Draw a straight line between the two points of (T_{av}, μ). The intersecting point with the SAE20 line is (117, 5.2).

So, $T_{av} = 117°F$ or $\Delta T_F = 34\ °F$ and $5.2\ \mu reyn$
(Keep in mind every time we have a new viscosity, we have to find a new Sommerfeld number)

The last part is as follows:
$T_{max} = 134°F < 250°F$

Figure 12-15: $\frac{h_0}{c} = 0.49$
$h_0 = (0.49)(0.0015) = 0.000\ 735"$
$h_{min} = 0.000\ 2 + 0.000\ 04d = 0.000\ 26"$

So, $h_0 > h_{min}$ and $(S, h_0/c)$ is inside the optimal zone.
(Where $S = 0.176$ and $h_0/c = 0.49$)

Figure 12-17: $(r/c)f = 4.2$
$$f = \frac{4.2}{500} = 0.0084 < 0.01$$

Figure 12-20: $\frac{P}{p_{max}} = 0.45$
$$p_{max} = \frac{222.2}{0.45} = 494\ psi$$

Power loss:
$$H_{loss} = \frac{fWrN}{1050} = \frac{(0.0084)(500)(0.75)(30)}{1050} = 0.0897\ hp$$

Example 3
A journal bearing has $r = l = 1.5"$, $c = 0.0015"$, $W = 1000\ lb$, and $N = 30\ rps$. Lubricant's inlet temperature is assumed to be $120°F$. The bearing is to be designed with high load capacity. Select a lubricant, and design and evaluate the bearing.

Solution:
The first part of the process is as follows:
$P = \frac{W}{ld} = \frac{1000}{1.5*3} = 222.2\ psi$, $N = 30\ rps$, $\frac{l}{d} = \frac{1}{2}$, and $\frac{r}{c} = \frac{1.5}{0.0015} = 1000$
Also, $T_1 = 120°F$

For the second part,
Figure 12-16: on the plot of $\frac{l}{d} = ½$, select a point located close to the "Max W" edge, say $S = 0.3$
$$S = 0.3 = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = (1000)^2 \frac{\mu(30)}{222.2}$$
So, $\mu = 2.22\ \mu reyn$

Figure 12-24 with $S = 0.3$:
$$\frac{9.70\Delta T_F}{p_{psi}} = 0.394\ 552 + 6.392\ 527S - 0.036\ 013S^2 = 2.309$$
$\Delta T_F = (2.309)(222.2)/(9.70) = 52.9°F$

So, $T_{av} = T_1 + \frac{\Delta T_2}{2} = 146°F$

Figure 12-12: locate the point (146, 2.22). It is below the SAE20 line. On the line, $\mu = 2.5\ \mu reyn$, which gives a Sommerfeld number $S = 0.34$. The design remains inside the optimal zone.

Or, the average temperature needs to be $153°F$ for the lubricant to have a viscosity of 2.22 $\mu reyn$. That means inlet temperature needs to be at $126.5°F$.

The last part gives the following results: (based on $S = 0.34$)
$h_0 = (0.43)c = 0.000\ 625"$
$h_{min} = 0.000\ 2 + 0.000\ 04d = 0.000\ 32" < h_0$

$f = (8.5)/(r/c) = 0.0085 < 0.01$

$Q = (4.8)/(rcNl) = 1394\ in^3/s$
$Q_s = 0.7\dot{2} \cdot Q = 1004\ in^3/s$

$p_{max} = P/0.375\ = 593\ psi$

12-10 Clearance
Why this section?

- Clearance c has a range due to manufacture and assembly
- It tends to increase due to wear

How to take into consideration change in clearance?
A suitable fit is assigned between journal and bushing. For example, H8,f7 (close to running fit) or H9/d9 (free running);

Then the range of clearance is determined (see Table 12-3, for example);

Performance of the bearing (As indicated by h_0, T_2 the outlet temperature, Q, and H the power loss, for example) is calculated and plotted against clearance c. See Figure 12-25.

The initial clearance band (i.e., the tolerance specified for manufacturing) should be located to the left of the peak of the $h_0 - c$ plot.

Chapter 13
Gears - General

Part 1: Geometry and Tooth System

13-1 … 13.8:

Types of Gears, …, The Forming of Gear Teeth

13-12: Tooth Systems

Part 2: Kinematics

13-13: Gear Trains

Part 3: (to be discussed with Chapters 14 and 15)

13-19 … 13-11:

Bevel Gears, Parallel Helical Gears, Worm Gears

13-14 … 13-17:

Force Analysis – Spur, Bevel, Helical and Worm Gears

13-1 Types of Gears

Why Gears?

Of constant-speed mechanical transmission elements, the frequencies of usage are:

- Gears: 50%
- Couplings: $\sim$20%
- Chain Drives: 10$-$20%
- Belt Drives: 10-12%
- Power screws, wire ropes, friction wheels, etc.: 5-10%

Types of Gears:

- Spur gears: most common; transmit power between two parallel shafts
- Helical gears: between two intersecting shafts
- Bevel gears: between intersecting shafts
- Worm gear sets: between non-parallel and non-intersecting shafts
- And many other types

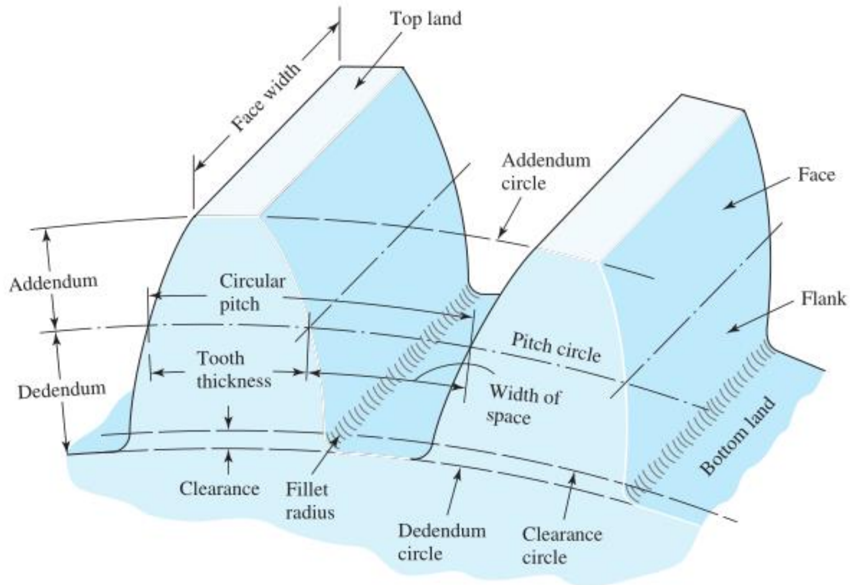
13-2 Nomenclature

13-12 Tooth Systems

Nomenclature: Figure 13-5, for spur gears only.

Figure 13–5

Nomenclature of spur-gear teeth.



Tooth System: refers to the standard that specifies the tooth geometry and so on. There are the metric system and the US customary system.

- Pitch circle
- Circular pitch p, pitch diameter d, in in or mm
- Number of teeth N
- Diametral pitch $P = N/d$, in teeth/in; or
- Module $m = d/N$, in mm
- P and m are standardized, see Table 13-2
- Metric gears and US customary gears are NOT interchangeable

Tables 13-1, 13-3, 13-4: formulas for spur gears, 20° straight bevel gears, and helical gears

Table 13-5: information for worm gearing

Typical values for face width

$$\frac{3\pi}{P} \leq F \leq \frac{5\pi}{P}$$

Or

$$3\pi m \leq F \leq 5\pi m$$

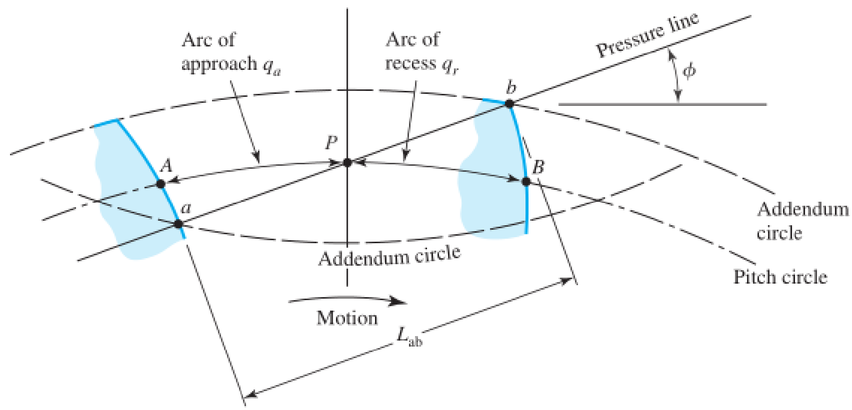
13-3 Conjugate Action
13-4 Involute Properties
13-5 Fundamentals

- The fundamental Law of Gearing:
  Angular velocity ratio between the gears of a gearset must remain constant throughout the mesh
- Involute tooth form meets the fundamental law, and has the advantage that error in center-to-center distance will not affect the angular velocity ratio.

13-6 Contact Ratio

Figure 13–15

Definition of contact ratio.



- Addendum circles
- Pressure line (passing through pitch circles)
- A part of gear teeth enters into contact at point a and exits from contact at point b on the same line
- The distance between these points in the length of action L_{ab} also labelled as Z.

Gear contact ratio m_c defines the average number of teeth that are in contact at any time,

$$m_c = \frac{L_{ab}}{p_b} = \frac{L_{ab}}{pcos\phi}$$

Where $p_b = p \cdot cos\phi$ is the base pitch and ϕ is the pressure angle.

Significance:
For a pair of gears the mesh properly, their diametral pitch or module, and pressure angle must be the same. In addition, contact ratio must meet certain requirements.

$$m_c = \frac{L_{ab}}{pcos\phi} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - csin\phi}{pcos\phi}$$

Or

$$m_c = \frac{L_{ab}}{\pi mcos\phi} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag^2} - r_{bg}^2} - csin\phi}{\pi mcos\phi}$$

Where:

r_{ap}, r_{bp}: radii of addendum circle and base circle of the pinion;
r_{ag}, r_{bg}: radii of addendum circle and base circle of the gear;
c: center-to-center distance;

The radius of base circle of a gear is $r_b = \frac{d}{2} cos\phi$, with d being the pitch diameter.

Example 1

A gear set has diametral pitch of $P = 10\ teeth/in$ and pressure angle of 20°. Teeth numbers are $N_p = 30$ and $N_g = 75$. Determine the contact ratio of the set. Assume full depth tooth profile.

Solution:

	Pinion	**Gear**
Pitch radius, in	1.5	3.75
Addendum radius, in	1.6	3.85
Base radius, in	1.4095	3.5238
Center-to-centre distance, in	5.25	

$$L_{ab} = 0.7572 + 1.5509 - 17956 = 0.5125"$$
$$p = \frac{\pi}{P} = 0.3142"$$
$$m_c = \frac{0.5125}{0.3142 \cdot \cos(20°)} = 1.74$$

Example 2

A set of stub-profiled gear has, $N_p = 18$ and $N_g = 72$. Module is $m = 5\ mm$. Determine the set's contact ratio. Pressure angle is 22.5°.

Solution:

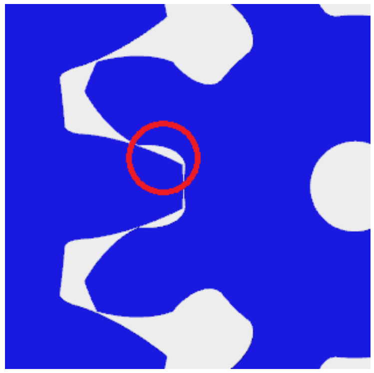
	Pinion	**Gear**
Pitch radius, mm	45	180
Addendum radius, mm	49	184
Base radius, mm	41.575	166.298
Center-to-centre distance, mm	225	

$$L_{ab} = 25.933 + 78.746 - 86.104 = 18.575\ mm$$
$$m_c = \frac{18.575}{n \cdot \pi \cdot \cos(22.5°)} = 1.28$$

13-8 The Forming of Gear Teeth

13-7 Interference

- Mainly, there are form cutting and generating cutting
  - Form cutting: the cutter is the exact shape of the tooth space; expensive
  - Generating cutting: the cutter has a shape different from the tooth space; more common
- Of interest to discussing interference is generating cutting which includes,
  - Shaping: pinion cutting (Figure 13-17) and rack cutting (Figure 13-18)
  - Hobbing: using hob, a worm-like cutting tool, to cut a blank (Figure 13-19)
- Interference refers to contact taking place on the non-involute portion of the tooth profile (inside base circle)
- Undercut refers to the removal of interfering material during generating cutting.



- To avoid interference, the pinion requires a minimum number of teeth ,while the gear has a restriction on maximum number of teeth.
- The text has three equations, (Eq. 13-10), (Eq. 13-11) and (Eq. 13-13) for determining the minimum number of teeth to avoid interference.
- (Eq. 13-11) is for general cases, and recommended. For a pinion-gear set, the minimum number of teeth on pinion without interference is, $(Eq. 13-11)$:

$$N_P = \frac{2k}{(1 + 2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1 + 2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

Where:
m is the gear ratio $m = N_G/N_p . m > 1$.
$k = 1$ for full-depth teeth, and 0.8 for stub teeth
ϕ is the pressure angle

- (Eq. 13-10) is for cases of one-to-one gear ratio.

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- (Eq. 13-13) is for cases of pinion meshing with a rack.

$$N_P = \frac{2(k)}{\sin^2\phi} \qquad (13\text{–}13)$$

- Maximum number of teeth on a gear mating with a specific pinion is determined by (Eq. 13-12)

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (13\text{–}12)$$

- A number of examples are shown within the section.

1st example: a set of gears, shapes by pinion cutter, 20° pressure angle, full-depth teeth; then smallest N_p is 1 3and largest N_G is 16.

2nd example: a set of gears, 20° pressure angle, full-depth teeth, cut by hobbing; then smallest N_p is 17 and largest N_G is 1309.

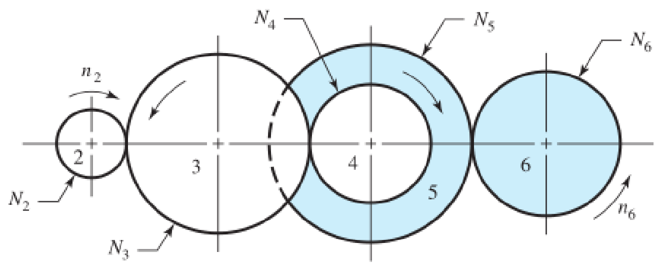
13-13 Gear Trains

Types of Gear Trains

- Simple of series trains, See Figure 13-27

Figure 13–27

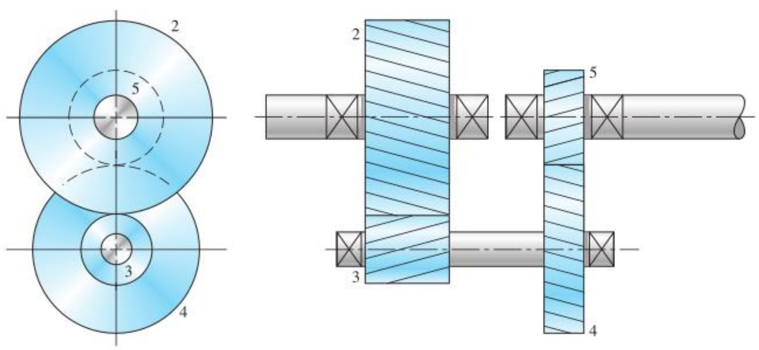
A gear train.



- Compound trains
  - Reverted (Figure 13-29)

Figure 13–29

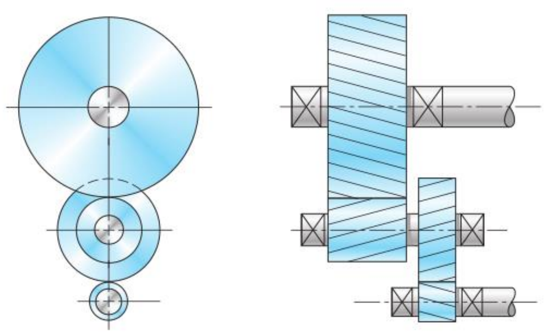
A compound reverted gear train.



  - Non-reverted (Figure 13-28)

Figure 13–28

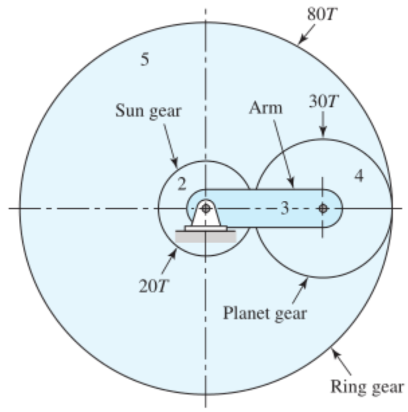
A two-stage compound gear train.



- Planetary or epicyclic trains (Figure 13-30)

Figure 13–30

A planetary gear train.



Train Value, Speed Ratio, Gear Ratio, and so on

- In the text, train value e is used. It is defined as

$$e = \pm \frac{product\ of\ driving\ tooth\ numbers}{product\ of\ driven\ tooth\ numbers}$$

  e is positive if the last gear rotates in the same sense as the first, and negative if the last gear rotates in the opposite sense.
  e is also the ratio of n_L, the speed of the last gear, over the speed of the first gear n_F.

$$e = \frac{n_L}{n_F}$$

- Speed ratio = velocity ratio = transmission ratio – train value.
- Gear ratio is commonly used in daily conversions. Gear ratio $= 1/e$

Problem-Solving

- Given a train, find velocity ratio;
- Given required gear ratio, determine the type of train and teeth numbers.
  See Examples $13-3 \sim 13-5$.

Velocity Ratio – Planetary Gear Trains

- Tabular method see "Dynamics of Machinery", R. L. Norton.
  - Follow the power flow
  - Velocity difference equation

$$\omega_{gear} = \omega_{arm} + \omega_{gear/arm}$$

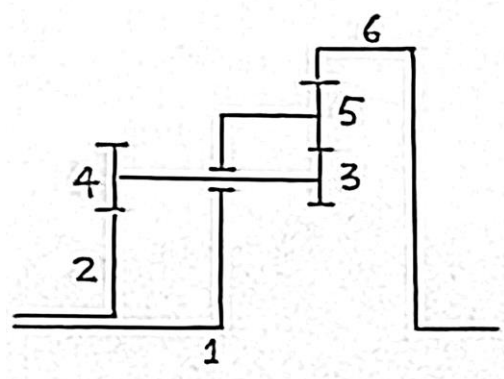
  (NOTE: The last term represents the velocity of gear relative to the arm.)
  - Relative velocity obeys

$$VR = \frac{[\omega_{gear/arm}]_{driven}}{[\omega_{gear/arm}]_{driver}} = \pm \frac{N_{driver}}{N_{driven}}$$

  (NOTE: Where the " + " is used with internal set and " − " is used with external set.)

Example 3:

The schematic of a planetary gear train is shown below, with “1” being the arm. Gears 2 and 6 rotate about the same axis as the arm. Input is to Gear 2.



(1) Given $N_2 = 30, N_3 = 25, N_4 = 45, N_5 = 30, N_6 = 160, \omega_2 = 50\, rad/s, \omega_{arm} = -75\, rad/s$
Find ω_6.
(2) Given $N_2 = 30, N_3 = 25, N_4 = 45, N_5 = 50, N_6 = 200, \omega_2 = 50\, rad/s, \omega_6 = 0$
Find $\omega_{arm}, \omega_3, \omega_4, \omega_5$

Solution:
(1): Power flow: 2 → 4 & 3 → 5 → 6

Gear	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	VR
2	50	-75	125	$-N_2/N_4$
4		-75	$(125)\left(-\frac{N_2}{N_4}\right)$	
3				$-N_3/N_5$
5		-75	$(125)\left(-\frac{N_2}{N_4}\right)\left(-\frac{N_3}{N_5}\right)$	
6	ω_6	-75	$(125)\left(-\frac{N_2}{N_4}\right)\left(-\frac{N_3}{N_5}\right)\left(\frac{N_5}{N_6}\right)$	N_5/N_6

$\therefore \omega_6 = -61.98\, rad/s$

(2): Power flow: 2 → 4 & 3 → 5 → 6

Gear	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	VR
2	50	x		
4		x		
3				
5		x		
6	0	x		

$\omega_{arm} = x = -4.545\, rad/s$
$\omega_3 = \omega_4 = -40.91\, rad/s$
$\omega_5 = 13.64\, rad/s$