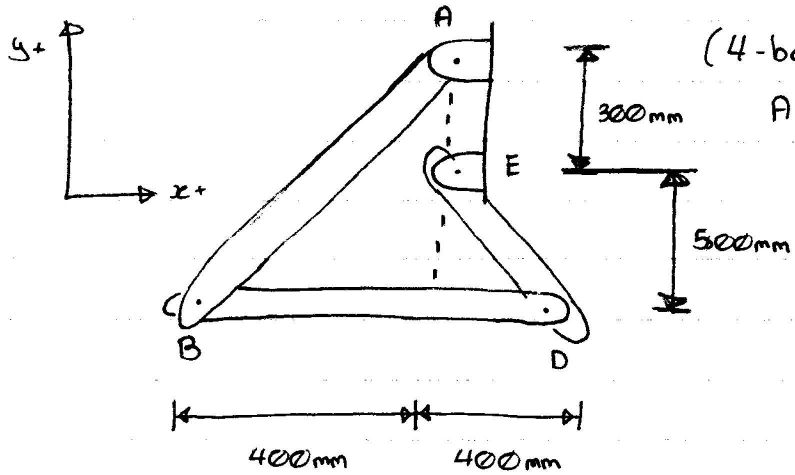
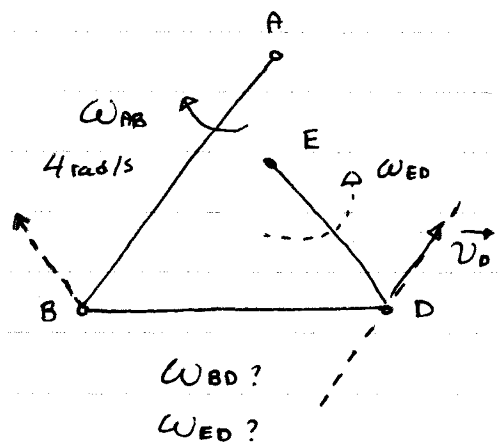
In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.



(4-bar linkage : AB, BD, DE and AE - Fixed bar linkage).

BAR AB, DE : Fixed-axis rot

BAR BD : General motion



$r_{B/A} \Rightarrow -0.4\vec{i} - 0.8\vec{j}$ (m)

$r_{B/D} \Rightarrow 0.8\vec{i}$ (m)

$r_{D/E} \Rightarrow 0.4\vec{i} - 0.5\vec{j}$ (m)

B on AB
$$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A} = (-4\vec{k}) \times (-0.4\vec{i} - 0.8\vec{j}) = -3.2\vec{i} + 1.6\vec{j} \ (m/s)$$

(Assume ccw ω_{BD}, ω_{ED})

D on ED
$$\vec{v}_D = \vec{\omega}_{ED} \times \vec{r}_{D/E} = (\omega_{ED}\vec{k}) \times (0.4\vec{i} - 0.5\vec{j}) = 0.5\,\omega_{ED}\vec{i} + 0.4\vec{j}$$

(For pin connection, same velocity)

Rigid Body BD:
$$\vec{v}_D = \vec{v}_B + \vec{\omega}_{BD} \times r_{BD}$$

$$\therefore\ 0.5\,\omega_{ED}\,\vec{i} + 0.4\,\omega_{ED}\,\vec{j}$$
$$= -3.2\vec{i} + 1.6\vec{j} + (\omega_{BD}\vec{k}) \times (0.8\vec{i})$$
$$= -3.2\vec{i} + 1.6\vec{j} + 0.8\,\omega_{BD}\,\vec{j}$$

$$\therefore \begin{cases} 0.5\,\omega_{ED} = -3.2 \\ 0.4\,\omega_{ED} = 1.6 + 0.8\,\omega_{BD} \end{cases}$$

$\therefore\ \omega_{ED} = -6.4$ (should be cw, NOT ccw)

$\therefore\ \omega_{BD} = -5.2$ (rad/s)

SAMPLE PROBLEMS GIVEN IN THE TEXTBOOK:

$\boxed{15.5}$ $\vec{v}_B = v_B(\cos 60^\circ\,\vec{i} + \sin 60^\circ\,\vec{j})$

$\vec{v}_D = \vec{v}_A + \vec{\omega} \times \vec{r}_{D/A}$

$\boxed{15.6}$ C is $v = 0$

$\boxed{15.7}$ β = law of sines

$\boxed{15.8}$ ~

§ 15.3 - Instantaneous Center of Rotation (Also known as the instantaneous center of zero velocity, labeled by C, or IC).

Why instantaneous center?

general motion = translation with A + rotation about A (as if A were fixed)

or $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

If A is chosen such that $\vec{v}_A = \vec{0}$ at the given time instant, or given position/configuration of the rigid body, then general motion is simply rotation about a point whose velocity is zero at the instant under consideration.

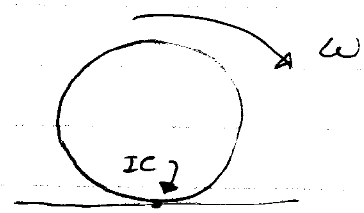
An instantaneous center is the point of a rigid body whose velocity is zero at the given instant / Position / configuration.

"instantaneous" to emphasize that velocity at the point is zero only at the given instant.

IC can be regarded as the axis of rotation at the given instant.

When known, IC can/should be chosen as the base point.

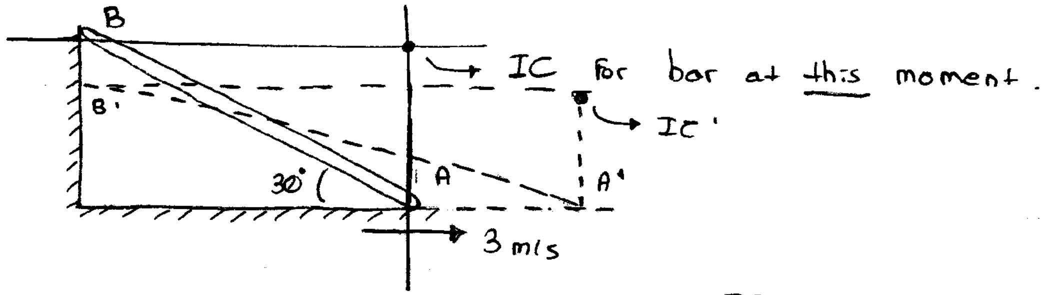
IC can be located by inspection, or by construction.

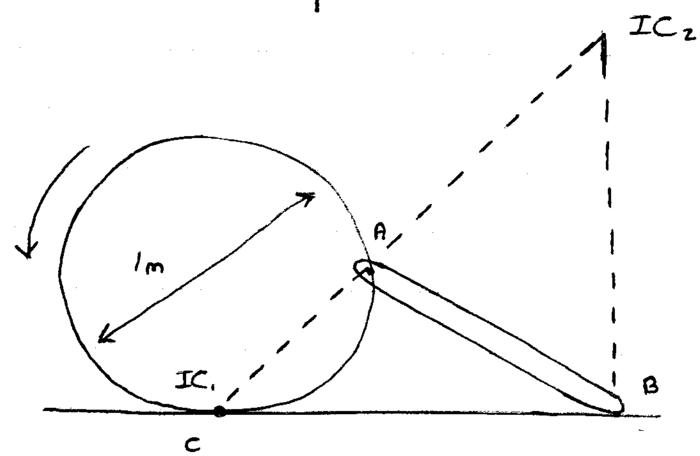


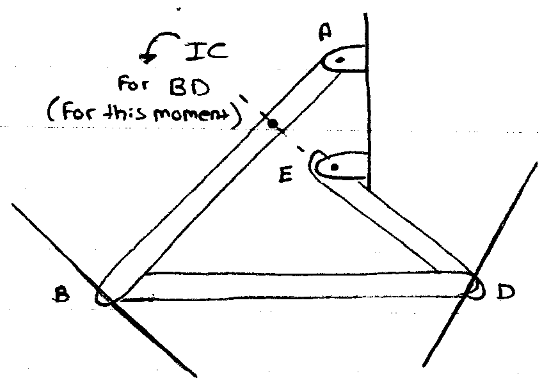
How to determine the location of IC
2 Points/particles whose lines of action of velocity are known, and non-parallel;
At each point, draw a line that is normal to the line of action of velocity, and passes through the point;

The intersecting point gives the location of IC;

It can be located beyond the physical dimensions of the rigid body.







Note: Member DE always rotates about E

Member BA always rotates about A

BD rotates about IC at this instant in time <u>only</u>

How to determine the location of IC

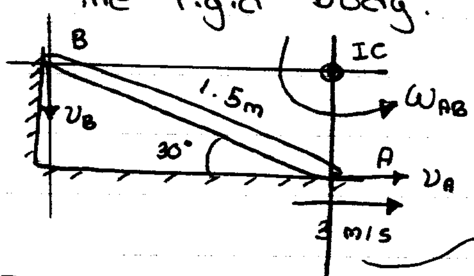
2 points/particles whose line of action of velocity are known, and non-parallel.

At each point, draw a line that is normal to the line of action of velocity, and passes through the point;

The intersecting point gives the location of IC;

It can be located beyond the physical dimensions of the rigid body.

1



ω_{AB} ?

v_B ?

"normal"

(draw two lines perpendicular to motion)

Solution :

(1) Locating the IC.

(2) $r_{A/IC} = 0.75$ m

$r_{B/IC} = 1.299$ m

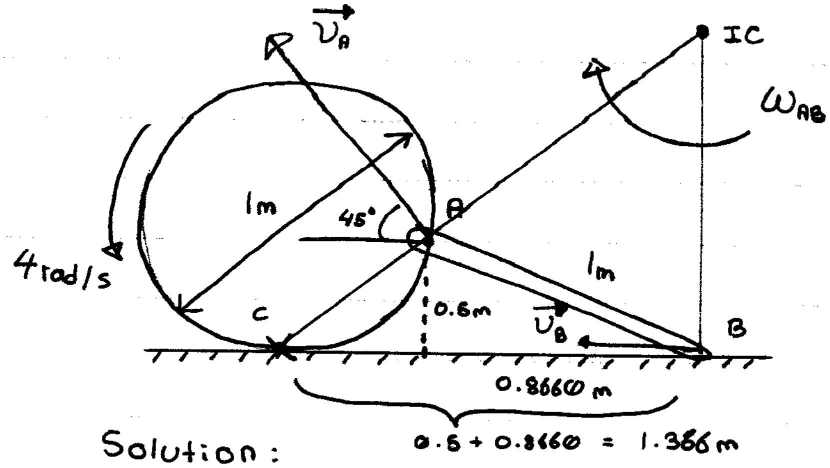
$\therefore v_A = r_{A/IC} \cdot \omega_{AB}$

$\therefore \omega_{AB} = v_A / r_{A/IC} = 4$ rad/s ↺

and $v_B = r_{B/IC} \cdot \omega_{AB}$

$v_B = (1.299)(4) = 5.196$ m/s ↓

WILL BE ONE QUESTION ON FINAL USING IC APPROACH. *



ω_{AB} ?

v_B ?

Solution:

(1) Disk : A on Disk

$$v_A = r_{A/C} \cdot \omega_{disc}$$
$$= (0.5)\sqrt{2} \cdot 4$$
$$= 2.828 \text{ m/s}$$

(2) Bar AB :

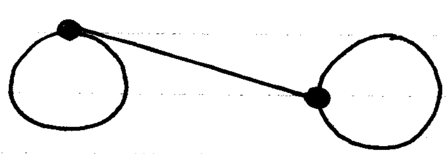
$$r_{A/IC} = \sqrt{2}(1.366) - \sqrt{2}(0.5) = 1.225 \text{ m}$$
$$r_{B/IC} = 0.5 + 0.8660 = 1.366 \text{ m}$$

A on AB : $v_A = r_{A/IC} \cdot \omega_{AB}$

$$\omega_{AB} = \frac{2.828}{1.225} \Rightarrow 2.309 \text{ rad/s}$$

and $v_B = r_{B/IC} \cdot \omega_{AB} = (1.366)(2.309)$

$$= 3.154 \text{ m/s} \leftarrow$$

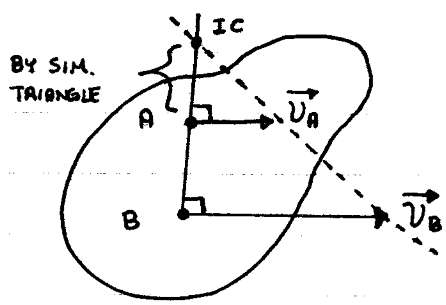


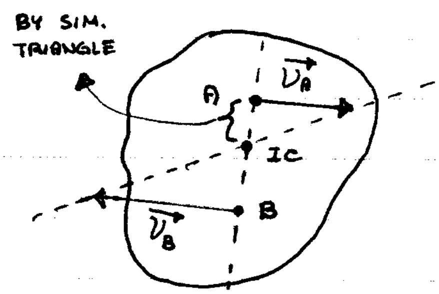
How to determine the location of IC
(when velocities at 2 points are parallel)

Requirements:

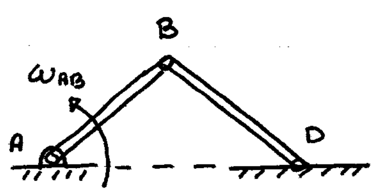
- v_A, v_B known
- the line connecting A and B is perpendicular to the velocities

At each point, draw the velocity vector, preferably to scale;
Connect the tips of velocity vectors by a straight line;
The intersecting point of the straight line and AB gives the location of IC.

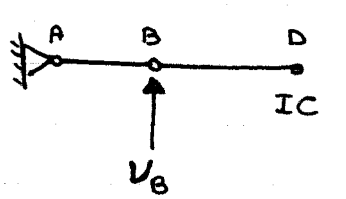


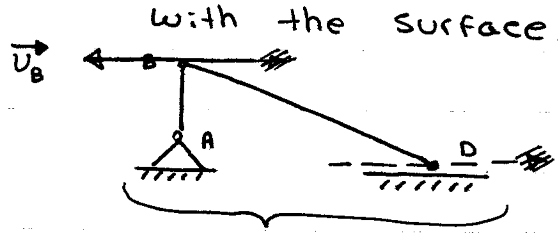


Example:



D slides over the horizontal surface and stays in contact with the surface.

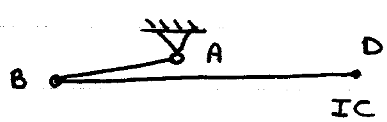


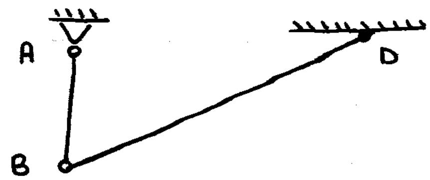


Instantaneous translation

$\therefore \ \omega_{BD} = 0$

$\vec{v}_D = \vec{v}_B$





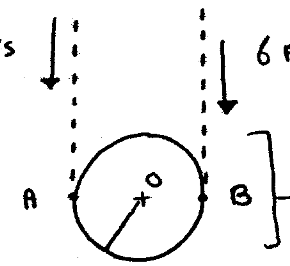
instantaneous translation

$\therefore \quad \omega_{BD} = 0$

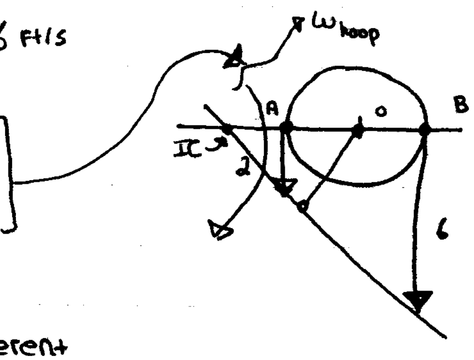
$\vec{v}_D = \vec{v}_B$

(Identify IC)

1



The Hoop
(Note two different ropes are wrapped around the hoop at A and B)



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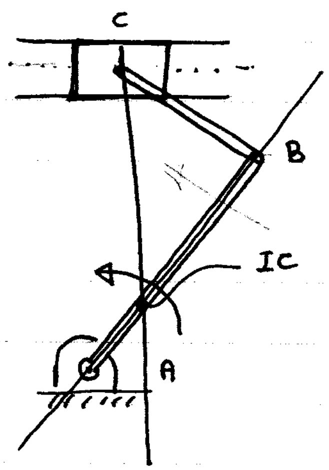

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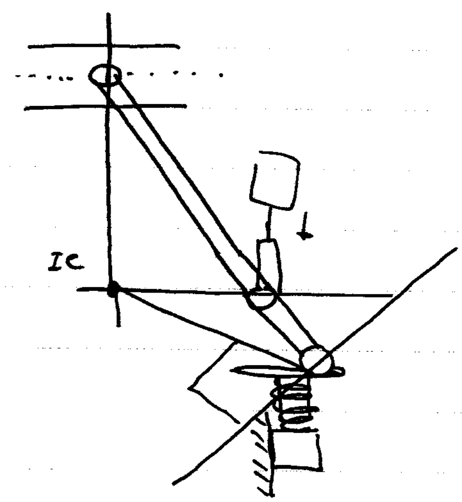

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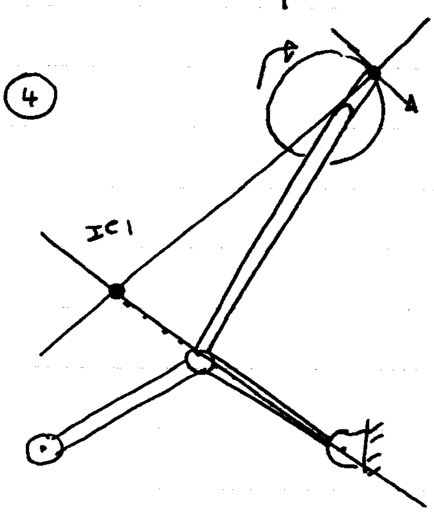

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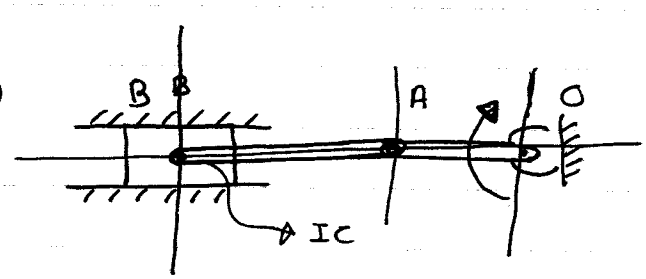
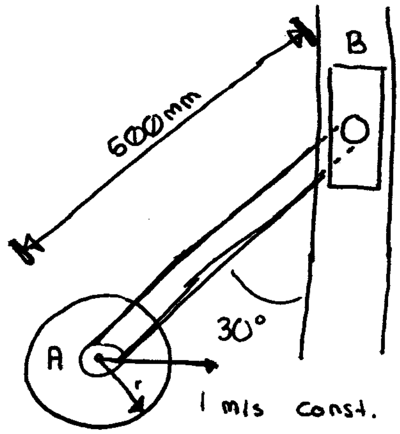




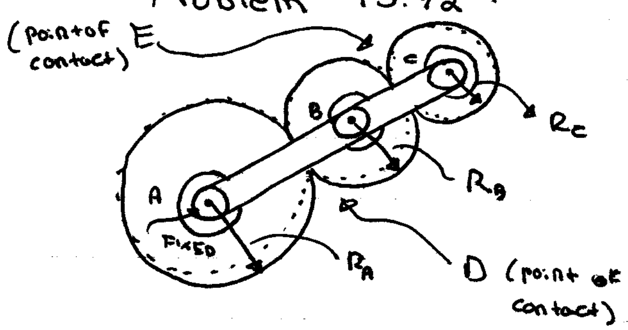
1) $\omega_{DISK} = 10$ rad/s

2) $\omega_{AB} = 2.309$ rad/s

3) $\alpha_{DISC} = 0$

4) $\alpha_{AB} = 3.706$ rad/s^2

Problem 15.72 :



A is Fixed axis of rotation of Gear A and Arm ABC

Gear A : Rotates CW, 40 rad/s

~~Gear~~ Arm ABC : rotates CCW, 30 rad/s

$r_A = 0.25$ m, $r_B = 0.15$ m,

$r_C = 0.1$ m

Determine ω_B and ω_C

Solution :

(1) Gear B

at center of B

$v_B = r_{B/A} \cdot \omega_{ABC}$

$= (0.4)(30)$

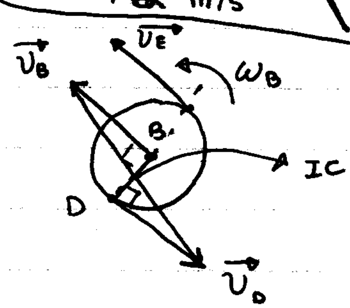
$= 12$ m/s

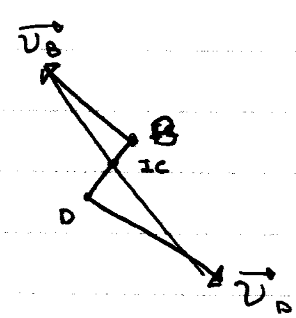
At contact point D :

$v_D = r_A \omega_A$

$= (0.25)(40)$

$= 10$ m/s





LET $r_{B/IC} = x$

$$\frac{12}{x} = \frac{10}{0.15 - x}$$

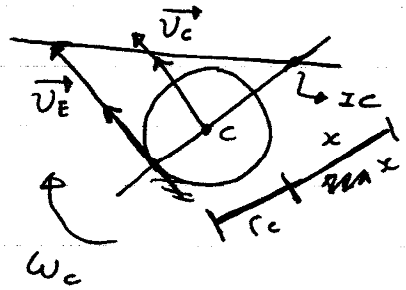
$\therefore x = 0.08182$ m

$\therefore\ v_B = x \cdot \omega_B \cdot$

$\therefore\ \omega_B = 146.7 \text{ rad/s}$

and: $v_E = (0.15 + x) \cdot \omega_B = 34 \text{ m/s}$

(2) Gear C



at contact point E:

$v_E = 34 \text{ m/s}$

at center of C:

$v_c = r_{B/A} \cdot \omega_{ABC}$

$x = 0.24\text{m}$

$\therefore\ v_c = x \cdot \omega_c$

$\therefore\ \omega_c = 100 \text{ rad/s}$

$r_A + r_B + r_B + r_c \cdot \omega_{ABC}$

$= 24 \text{ m/s}$