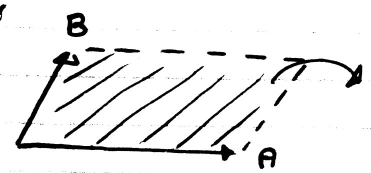
Vectors (Review)

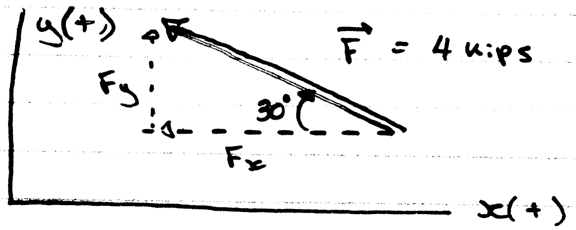
1) Given vectors $\vec{A}$ and $\vec{B}$, determine graphically
$\vec{A} \pm \vec{B}$
$a\vec{A}$ when $a > 0$, $a < 0$, and $a = 0$

2) Determine $\vec{A} \times \vec{B}$



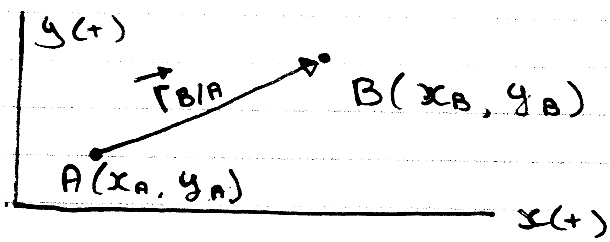
(magnitude of two vectors)

3) Express the following vectors in rectangular components



$\vec{F}$ = 4 kips
$F_x = \cos(30°)4 \Rightarrow -3.464\vec{i}$
$F_y = \sin(30°)4 \Rightarrow 2\vec{j}$

$\vec{F} = -3.464\vec{i} + 2\vec{j}$ kips

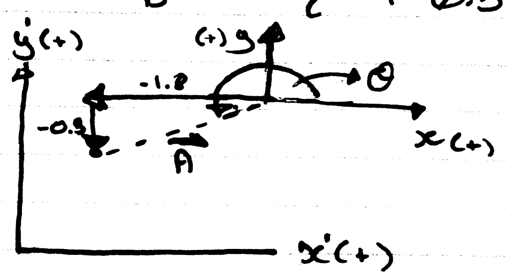


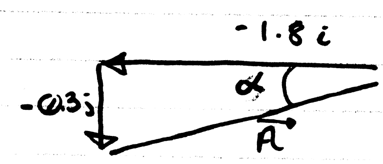
(x_A, y_A) known
(x_B, y_B)

$r_{B/A} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j}$

4) Visualize (Draw) the following vectors, and determine the angles that the vectors make with respect to the x-axis.

$\vec{A} = -1.8\vec{i} - 0.3\vec{j}$
$\vec{B} = -\vec{i} + 0.5\vec{j}$





$\alpha = 9.5$
$\theta = \alpha + 180°$
$\theta = 189.5°$

5) For the $\vec{A}$ and $\vec{B}$ vectors given in 4), evaluate $\vec{A} \cdot \vec{B}$, and $\vec{A} \times \vec{B}$

$$\vec{A} \cdot \vec{B} = (-1.8 \times -1) + (-0.3 \times 0.5)$$
$$= 1.65$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j \\ -1.8 & -0.3 \\ -1 & 0.5 \end{vmatrix} = -1.2\,\vec{k} \quad \text{(review)}$$

6) What is the unit vector associated with $\vec{B}$ in 4)

$$\vec{B} = -\vec{i} + 0.5\,j \Rightarrow \|B\| = 1.118$$
$$u_B = \frac{-\vec{i} + 0.5\vec{j}}{1.118}$$
$$u_B = -0.894\,\vec{i} + 0.447\,\vec{j}$$

7) What are the unit vectors associated with co-ordinate axes $x-y-z$?

↳ i, j, and k

Statics (Review)

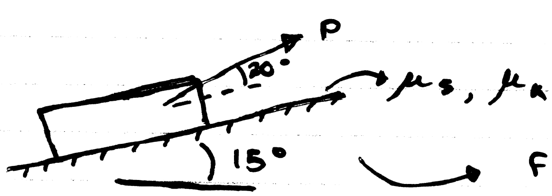
FBD : Part of Problem Solving in Ch. 12 and Ch. 16

Friction : present in the real world

Forces in components : For ease in problem solving

Example :

The coefficients of static and kinetic friction between the 100 kg block and inclined surface are 0.3 and 0.2, respectively. If P = 700 N, would the block be stationary, or in motion?



→ Finish for tutorial

Time-differentiation and Integration

Time-differentiation means to differentiate with respect to time t.

For example, $x = \cos t$, then

$$dx/dt = -\sin t$$

Time differentiation involving composition of Functions, or application of chain rule.

(1) Example : $y = x^2$, $x = \cos t$, $dy/dt = ?$

(2) Example : $z = (e^{\sin t})^2$, $dz/dt = ?$

(3) Example : $z = y^2$, $y = e^x$, $x = x(t)$, $dz/dt = ?$

Techniques of Differentiation

Implicit differentiation in particular

Example (1) = $y = (\cos t)^2$

$$dy/dt = -\sin(2t)$$

Example (2) = $z = (e^{\sin t})^2$

$$dz/dt = 2\cos t \underbrace{e^{2\sin t}}_{(e^{\sin t})^2}$$

Example (3) =

$$\frac{dz}{dt} = \frac{dz}{dy} \cdot \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$= 2y \cdot e^x \cdot \frac{dx}{dt}$$

$$= 2e^{x(t)} \cdot e^{x(t)} \cdot \frac{dx}{dt}$$

$$= 2e^{2x(t)} \cdot \frac{dx}{dt}$$

Two types of integrals

$\begin{cases} \text{indefinite integrals, or anti-derivatives: } \int f(x)\,dx \\ \text{definite integrals: } \int_a^b f(x)\,dx \end{cases}$

1) Indefinite Integrals

a) if $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$ where C is any constant

b) indefinite integral of $f(x)$ is a function and answers the question, "what function, when differentiated gives $f(x)$?"

c) some basic indefinite integrals

Power functions : $f(x) = x^n \quad (n \neq -1)$

Polynomials : $f(x) = P_0 + P_1 x + \dots P_n x^n$

Trig functions : $f(x) = \sin x$, or $\cos x$

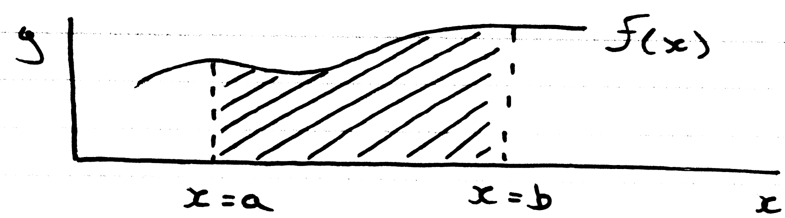
Exponential func : $f(x) = e^x$

and : $\int \frac{1}{x} dx = \ln|x| + C$

remember absolute signs

2) Definite Integrals

a) definite integral $\int_a^b f(x)dx$ is a number, and represents the area under the curve $f(x)$ from $x = a$ to $x = b$



b) "a" and "b" are called the lower and upper limit of integration, respectively.

c) By FTC, $\int_a^b f(x)dx = F(b) - F(a)$

d) The limit a or b can be a variable; as a result, definite integral gives a function.

$$\int_a^x f(x)dx = F(x) - F(a)$$

Ch. 11 Kinematics of Particles

Organization of Chapter 11

Rectilinear motion of a particle

§ 11.1 ~ § 11.3

Focus on only § 11.1, § 11.2A + B, § 11.3

Curvilinear motion of a particle

§ 11.4 ~ § 11.5

Rectangular components

Tangential + normal components

Radial + Transverse components

Introduction (P. 616)

Dynamics includes two branches

1. Kinematics

Study of the geometry of motion, such as displacement, velocity, acceleration in relation to time, without reference to the cause of motion.

2. Kinetics

Study of the relation between forces acting on an object, the mass of the object and the motion of the object. The object can be a particle (Ch. 12) or a rigid body (Ch. 16)

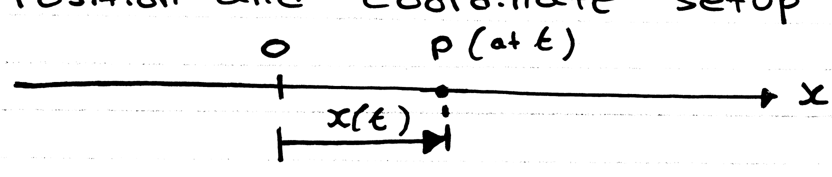
§ Ch. 11.1 Rectilinear Motion of Particles

11.1A Position, Velocity, and Acceleration

1. Rectilinear Motion

↳ A Particle is said to be in rectilinear motion if it moves along a straight line.

2. Position and Coordinate setup



(From previous):
x-axis:
the straight line along which the particle is moving

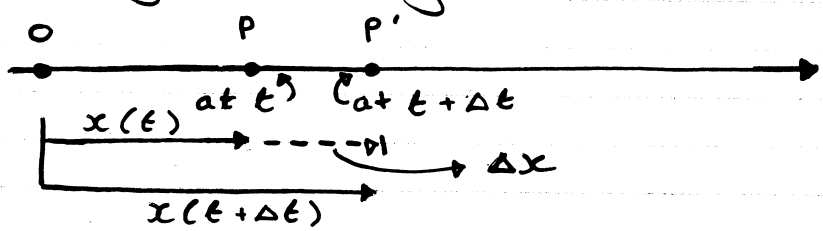
origin O:
Fixed on the straight line

Position of particle at time t
by position coordinate $x(t)$

For example, $\underline{x(t) = 6t^2 - t^3 \ (m)}$
units: time t: seconds, or s
x: SI: meter, or m
US customary: Foot, or ft
inch, or in

$x(t) = (6\,m/s^2)\cdot t^2 - (1\,m/s^3)\,t^3$

3. Average velocity and instantaneous velocity



Average velocity $= \dfrac{\Delta x}{\Delta t}$

Instantaneous velocity (or simply velocity)
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

1) Units: m/s
ft/s or in/s

2) $v > 0$: Particle moves in positive direction
< 0: Particle moves in negative direction

3) Speed $= |v|$ magnitude of velocity

4) irreversible motion : one in which the velocity does not change the sign

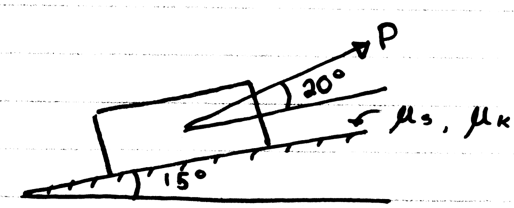
reversible motion : one in which the velocity changes sign, at least once

5) For reversible motion to occur, $v = 0$ must be true, at least once.

JAN. 12 /17

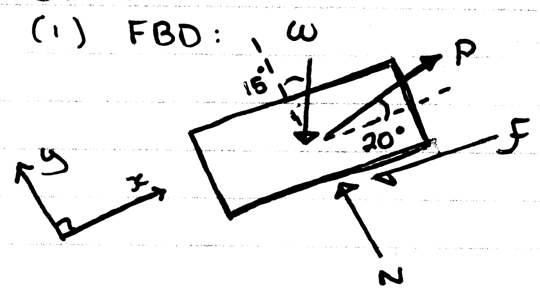
EXAMPLE:

The coefficients of static and kinetic friction between the 100-kg block and the inclined surface are 0.3 and 0.2, respectively. If $P = 700\,N$, would the block be stationary, or in motion?



Solution:

(1) FBD:



$w = 100\ kg \cdot 9.81\ m/s^2$

$w = 981\ N$

(2) Assume the block is stationary:

$P = 700\ N$

Unknowns: $N?$, $f?$

$\Sigma F_x = 700 \cdot (\cos 20°) - 981 \cdot (\sin 15°) - f = 0$

$\therefore f = 403.88\ N$

$\Sigma F_y = N - 981 \cdot (\cos 15°) + 700 \cdot (\sin 20°)$

$\therefore N = 708.16\ N$

$f/N > 0.3$

$\therefore$ This assumption is not true.

(3) Overcoming impending motion:

$f = f_{max} = 0.3 \cdot N$

Unknown: P? N?

Solving leads to: $P = 516.5$ (N)

$N = 771.0$ (N)

∴ block is in motion

From example 3: (Previous notes)

$$\frac{dz}{dt} = 2e^{2x(t)}\frac{dx}{dt}$$

? - by product rule, chain rule

$$\frac{d^2z}{dt^2} \Rightarrow 2\left[\frac{d}{dt}\left(e^{2x(t)}\right)\cdot\frac{dx}{dt} + e^{2x(t)}\cdot\frac{d}{dt}\left(\frac{dx}{dt}\right)\right]$$

$$\Rightarrow 2\left[e^{2x(t)}\cdot 2\cdot\frac{dx}{dt}\cdot\frac{dx}{dt} + e^{2x(t)}\cdot\frac{d^2x}{dt^2}\right]$$

$$\Rightarrow 2e^{2x(t)}\left[2\left(\frac{dx}{dt}\right)^2 + \frac{d^2x}{dt^2}\right]$$

Note: $\frac{d}{dt}\left(e^{2x(t)}\right) \rightsquigarrow e^{2x(t)}\cdot 2\cdot\frac{dx}{dt}$

and: $\frac{d}{dt}\left(\frac{dx}{dt}\right) \rightsquigarrow \frac{d^2x}{dt^2}$

$$I_1 = \int\left(x - \frac{lx}{\sqrt{l^2+x^2}}\right)dx$$ (Prob. 11.20)

$$I_2 = \int\left[-\frac{32.2}{(1+(y/20.9\times10^6))^2}\right]dy$$ (Prob. 11.29)

(Note l is treated as a constant.)

$$I_1 = \frac{x^2}{2} - l\sqrt{l^2+x^2} + c$$

[For I_1: $u = l^2 + x^2$

$du = 2x\,dx$

∴ $x\,dx = \frac{1}{2}du$]

$$I_2 = (672.98\times10^6)\cdot\left(\frac{1}{1+(y/20.9\times10^6)}\right) + C$$

[For I_2: $u = 1 + \frac{y}{20.9\times10^6}$

$\int f(x)\cdot dy = f(x)\cdot y + c$]