Chapter 1 – Introduction

1.3 Generalized Coordinates

They are a set of coordinates $(q_1, q_2, q_3, \dots)$ that describe the configuration (or positions) of a dynamic system.

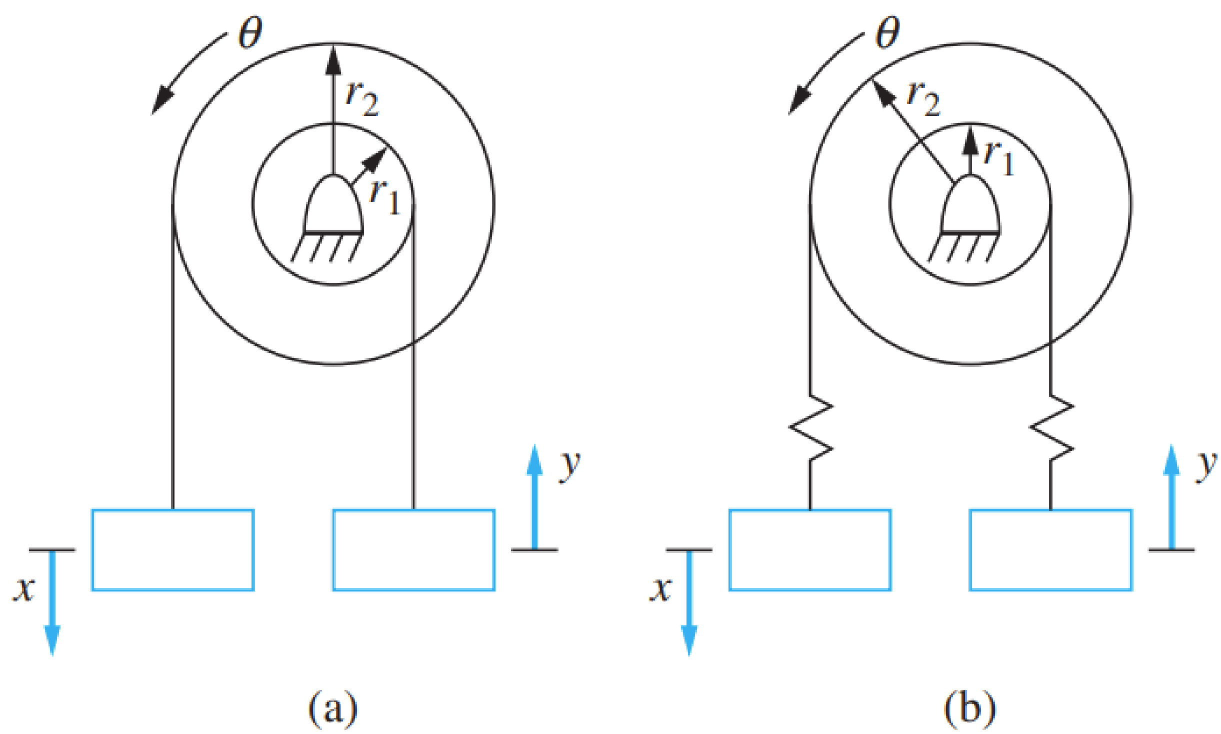
For any given system, the choice of generalized coordinates is not unique, but the number of independent coordinates is unique.

The number of independent coordinates equals the Degrees-of-Freedom (DOFs) needed/used to completely specify the configuration of the system.

When determining degrees of freedom, check if both ends of a 'device' would have the same displacement. If they don't, then it's a new degree of freedom – such as when slip exists or a spring.

Example 1.1
(TODO)

Figure 1.5



(1) Cables are inextensible and no slips between pulley and cables
(2) Cables are inextensible with slips between pulley and cables
(3) Cables are extensible (modeled as springs) and no slips
(4) Cables are extensible with slips

Parts (1) and (2) belong to (a)
Parts (3) and (4) belong to (b)

Determine DOF and q_i (coordinates) of the cases above.

(1) DOF = 1 ; x or y or θ
(2) DOF = 3; x, y, θ or u_1, u_2, θ
(3) DOF = 3; x, y, θ
(4) DOF = 5; x, y, θ, u_1, u_2 (where u_i is typically used for slip)

1.7 Review of Dynamics

1.7.1 Kinematics

Rigid bodies in general motion

1.7.2 Kinetics

Newton's 2nd law of motion

For a particle, eq. (1.32)

$$\sum F = ma$$

For a rigid body in general motion, eqs. (1.33), (1.34)

$$\sum F = m\bar{a} \quad | \quad \sum M_G = \bar{I}\alpha$$

For a rigid body in fixed-axis rotation, eqs. (1.33), (1.35)

$$\sum F = m\bar{a} \quad | \quad \sum M_o = I_o\alpha$$

The difference between (1.34) and (1.35):
$\sum M_G$ is for a rigid body undergoing planar motion – G is the mass center of the rigid body, and $\bar{I}$ is the mass moment of inertia about an axis parallel to the z-axis that passes through the mass center.

$\sum M_o$ is used when the axis of rotation is fixed, and I_o is the moment of inertia about the axis of rotation.

1.7.3 Principle of Work and Energy

- Kinetic Energy T:

For a rigid body, eq. (1.38); use the first term for a particle (there are 2 terms, one for translation and 1 for rotation)

$$T + \left(\frac{1}{2}\right) m\bar{v}^2 + \left(\frac{1}{2}\right)\bar{I}\omega^2$$

For a rigid body in fixed-axis rotation, eq. (1.39)

$$T = I_o\omega^2$$

- Work done by a force $U_{A\to B}$:

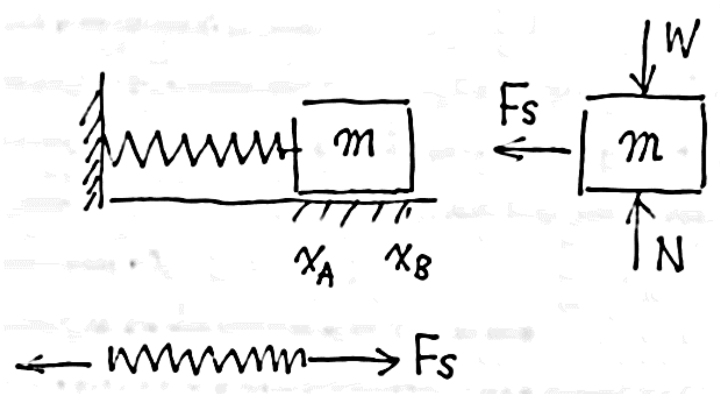
On a particle or a rigid body: eq. (1.40)

$$U_{A\to B} = \int_{\tau_A}^{\tau_B} F\, d\tau$$

- Work done by a moment $U_{A \to B}$:

On a rigid body: eq. (1.41)

$$U_{A \to B} = \int_{\theta_A}^{\theta_B} M \, d\theta$$

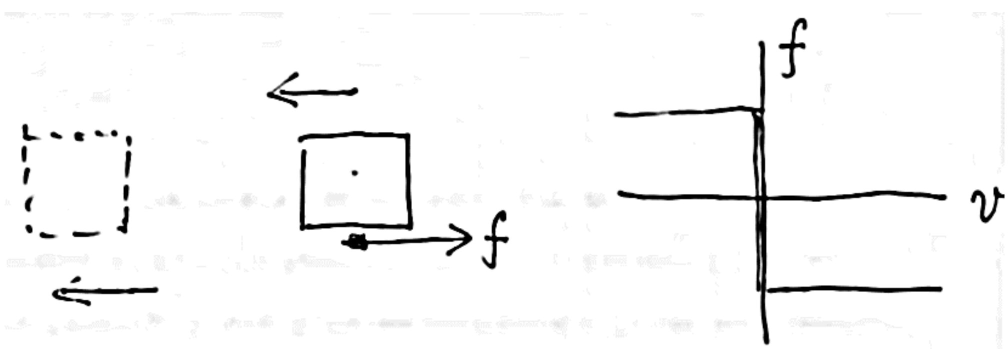


- Conservative Forces and Non-Conservative Forces:

Conservative forces store and release energy.

Typical conservative forces include spring forces (linear or nonlinear; but must be elastic), gravitational forces, and central forces.

The work done by such forces is independent of the path taken from A to B.



Non-conservative forces dissipate energy.
Samples include friction and air resistance.

The work done by such forces is dependent of the path taken from A to B.

Potential energy function V
V is related to conservative forces, and the work done by such forces.

For example:
The potential energy of a gravitational force is $V = mgh$, where h is positive if above the datum.

The potential energy of a linear spring is, $V = (1/2)kx^2$, where x is the elongation or compression from the natural length of the spring.

- Conservative force in terms of V:

$$\vec{F} = -\nabla V = -\left(\frac{\delta}{\delta x}\vec{i} + \frac{\delta}{\delta y}\vec{j} + \frac{\delta}{\delta z}\vec{k}\right)V$$

- Work done by a conservative force:

$$U_{A\to B} = V_A - V_B$$

- The principle of energy conservation, eq. (1.45)

$$T_A + V_A = T_B + V_B$$

- The principle of work and energy, eq. (1.47)

$$T_A + V_A + U_{A\to B,NC} = T_B + V_B$$

Where $U_{A\to B,NC}$ is the work done by non-conservative forces from A to B.

In general, friction contributes to $U_{A\to B,NC}$. However, for cases of rolling without slip, friction does no work.

Examples 1.4, 1.5, and 1.6
(TODO)

1.7.4 Principle of Impulse and Momentum
Definition of linear impulse $I_{1\to 2}$ (a vector): eq. (1.48)

$$I_{A\to B} = \int_{t_1}^{t_2} F\,dt$$

Definition of angular impulse about O, $J_{O,1\to 2}$: eq. (1.49)

$$J_{o_{1\to 2}} = \int_{t_1}^{t_2} \Sigma M_o\,dt$$

Definition of linear momentum L (a vector): eq. (1.50)

$$L = m\bar{v}$$

Definition of angular momentum about G, H_G: eq. (1.51)

$$H_G = \bar{I}\omega$$

The principle of linear momentum: eq. (1.52)

$$L_1 + I_{1\to 2} = L_2$$

The principle of angular momentum: eq. (1.53)

$$H_{G_1} + J_{G_{1\to 2}} = H_{G_2}$$

1. 8 Two Benchmark Examples

Problem 1.18

(TODO)

Problem 1.20 (changed the applied force)

(TODO)

Problem 1.22

(TODO)