Example 2

Select bearings A and B for the shaft of Example 7-2. They are to be used for a minimum of 1,000 hours of continuous operation. Shaft rpm is 450. Radial loads are, $F_A = 375\ lb$ and $F_B = 1918\ lb$. Shaft diameter at both locations is 1" (D_1 and D_7 in Figure 7-10). Assume 90% reliability.

Solution:

Since there is no thrust load, deep-groove ball bearings are first considered. Table 11-2 has a list of 02-series deep groove ball bearings.

Table 11–2

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm d_S	d_H	Load Ratings, kN Deep Groove C_{10}	C_0	Angular Contact C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

$a_1 = 1;$

$k_a = 1.2\ (Table\ 11-5,\ \ commercial\ gearing,\ \ 1.1{\sim}1.3);$

$L_{10} = 10^6\ revs;$

$L_D = 1000 \cdot 60 \cdot 450 = 27 \cdot 10^6\ revs;$

Table 11–5

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

Bearing B: $F_D = F_B = 1918\ lb = 8535\ N$. From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

It's found that $C_{10} = 30{,}726\ N$. Note $a = 3$.

Table 11-2 Shows that the smallest (in dimensions) bearing meeting the requirements is the one with bore diameter of 40 mm and its C_{10} is 30.7 kN.

Switch to roller bearing (Table 11-3). With $a = 10/3$, then $C_{10} = 27{,}529\ N$.
From Table 11-3, under 02-series, the bearing with 35-mm bore has $C_{10} = 31.9\ N$;
Under 03 series, the bearing with 25-mm bore has a $C_{10} = 28.6$ kN.

Table 11–3

Dimensions and Basic Load Ratings for Cylindrical Roller Bearings

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

Select 03-series, $bore = 25\ mm$, $OD = 62\ mm$, $width = 17\ mm$, and $C_{10} = 28.6\ kN$

Assessing reliability: $a_1 = 0.88$, and $R = 91.7$
Bearing A: $F_D = F_A = 375\ lb = 1669\ N$. Use the same bearings as at B.

Assessing reliability: $a_1 = 0.00382$, $R = 99.9975$. So $R \geq 99$.
The two bearings combined will have a reliability of $(0.917) * (0.99) = 0.91$

11-6 Combined Radial and Thrust Loading
Equivalent Radial Load F_e

- C_{10}, the Basic Dynamic Load Rating, is a radial load.
- A ball or roller bearing s typically capable of taking radial as well as some thrust loads.
- Thrust loads shortens a ball or roller bearing's life faster than radial load; that is, thrust load does more damage.
- Equivalent radial load F_e is

$$F_e = XVF_r + YF_a$$

Where:
F_r and F_a are the radial and thrust loads applied to the bearing (from FBD)
F_e is the equivalent radial load; => F_D
V, X and Y are factors whose values depend on specific bearing.
V: the rotation factor
X: the radial load factor
Y: the thrust load factor

For X and Y:
Table 11-1 lists the X and Y values for ball bearings. X and Y depend on e, which in turn depends on F_a/C_0. C_0 is the **Basic Status Load Rating.**

Table 11-1

Equivalent Radial Load Factors for Ball Bearings

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

$F_a/(VF_r) \le e$	$F_a/(VF_r) > e$

For straight or cylindrical roller bearings, $Y = 0$.

It is highly recommended to use the X and Y values published by the manufacturer. Methods or processes to evaluate X and Y vary with manufacturers.

For V:
$V = 1$ if inner ring rotates;
$V = 1.2$ if outer ring rotates
$V = 1$ if using self aligning bearings.

Basic Static Load Rating C_0

- **Basic Static Load Rating** or **Basic Static Rated Load** C_0 is defined as the load that will produce a total permanent deformation (in the rolling elements and raceways) at any contact point of 0.0001 times the diameter of the rolling elements.
- C_0 can also be defined as the load that will produce a maximum contact (compressive) stress of $4\ GPa$ $(580\ ksi)$ at the contact point.
- C_0 can be exceeded. It takes about 8 times of C_0 to fracture a bearing.

11-8 Selection of Ball and Roller Bearings
General Principles/Considerations

- Always refer to a catalog and read the engineering section
- Type of load to be carried
    - Radial
    - Axial
    - Radial + Axial
- Rating Loads (dynamic and static)
- Limiting Speed
- Permissible Alignments
- Space Limitation (bore, OD, width)
- Mounting/Dismounting, and Enclosure

General Procedure (not meant to be followed mechanically)

1. Use FBD of the shaft to determine the radial load F_r, and thrust (axial) load F_a.
2. Set design life L_D; Table 11-4 lists typical values.
3. Set reliability factor, application factor (and other factors if required).
4. Select type of bearing.
5. Pre-select a bearing, say, based on bore diameter and/or permissible misalignment, and/or permissible speed, so as to facilitate selecting X and Y values.
6. Evaluate the equivalent dynamic (radial) load F_e.
7. Determine C_{10}.
8. Select appropriate bearing(s) from the catalog.
9. If necessary, iterate (back to step 6) until a suitable bearing is chosen. Note that all bearings require iterations.
10. Once a bearing is chosen, assess its reliability.

Example 3

A bearing is subject to $F_r = 5400\ N$ and $F_a = 1900\ N$. The following is known $bore = 35\ mm$, at least $30 \cdot 10^6\ revs$, 90% reliability, and $k_a = 1.5$. Select a suitable bearing from $Table\ 11-2$.

Solution:

From Table 11-2, 02-series single-row deep-groove ball bearing with a $35-mm$ bore has $C_{10} = 25.5\ kN$ and $C_0 = 13.7\ kN$.

$F_a/VF_r = 0.352$ and $F_a/C_0 = 0.139$

From Table 11-1, F_a/C_0 is between 0.11 and 0.17. So, $e < 0.34$.

That means, $F_a/VF_r = 0.352 > e$, therefore $X_2 = 0.56$ and $Y_2 = 0.138$ by linear interpolation.

So, $F_e = XVF_r + YF_a = 5646\ N$.

Use the basic bearing equation to determine C_{10}:

$$\frac{C_{10}}{(1.5)(5646)} = \left(\frac{(30)(10^6)}{(1)(10^6)}\right)^{1/3}$$

So, $C_{10} = 26{,}315\ N$. Therefore the 02-series single-row deep-groove ball bearing is not suitable.

Consider 02-series angular contact ball bearing with the same bore diameter. It has $C_{10} = 27.0\ kN$ and $C_0 = 15.0\ kN$.

Since $F_a/C_0 = 0.127$, then $e < 0.34$ and $F_a/VF_r > e$.

Then X_2 remains at 0.56, but Y_2 becomes 1.41.

Now $F_e = 5703\ N$, and C_{10} is found to be 26,581 N, which is less than the catalog's 27.0 kN.

So, the 02-series single-row angular contact ball bearing is suitable.

The actual reliability is 90.6% under the given loads and a life of 30-million revs.

11-7 Variable Loading

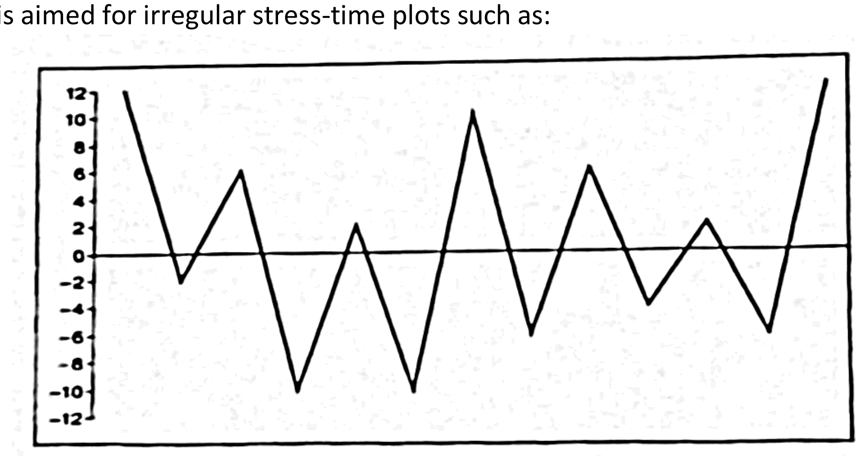
6-15 Cumulative Fatigue Damage

Sec. 11-7 is simply Sec. 6-15 as applied to bearings.

Cumulative Fatigue Damage

The first half of Sec 6-15 is about the *rain-flow counting technique* which is used to determine the minimum and maximum stresses at different load cycles.

The technique is aimed for irregular stress-time plots such as:



Details of the technique can be found in ASTM E1049-85(2017), “Standard Practices for Cycle Counting in Fatigue Analysis.”

The outcome of rain-flow counting is a list of information such as cycle index i, maximum and minimum stresses σ_{max_i} and σ_{min_i}, and the number of cycles n_i, within a repetitive time block.

For example, the stress-time plot of $Q1$ of $A1$ can be summarized as follow:

Cycle index	σ_{max_i}	σ_{min_i}	n_i
1	8	-8	13
2	16	-16	4
3	20	-20	2
4	28	-28	1

The orders of occurrence of the stress cycles are not taken into account. This simplified the cumulative fatigue damage analysis, but is however a well recognized drawback of the analysis.

The second half of Sec. 6-15 contains the Palmgren-Miner rule (or known as Miner’s rule in North America) for cumulative fatigue damage analysis.

The basic premise is, if it takes N cycles to fail (by fatigue) a component, then each cycle contributes towards the eventual failure, or does damage, by the amount of 1/N. When the damage adds up to 1 (i.e. 100%), the component fails.

The rule states that:

$$\sum \frac{n_i}{N_i} = c \qquad (6\text{–}57)$$

Where n_i is the number of cycles at stress level σ_i, and N_i is the number of cycles to failure as if all cycles were loaded at stress level σ_i. n_i is determined from load-time plot; N_i is by finite life calculation.

The constant c is determined experimentally. It is found that $c = 0.7{\sim}2.2$. But $c = 1$ is typically used.

$$D = \sum \frac{n_i}{N_i} \qquad (6\text{–}58)$$

where D is the accumulated damage. When $D = c = 1$, failure ensues.

When evaluating N_i, if the component is found to have infinite life, N_i is set to ∞ and $n_i/N_i = 0$.

Finally, stress level σ_i means σ_{rev_i}, the i-th σ_{rev}.

Bearings under Variable Loading
Applying Eq. (6-58) to bearings, the result is

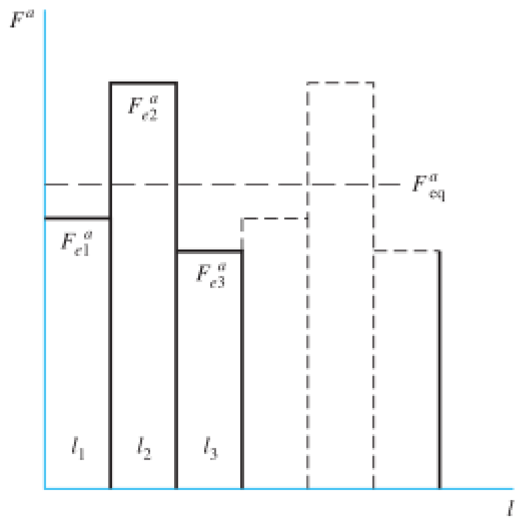
$$\sum \frac{l_i}{L_i} = 1 \qquad (11\text{–}16)$$

Where L_i is the bearing life (in revs) under load level F_{ei}, and I_i is the number of revs under load F_{ei}.

- (Eq. 11-16) is only applicable to piecewise constant (including zero) loadings, see Figure 11-10.

Figure 11–10

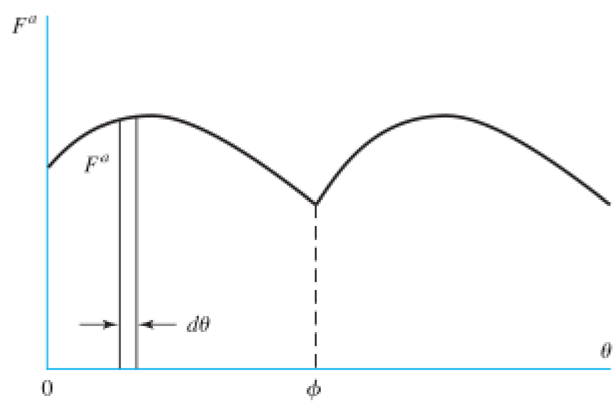
A three-part piecewise-continuous periodic loading cycle involving loads F_{e1}, F_{e2}, and F_{e3}. F_{eq} is the equivalent steady load inflicting the same damage when run for $l_1 + l_2 + l_3$ revolutions, doing the same damage D per period.



- If the load is continuous (see Figure 11-11), the summation is to be replaced by integral. Example 11-6 shows how it is done.

Figure 11–11

A continuous load variation of a cyclic nature whose period is ϕ.



- (Eq. 11-16) can be used to find
  - The life of a bearing under variable loading
  - The equivalent radial load under variable loading

$$F_{eq} = \left[\sum f_i (a_{fi} F_{ei})^a\right]^{1/a} \qquad L_{eq} = \frac{K}{F_{eq}^a} \tag{11–15}$$

Where f_i is the fraction of revs under F_{ei}, the equivalent radial load in load cycle i; and k_{ai} is the load-application factor associated with loading condition for load cycle i.

F_{eq} is to replace all the individual F_{ei}'s with the aim of simplifying the calculation. The load application factor on F_{eq} is 1.

Example 11-5

A ball bearing is run under four piecewise continuous steady loads. Information is given in tabular format, cols. (1), (2), (5) to (8) in particular. Other columns are from calculations.

(1) Time Fraction	(2) Speed rev/min	(3) Product Column	(4) Turns Fraction,	(5) F	(6) F	(7) F	(8) a_{fi}	(9) (7)x(8)
0.1	2000	200	0.077	600	300	794	1.10	873
0.1	3000	300	0.115	300	300	626	1.25	795
0.3	3000	900	0.346	750	300	878	1.10	966
0.5	2400	1200	0.462	375	300	668	1.25	835
Σ		2600	1.000					

Note:

The a_{fi} in Col. (8) are load application factors k_{ai}, which are either given, or selected from Table 11-5.
Last row of Col. (3) gives the equivalent rpm;
Col. (4) gives the f_i

Example 4

The table below lists the information relating to four piecewise constant loads that a ball bearing is subjected to.

Load case index i	f_i	F_{ei}, lb	k_{ai}	$(k_{ai}F_{ei})^a$x10^3 lb^3	$f_i(k_{ai}F_{ei})^a$x10^6 lb^3
1	0.08	794	1.10	666.25	53.30
2	0.115	626	1.25	479.13	55.01
3	0.35	878	1.10	900.87	315.3
4	0.455	668	1.25	582.18	264.9
Σ					688.5

The chosen bearing has $C_{10} = 9.56\ kN$ (rated at 10^6 revs). Determine the life of the bearing. Reliability is 95%, All other conditions (such as rotating inner ring, room temperature, non-corrosive environment, and so on) are assumed typical.

Solution:

There are two ways to find L_D.

(1) $F_{eq} \to L_D$ by the basic bearing equation. For F_{eq}, we need $f_i(k_{ai}F_{ei})^a$

(Eq. 11-15)a:

$$F_{eq} = \sqrt[3]{688.6(10^6)} = 833.0\ lb = 3929\ N \to F_D$$

Life of the bearing is then evaluated:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

$$\frac{9560}{(1)(3929)} = \left(\frac{L_D}{(0.62)(10^6)}\right)^{1/3}$$

So, $L_D = 8.93$ million revs.

(2) $F_{ei} \rightarrow L_i$ for each load case by the basic bearing equation. Then use the Miner's rule to find L_D.

Load case index i	f_i	F_{ei} lb	k_{ai}	L_i x10^6, revs
1	0.08	794	1.10	9.23
2	0.115	626	1.25	12.83
3	0.35	878	1.10	6.82
4	0.455	668	1.25	10.56

If L_D denotes the life of the bearing under the given variable loading, then $l_i = f_i L_D$

$(Eq. 11-16)$ becomes:

$$\sum \frac{l_i}{L_i} = \frac{0.08L_D}{9.23(10^6)} + \cdots + \frac{0.455L_D}{10.56(10^6)} = 1$$

So $L_D = 8.93$ million revs.

11-10 Design Assessment for Selected Bearings

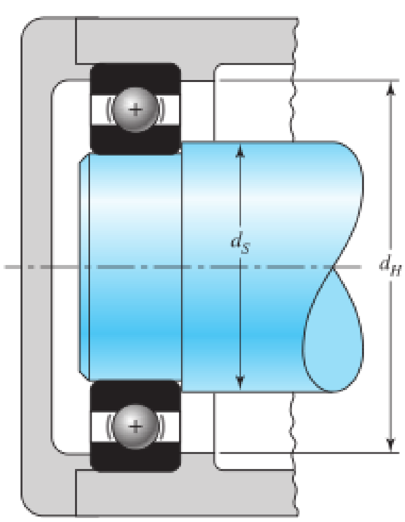
Once a selection was made, assessing for the following:

- Reliabilities of individual bearings, and of pairs of bearings;
- Shouldering on shaft and housing;
- Journal's and housing's tolerance;
- Lubrication; (see Sec. 11-11)
- Pre-load, where applicable. (see Sec. 11-12)

Reliability

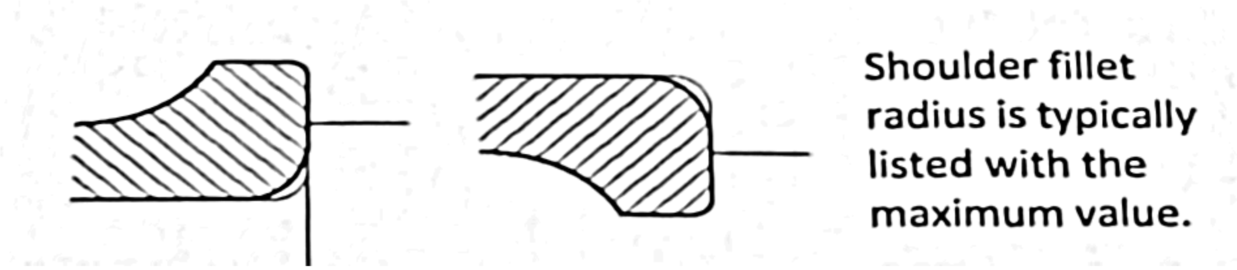
- The reliability of a pair of bearings A and B is $R = R_A R_B$;
- If $R \geq 90\%$ is required, ten one bearing (or both bearings) has (or have) to have more than 90% reliability. For example, $(0.9)(0.99) \approx 0.9$; $(0.95)(0.95) \approx 0.9$; On the other hand, $(0.9)(0.9) \approx 0.81$

Shoulders and Fillets



d_S is the shaft shoulder diameter. Catalogs may list the min and max values of d_S. If only one value is given, it is typically the minimum value.

d_H is the housing shoulder diameter. Catalogs may list the min and max values of d_H. If only one value is given, it is typically the maximum value.



Fits between Journal Bearing, and Bearing and Housing
Rotating ring: press fit;
Nonrotating ring: push fit or tap fit;

Tolerances on Journal and Housing
Bearings manufacture tolerances are specified by ABEC (Annular Bearing Engineering Committee). There are 5 AEBC grades/scales, 1, ,3 , 5, 7 and 9. The higher the grade, the higher the precision.

Catalogs provide details of bearing tolerances.

Tolerances are then assigned to the journal and housing, based on the required or chosen fits.

In general, with rotating journals, choices are j6 (or k6, m6 n6, j5) Note that these are transition fits.

For stationary housing, we may choose H7 (or G7, F7, H8). They are clearance fits.

Catalogs have detailed recommendations.

Table 2

Fits for solid steel shafts

Radial bearings with cylindrical bore

Conditions	**Examples**	**Shaft diameter, mm** Ball bearings	Cylindrical and taper roller bearings	CARB and spherical roller bearings	**Tolerance**
Rotating inner ring load or direction of load indeterminate					
Light and variable loads ($P \leq 0{,}06\ C$)	Conveyors, lightly loaded gearbox bearings	(18) to 100	≤ 40	–	j6
		(100) to 140	(40) to 100		k6
Normal and heavy loads ($P > 0{,}06\ C$)	Bearing applications generally, electric motors, turbines, pumps, internal combustion engines, gearing, woodworking machines	≤ 18	–	–	j5
		(18) to 100	≤ 40	≤ 40	k5 (k6)[1]
		(100) to 140	(40) to 100	(40) to 65	m5 (m6)[1]
		(140) to 200	(100) to 140	(65) to 100	m6
		(200) to 280	(140) to 200	(100) to 140	n6
		–	(200) to 400	(140) to 280	p6
		–	–	(280) to 500	r6[2]
		–	–	> 500	r7[2]

11-11 Lubrication

- Purpose of lubrication;
- Types of lubricant (grease and oil);
- When to use what?
  See the rules listed near the end of Section 11-11.

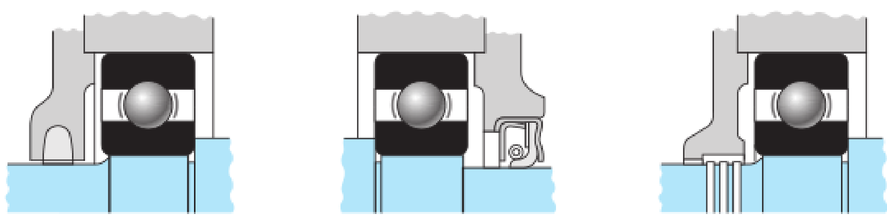
Use Grease When	Use Oil When
1. The temperature is not over 200°F.	1. Speeds are high.
2. The speed is low.	2. Temperatures are high.
3. Unusual protection is required from the entrance of foreign matter.	3. Oiltight seals are readily employed.
4. Simple bearing enclosures are desired.	4. Bearing type is not suitable for grease lubrication.
5. Operation for long periods without attention is desired.	5. The bearing is lubricated from a central supply which is also used for other machine parts.

11-12 Mounting and Enclosure

- An enclosure is used to prevent dirt and foreign matter from a entering a bearing, and to retain lubricant.
- Figure 11-26 shows typical means of external (to bearing) seals. That is, shaft and components inside the housing will be protected as well.

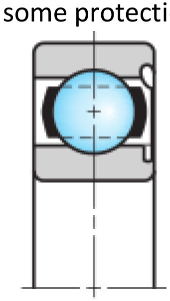
Figure 11–26

Typical sealing methods. *(General Motors Corp. Used with permission, GM Media Archives.)*



(*a*) Felt seal (*b*) Commercial seal (*c*) Labyrinth seal

- Shielded bearings (Figure 11-2d) provide some protection against dirt, but not a complete closure.



(*d*)
Shielded

- A sealed bearing, when sealed on both sides (Figure 11-2e), keeps lubricant in, for life, but can be relubricated.

Mounting

- Mounting bearings in a trouble-free and low-cost way is an important but challenging part of any design.
- Bearing catalogs area good source of information and guide, giving details for many design situations.
- The following is to be considered:
    - Axially locating a single bearing on a shaft
    - Locating and non-location arrangements of a pair of bearings
    - Misalignment
    - Preloading

Axially Locating a Single Bearing

Shoulder or spacer, and locknut or end plate or snap ring or retaining ring, for example.

Locating and Non-location Arrangements

- Such arrangements are the methods to locate the shaft as well as to allow for thermal expansion or contraction in the axial direction.
- The principle behind is: thrust load in each direction much be carried by *one* and *only one* bearing.
- Figure 11-20 and Figure 11-21 illustrate the locating and non-location arrangements, respectively, for two situations where the shaft is to be supported by two bearings, one at or near each end of the shaft.

Figure 11–20

A common bearing mounting.

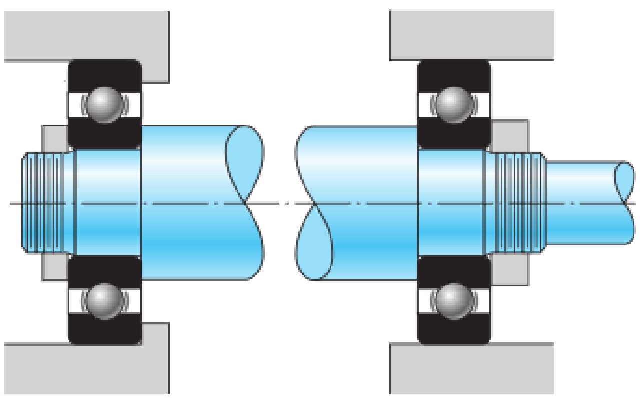


Figure 11–21

An alternative bearing mounting to that in Fig. 11–20.