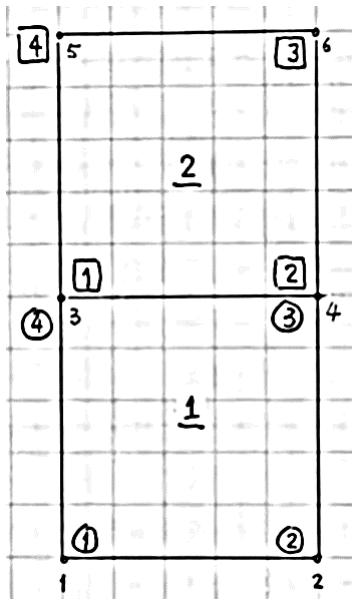
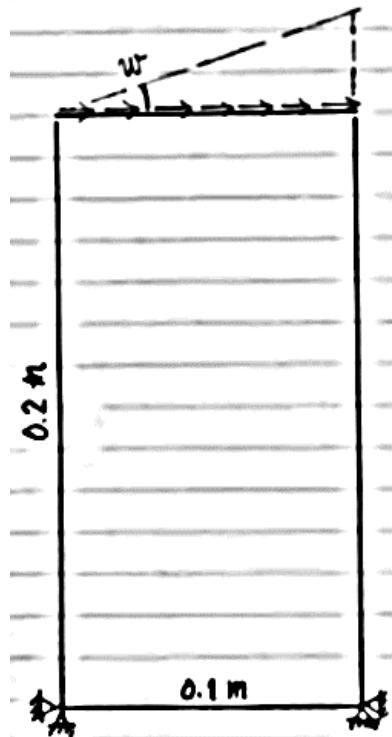


$[k]$ : singular, symmetric

$$[K] = \sum_{i=1}^{NE} [k] : \text{symmetric}$$

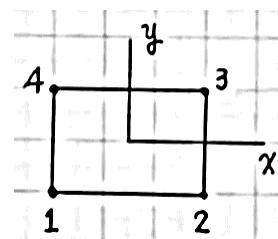
singular before applying B.C.'s



1,1	1,2	1,3	1,4
2,2	2,3	2,4	
3,3	3,4		
	4,4		

Symmetry

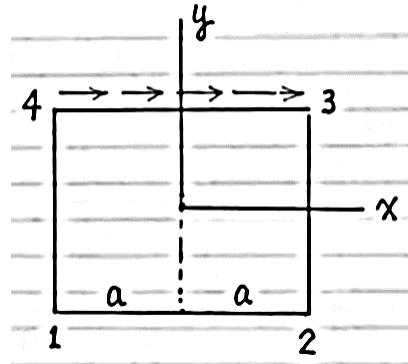
8x8



$$[k] = 10^8 \times$$

$$\left[ \begin{array}{cc|cc} 494.51 & 178.57 & -302.20 & -13.736 \\ 178.57 & 494.51 & 13.736 & 54.945 \\ \hline & & 494.51 & -178.57 \\ & & -178.57 & 494.51 \end{array} \right]$$

sym.



Surface load on edge "4 – 3":

$$\Phi = \begin{Bmatrix} \Phi_x \\ \Phi_y \end{Bmatrix} = \begin{Bmatrix} \Phi_x \\ 0 \end{Bmatrix}$$

$$\Phi_x = w(x + a)$$

w: force/length<sup>3</sup>

On the other hand, shape functions are, when evaluated at the edge where  $y = b$ ,

$$\bar{N}_1 = \bar{N}_2 = 0$$

$$\bar{N}_3 = (a + x)/(2a)$$

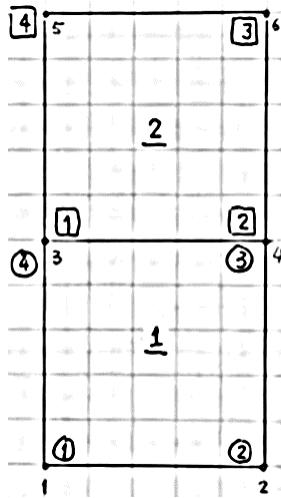
$$\bar{N}_4 = (a - x)/(2a)$$

$$\therefore \{f_{eq}\} = \int_{-a}^a [\bar{N}]^T \{\Phi\} t \, dx$$

$$[\bar{N}] = \begin{bmatrix} \bar{N}_1 & \bar{N}_2 & \dots & \bar{N}_4 \\ \bar{N}_1 & \bar{N}_2 & \dots & \bar{N}_4 \end{bmatrix}_{2 \times 8}$$

$$\therefore \{f_{eq}\} = \begin{bmatrix} 0, 0, 0, 0, \frac{4}{3}wta^2, 0, \frac{2}{3}wta^2, 0 \end{bmatrix}^T$$

for element 2



$$[K]_{12 \times 12} = [k]_{6 \times 6}$$

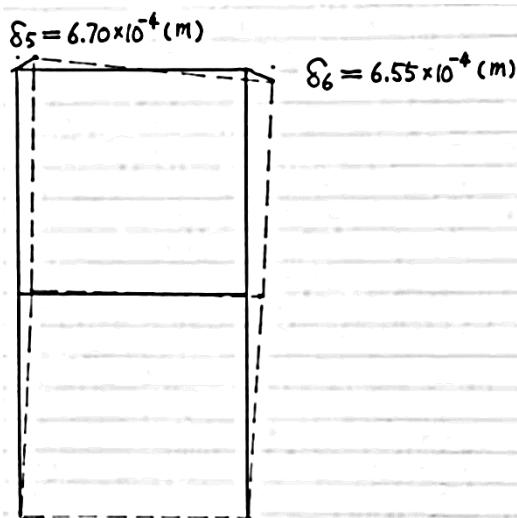
symm.

$$[k]_{6 \times 6}$$

(1,1)	(1,2)	(1,4)	(1,3)				1
	(2,2)	(2,4)	(2,3)				2
		(4,4)	(4,3)	(1,4)	(1,3)		3
		+ (1,1)	+ (1,2)				
		(3,3)	(2,2)	(2,4)	(2,3)		4
				(4,4)	(4,3)		5
					(3,3)		6

symm.

Results:



Stresses at Node 3 & 4 (in MPa):

$$\begin{array}{c} \left\{ \begin{array}{c} 151.2 \\ 487.4 \\ 201.4 \end{array} \right\} \\ 3 \end{array} \quad \begin{array}{c} \left\{ \begin{array}{c} -137.7 \\ -475.6 \\ 189.5 \end{array} \right\} \\ 4 \end{array}$$
  

$$\begin{array}{c} \left\{ \begin{array}{c} 29.8 \\ 82.4 \\ 166.6 \end{array} \right\} \\ 3 \end{array} \quad \begin{array}{c} \left\{ \begin{array}{c} -16.4 \\ -71.0 \\ 168.5 \end{array} \right\} \\ 4 \end{array}$$