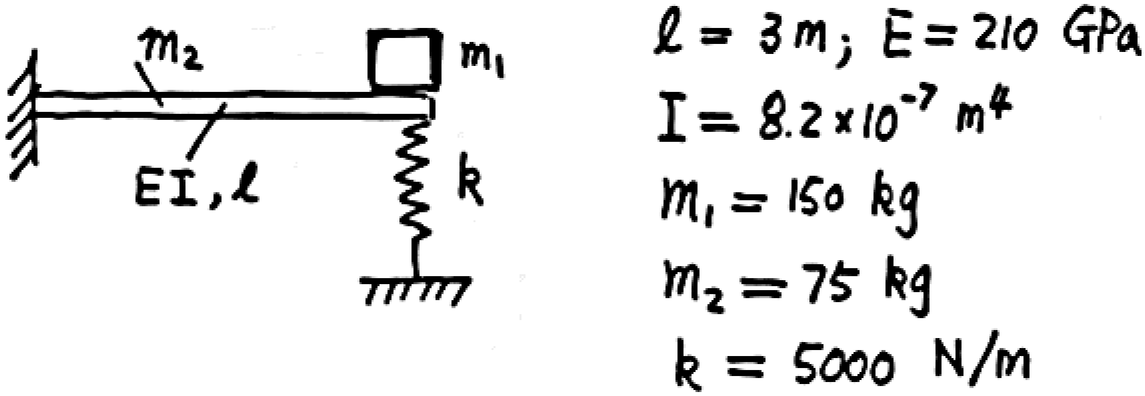
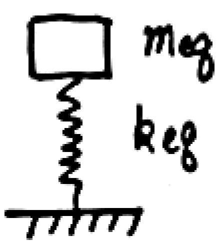
Example 2.13 (a beam as a spring)

Example: Evaluate the equivalent stiffness and equivalent mass of the system shown in Figure P2.20, where the beam has a mass of 75 kg.



Solution:



k_b: stiffness of the beam

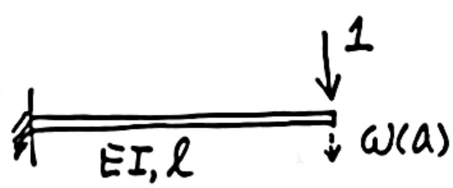


Table D.2, Case 1

$C_1 = -1$
$C_2 = a = l = 3$
$C_3 = C_4 = 0$
$y(z) = \frac{1}{6EI}(9.z^2 - z^3)$
$\omega(a) = y(a) = y(3) = \frac{9}{EI}$

$$\therefore k_b = \frac{EI}{9} = 19133\ (N/m)$$
$$and\ k_{eq} = k + k_b = 24133\ (N/m)$$

Now,
$m_{eq} = m_1 + m_b$

Where,
Assume ρ being the mass density per unit length, such that $m_2 = \rho \cdot l$

Dynamic deflection, in terms of (z, t)

$$X(z,t) = x(t) \cdot Y(z)$$

Where:
$x(t)$ is the response of the system
$Y(z)$ is chosen as the static deflection meeting the requirement of $Y(a) = 1$.

∴ scaling $y(z)$ such that at the tip, the static deflection is unity.

$$\therefore Y(z) = k_b \cdot y(z) = \frac{1}{54}(9z^2 - z^3)$$

$dz \rightarrow \rho dz$

[→ at z, dm has velocity of $\frac{d}{dt} X(z,t) = \dot{x} \cdot Y(z)$]

$$\therefore T_b = \frac{1}{2}\int_0^l \rho Y^2(z)\,\dot{x}^2\,dz$$
$$= \frac{\rho \dot{x}^2}{2}\int_0^l Y^2(z)\,dz$$
$$= \frac{1}{2}\underbrace{(0.23571 \cdot \rho \cdot l)}_{m_b = 0.23571\, m_z}\dot{x}^2$$

$\therefore m_b = 17.679\ (kg)$
$\therefore m_{eq} = 167.7\ (kg)$

2.7.3 Added mass

If the particle is submerged in an inviscid fluid, the movement of the particle causes movement of the surrounding fluid, see figure 2.29. The added mass is to include the inertia effect of the fluid.

Kinetic energy of the system is:

$$T = T_m + T_f$$

T_f, the kinetic energy of the fluid, is:

$$T_f = \frac{1}{2} m_a \dot{x}^2$$

Or,

$$T_f = \left(\frac{1}{2}\right) I_a \omega^2$$

See Table 2.2 or m_a, or Table 2.3 for I_a.

The equivalent mass is then $m_{eq} = m + m_a$ or $I_{eq} = I + I_a$.

2.8 External Sources

Excitation can be a force (or moment), or a motion input.

Work done by a force moment is, eq.(2.63)

$$U_{1\to 2} = \int_{x_1}^{x_2} F(t)\, dx = \int_{t_1}^{t_2} F(t)\dot{x}\, dt$$

Work done by a number of forces/moments is, eq. (2.64)

$$U_{1\to 2} = \sum \int_{t_1}^{t_2} F_i(t) \cdot \dot{x}_i\, dt$$

Assume $\dot{x}_i$ and $\dot{x}$ are directly proportional to each other $\frac{\dot{x}_i}{\dot{x}} = \gamma_i$

$$= \sum \int_{t_1}^{t_2} F_i(t) \cdot \gamma \cdot \dot{x}\, dt$$

The equivalent force is,

$$\sum \int_{t_1}^{t_2} F_i(t) \cdot \gamma_i \cdot \dot{x}\, dt$$

$$\int_{t_1}^{t_2} F_{eq}(t) \cdot \dot{x}\, dt$$

Examples 2.14 and 2.15

Summary of principles behind equivalent stiffness, damping, mass, and excitation:

Equivalent stiffness: potential energy of the original system = potential energy of the equivalent stiffness

Equivalent damping: energy dissipated in the original system = energy dissipated by the equivalent damper.

Equivalent mass: kinetic energy of the original system= kinetic energy of the equivalent mass

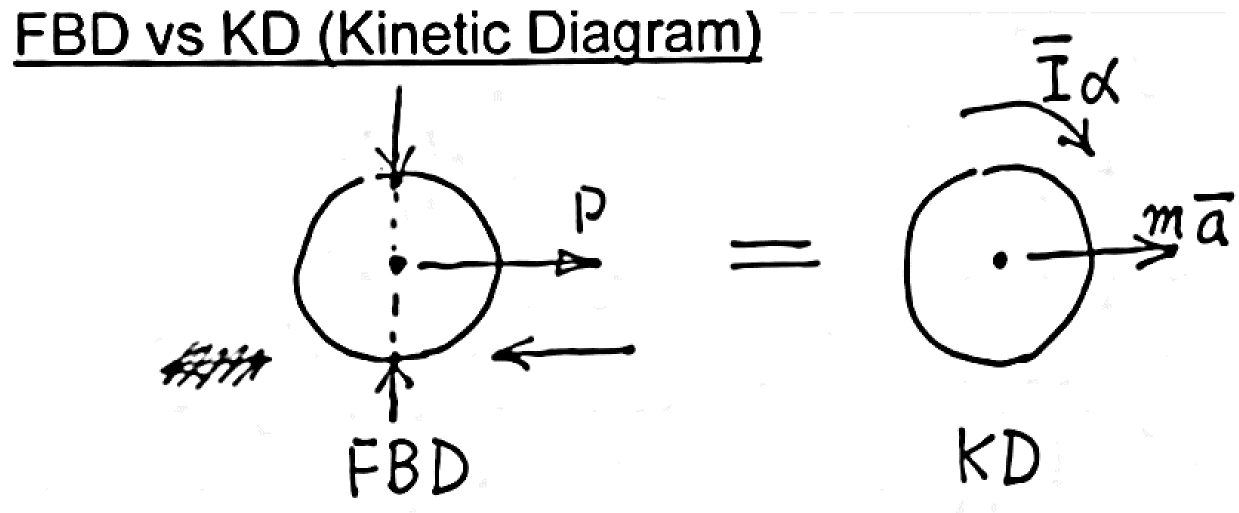
Equivalent excitation: work done by external forces (excitations) in the original system = work done by the equivalent excitation

2.9 FBD Method

This method combines FBD and Newton's Laws of motion. It is the fundamental way of arriving at the equation of motion, or the E.O.M.

Advantages: universal; able to deal with most engineering systems; able to determine reaction forces/moments in terms of the generalized coordinate.

Disadvantages: tedious, involving many equations when dealing with a system of rigid bodies.



2.12 Equivalent Systems Method

It is based on the general form of the principle of work and energy:

$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2$$

Note that $U_{1\to 2,NC}$ is usually how it's written, where NC is non-conservative

Where $U_{1\to 2}$ includes the work done by the viscous damping forces and the excitation forces.

State 1: pertaining to a specific or known configuration, for example, the static equilibrium configuration. That is,

$$T_1 = const. \;\; ; \;\; V_1 = const.$$

State 2: pertaining to an arbitrary configuration. That is,

$$T_2 = T(t) \;\; ; \;\; V_2 = V(t)$$

Then,

$$T_1 + V_1 + U_{1\to 2} = T + V$$

And

$$\frac{d}{dt}(T + V) = \frac{d}{dt}(U_{1\to 2})$$

$$T = \frac{1}{2} m_{eq} \dot{x}^2$$
$$\frac{dT}{dt} = \frac{1}{2} m_{eq} 2\dot{x}\ddot{x}$$

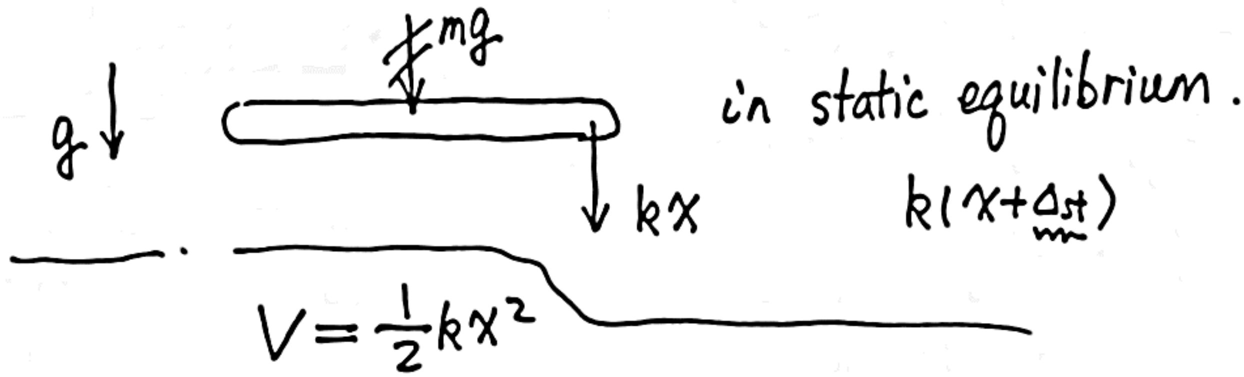
Following the few steps shown in eqs. (2.78) through (2.83), eq. (2.84) which is the equation of motion, is then obtained:

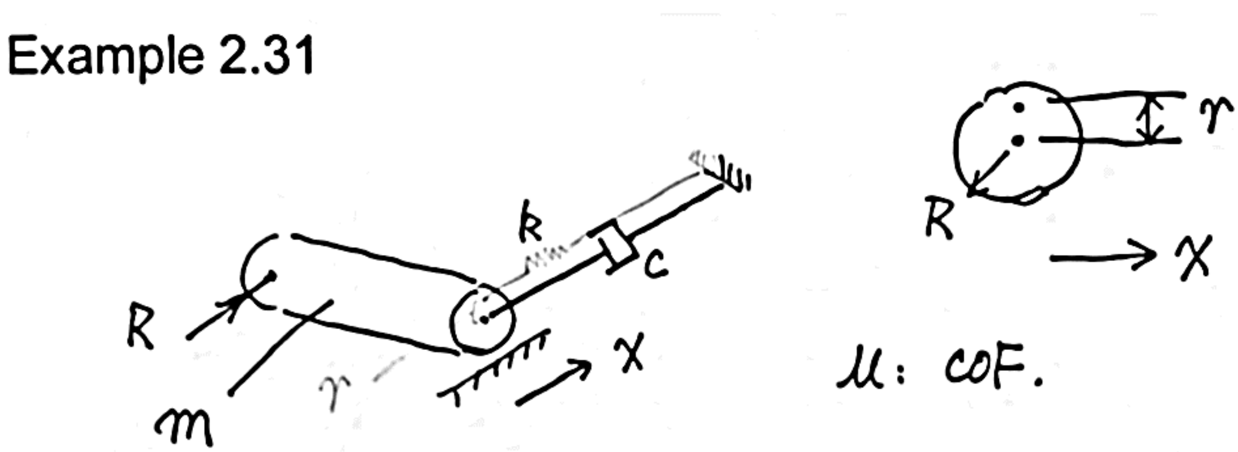
$$m_{eq}\ddot{x} + c_{eq}\dot{x} + kx = F_{eq}(t)$$

Or Eq. (2.85) if the generalized coordinate is an angle.

Examples 2.25~2.29
(TODO)

Static deflection and gravity (for Section 2.9 and Section 2.12)





(a) Rolling without slip

2.12 m_{eq}: need T

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2}\bar{I}\dot{\theta}^2$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\bar{I}\left(\frac{1}{2}mR^2\right)\left(\frac{\dot{x}}{R}\right)^2$$
$$T = \frac{1}{2}\left(\frac{3}{2}m\right)\dot{x}^2$$

$\therefore m_{eq} = \frac{3}{2}m$

$\therefore m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = 0$