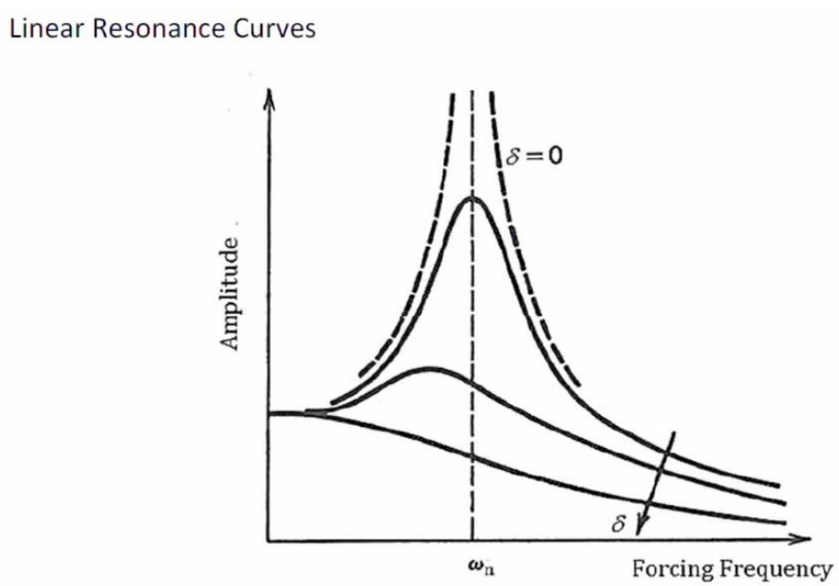
1.2 Forced Duffing Oscillator

$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = f_0 \cos(\omega t)$$

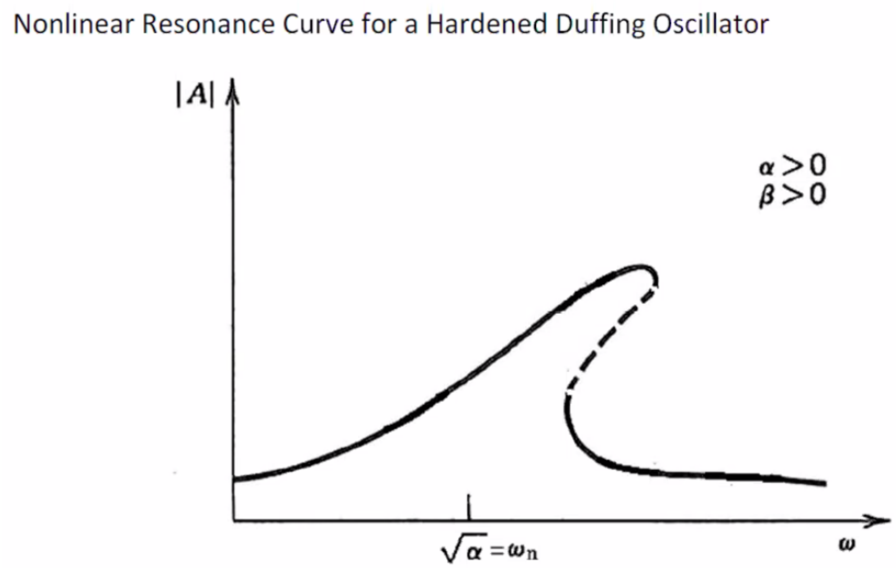
In addition to having nonlinear restoring forces, the forced Duffing oscillators are often used when the system demonstrates hysteresis (or the state variables' dependence of history)

The amplitude-frequency relation if $f(t) = f_0 \cos(\omega t)$:

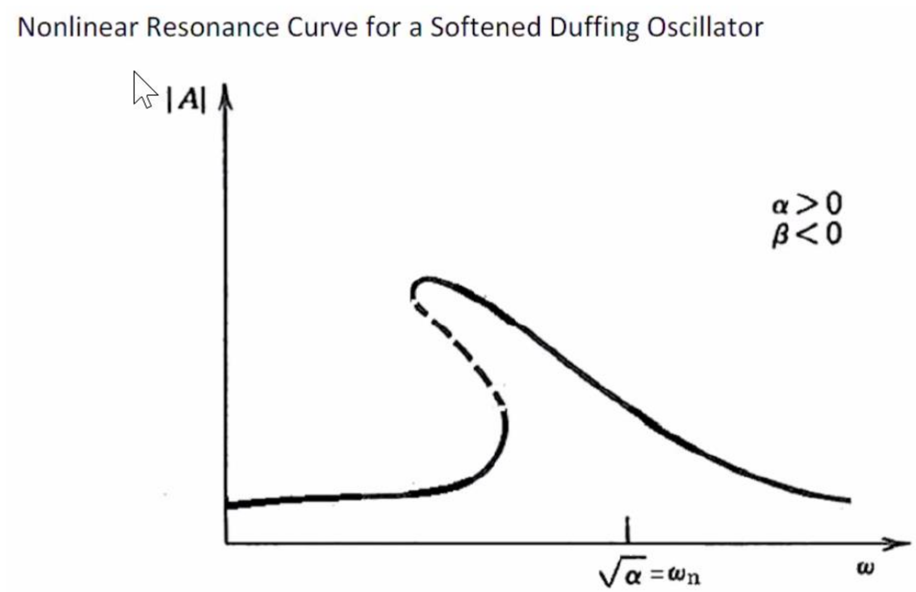
- The jump phenomenon
- The upsweep and downsweep paths



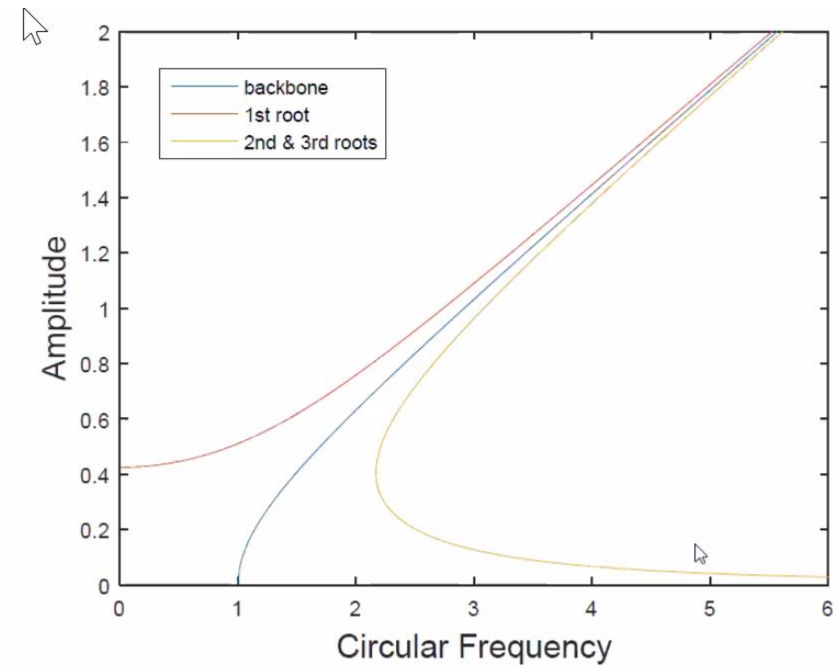
Revised from *Chaotic Vibrations An Introduction for Applied Scientists and Engineers,* Moon



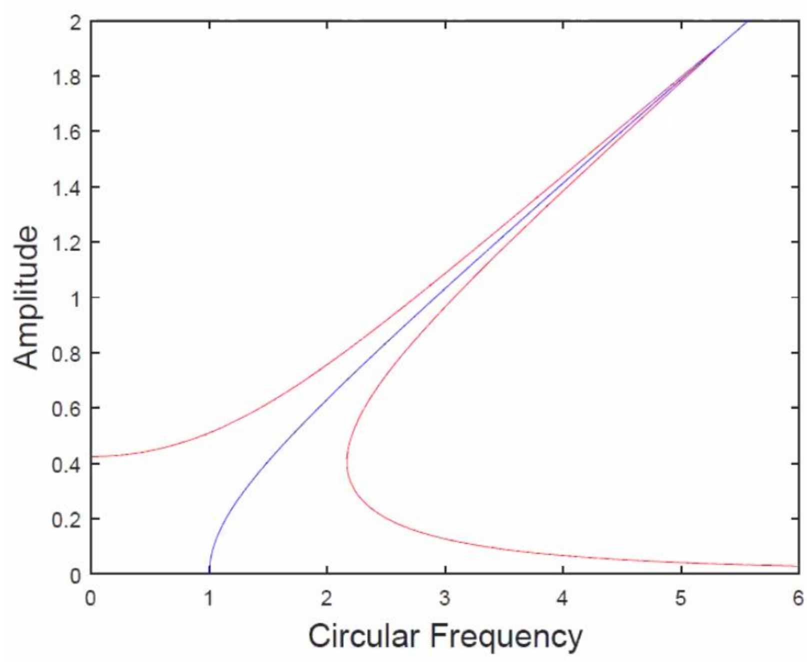
Revised from *Chaotic Vibrations An Introduction for Applied Scientists and Engineers,* Moon



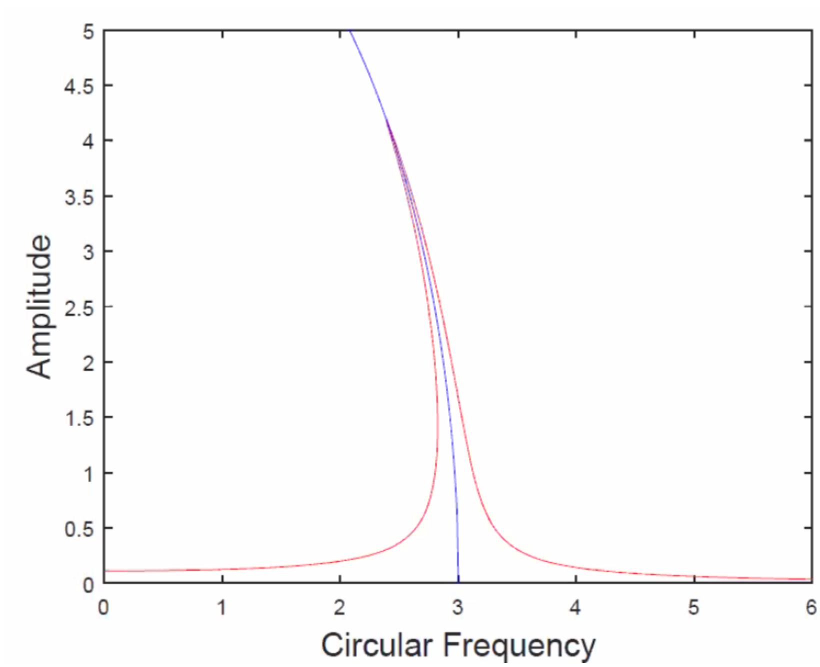
Revised from *Chaotic Vibrations An Introduction for Applied Scientists and Engineers,* Moon



Plot of duffing oscillator with excitation, but damping is zero. Backbone can be drawn from the natural frequency and seems to become the asymptote.



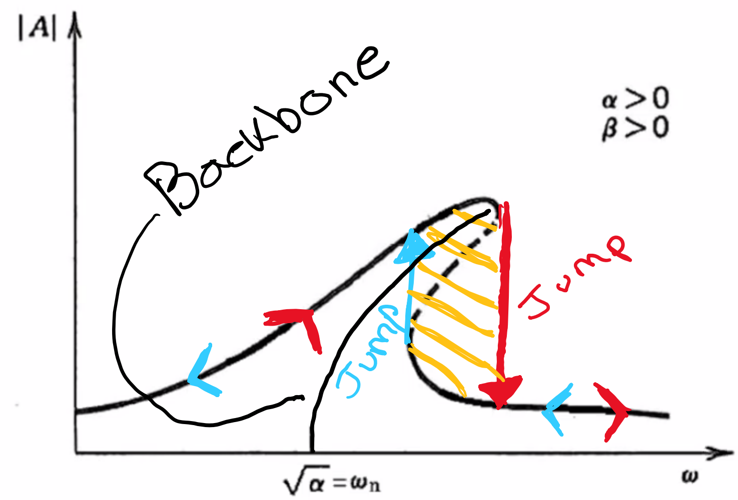
Plot of duffing oscillator where we have damping. The curve intersects the backbone at some point, then turns back.



1.3 Typical Analytic Approaches for Duffing Oscillator

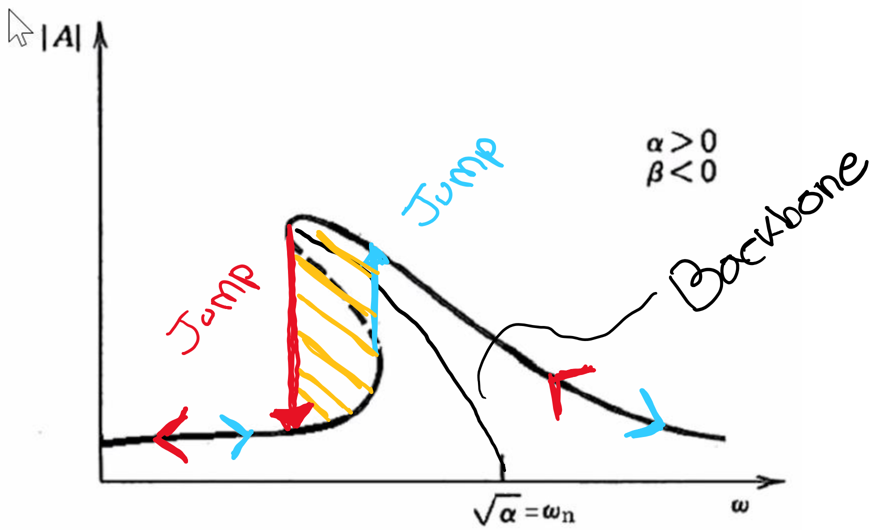
Perturbation methods (straightforward expansion, Lindstedt-Poincare method, …)
Harmonic balance method
Averaging method
…

Nonlinear Resonance Curve for a Hardened Duffing Oscillator



The red arrows indicate the upsweep path, and the blue arrows indicate the downsweep path. Notice the 'Jump,' and thus physically the dashed portion does not exist – however it is needed computationally. The hatched area indicates the hysteresis (the entire area enclosed by the curves).

Nonlinear Resonance Curve for a Softened Duffing Oscillator



Again, the red arrow indicates the upsweep path, and the blue arrow indicates the downsweep path. The dashed portion doesn't exist, and the enclosed hatched area is the hysteresis.

Equilibrium points, and Energy Curves

$$\ddot{x} + \alpha x + \beta x^3 = 0$$

$$\ddot{x} - f(x) = 0$$

Kinetic energy

potential energy

$$\dot{x} = y$$

Total energy $E(x, y) = H(x, y)$

Where H is the Hamiltonian

$$= \frac{1}{2} y^2 + \left(\frac{1}{2} \alpha x^2 + \frac{1}{4} \beta x^4 \right)$$

Then:

$$\dot{x} = \frac{\partial H}{\partial y} = y$$

$$\dot{y} = -\frac{\partial H}{\partial x} = -\alpha x - \beta x^3$$

Equilibrium points:

$$\begin{cases} \dot{x} = 0 \rightarrow y = 0 \\ \dot{y} = 0 \rightarrow \alpha x + \beta x^3 = 0 \end{cases}$$

H values at equilibrium points:

$$H(0,0) = 0$$
$$\vdots$$

Separatrices:

$$\int \frac{\partial H}{\partial y} dy + \frac{\partial H}{\partial x} dx = H$$

$$\uparrow \ at\ equilibirum\ points \downarrow$$

$$F(x, y) = H$$

Other energy curves:

$$F(x, y) = C \leftarrow energy\ level$$