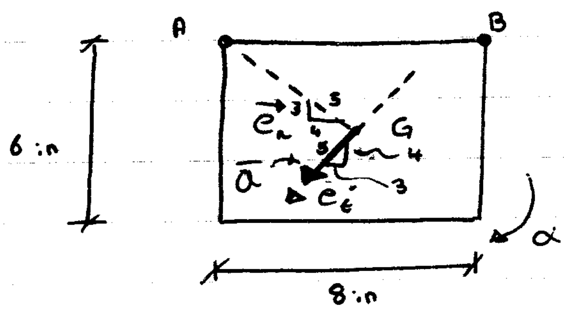
$$\Sigma M_G = \bar{I}\alpha = 0$$

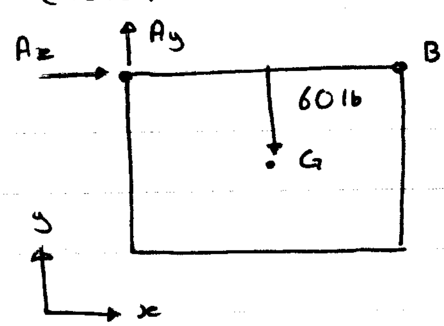
$$-0.03840 F_{EA} - 0.2116 F_{FD} = 0 \qquad (*)$$

$$(1) + (2): \quad F_{EA} = 47.94 \ N \ (T)$$

$$F_{FD} = 8.699 \ N \ (C)$$

SAMPLE PROB 16.8 (rotation)





$$\bar{a} = 5\alpha$$

$$\Sigma F_x = m\bar{a}_x$$

$$\hookrightarrow A_x = \left(\frac{60}{386}\right)\left(\frac{-3}{5}\bar{a}\right) = \left(\frac{60}{386}\right)\left(-\frac{3}{5} 5\alpha\right)$$

$$\therefore \ A_x = -0.4663\,\alpha \qquad (1)$$

$$\Sigma F_y = m\bar{a}_y$$

$$\hookrightarrow A_y - 60 = \left(\frac{60}{386}\right)\left(-\frac{4}{5}\bar{a}\right) = \left(\frac{60}{386}\right)\left(-\frac{4}{5} 5\alpha\right)$$

$$\therefore \ A_y - 60 = -0.6218\,\alpha \qquad (2)$$

$$\overset{+}{\circlearrowright} \sum M_G = \bar{I}\alpha$$

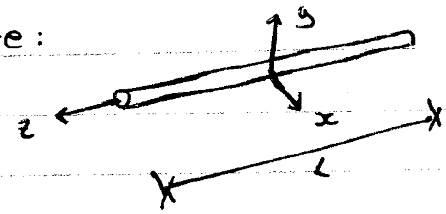
$$3A_x + 4A_y = 1.295 \cdot \alpha \qquad (3)$$

Solving : $\alpha = 46.32 \text{ rad/s}^2 \circlearrowright$

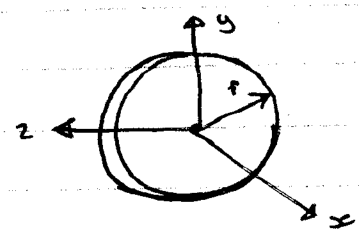
$A_x = 21.60 \text{ lb} \leftarrow$

$A_y = 31.20 \text{ lb} \uparrow$

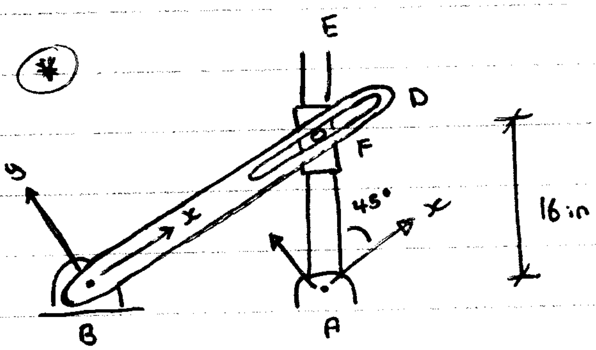
Note :



$I_x = \frac{1}{12} mL^2$



$I_x = \frac{1}{2} mr^2$



Locate Oxy

$\vec{v}_p$, $\vec{a}_p$

Solution:

$\vec{v}_p = 4388\,\vec{i}$ (in/s)

$\vec{a}_p = 407.3\,\vec{i} - 1222\,\vec{j}$ (in/s²)

(1) P w.r.t. AE

$\Omega = 6$ rad/s ↻

$\dot{\Omega} = \emptyset$

$v_{rel} = 8$ Ft/s ↑

$a_{rel} = \emptyset$

(2) P w.r.t. BD

Velocity analysis, $\Omega_{BD} = \emptyset$, $v_{rel} = 4388$ in/s

acceleration analysis, $\dot{\Omega}_{BD} = 54$ rad/s² ↻

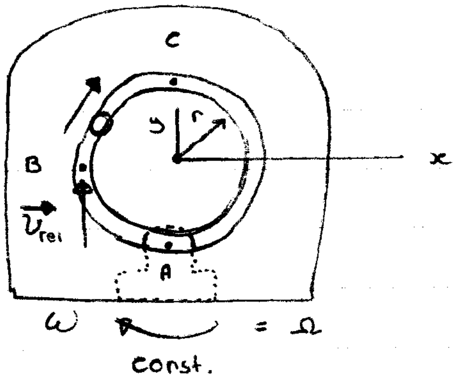
$a_{rel} = 407.3$ in/s² ↗

$$\vec{v}_p = \vec{v}_o + \vec{\Omega} \times \vec{r} + \vec{v}_{rel}$$

$$\vec{a}_p = \vec{a}_o + \dot{\vec{\Omega}} \times \vec{r} - \Omega^2 \vec{r} + 2\vec{\Omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

O : base point ;

$\vec{v}_o$, $\vec{a}_o$ may not be zero.



$\vec{v}_o \neq \vec{c}$, $\vec{a}_o \neq 0$

Ω = const

u = const

$\vec{a}_o = -900\,\vec{j}$ (mm/s²)

P at B

$$\vec{a}_P = \vec{a}_o + \cancel{\dot{\vec{\Omega}}}^{\,0} \times \vec{r} - \Omega^2 \vec{r} \qquad (\vec{r}_{P/O})$$

$$+ 2\vec{\Omega} \times \vec{v}_{rel} + \vec{a}_{rel} \qquad \left(a_n = \frac{v^2}{\rho}\right)$$

$$= (-900\vec{j}) + 0 - (-3)^2(-100\vec{i})$$

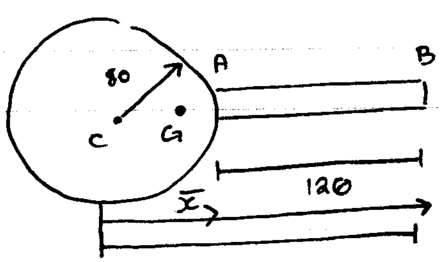
$$+ 2(-3\vec{k}) \times (90\vec{j})$$

$$- \frac{(90)^2}{100}\vec{i}$$

$$= 1521\,\vec{i} - 900\,\vec{j} \quad (\text{mm/s}^2)$$

At A : $\vec{a}_P = 621\,\vec{j}$ (mm/s²)

C : $\vec{a}_P = -2420\,\vec{j}$ (mm/s²)



Disc : 5kg, 80mm radius

Bar : 1.5kg, 120mm length

Find $\bar{I}$

Sol'n : $\bar{x} = 0.03231$ m

Disk : $I_1 = \frac{1}{2}(5)(0.08)^2 + (5)(\bar{x})^2$

$= 0.02122$ (kg·m²)

Bar : $I_2 = \frac{1}{12}(1.5)(0.12)^2 + (1.5)(0.14 - \bar{x}^2)^2$

$= 0.01920$ (kg·m²)

$\therefore \bar{I} = I_1 + I_2$

$= 0.04042$ kg·m²