From previous example:

$\begin{cases} \text{Table A} - 9 \\ \text{Case } 10 \end{cases}$

$$y_{12} = y_{AB}|_{x=0.45} = \frac{(-700)(0.225)(0.45)(0.9^2 - 045^2)}{(6 * 0.9)EI}$$
$$= -\frac{7.9738}{EI}$$

$$y_{22} = y_t = -\frac{(-700)(0.225)^2(0.9 + 0.225)}{3EI} = \frac{13.289}{EI}$$

$\begin{cases} \text{Table A} - 9 \\ \text{Case } 5 \end{cases}$

$$y_{11} = y_{max} = -\frac{(900)(0.9)^3}{48EI} = -\frac{(13.669)}{EI}$$
$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$\theta_{AB} = \frac{dY_{AB}}{dx} = \frac{F}{16EI}(4x^2 - l^2) \quad ; \quad 0 \leq x \leq \frac{l}{2}$$
$$\theta_A = \frac{(900)(-0.9^2)}{16EI} = -\frac{45.563}{EI}$$
$$y_{21} = (-\theta_A)(0.225) = \frac{10.252}{EI}$$

Or:

$$y_{21} = (\theta_C)(0.225) = \frac{10.252}{EI}$$

$$\therefore y_1 = y_{11} + y_{12} = -\frac{21.643}{EI}$$
$$y_2 = y_{21} + y_{22} = \frac{23.541}{EI}$$

Rayleigh's Method:

$$\therefore \omega_1 = \sqrt{\frac{g(\sum w_i |y_i|)}{\left(\sum w_i {y_i}^2\right)}}$$
$$= \sqrt{(9.81)\frac{(900)\left(\frac{21.643}{EI}\right) + (700)\left(\frac{23.541}{EI}\right)}{(900)\left(\frac{21.643}{EI}\right)^2 + (700)\left(\frac{23.541}{EI}\right)^2}}$$
$$= 0.66011\sqrt{EI}$$

Dunkerley's Method:

$$y_{11} = -\frac{13.669}{EI}$$
$$y_{22} = \frac{13.289}{EI}$$
$$\omega_{11}^2 = \frac{g}{|y_{11}|} = 0.71768\ EI$$

$$\omega_{22}^2 = \frac{g}{|y_{22}|} = 0.73820\ EI$$

$$\therefore \frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} = \frac{2.7480}{EI}$$

$$\omega_1 = \sqrt{\frac{EI}{2.7480}} = 0.60324\sqrt{EI}$$

Take $E = 200\ GPa, d = 25\ mm$

$\sqrt{EI} = 1261.6\ (N \cdot m^2)$

$\therefore\ \omega_{1(Dunkerley)} = (0.60324)(1261.6)$

$= 761.0\ rad/s$

$Or\ n_{1(Dunkerley)} = 7267\ rpm$

$Also\ \omega_{1(Rayleigh)} = 832.8\ rad/s$

$Or\ n_{1(Rayleigh)} = 7952\ rpm$

$$\therefore Operating\ Speed \le \frac{1}{3}\omega_1$$

$Or\ n \le 2422\ rpm$

**7-3 Shaft Layout**

- Between a shaft and its components (e.g., gears, bearings, pulleys, etc.), the latter must be located axially and circumferentially.
- Means to provide for torque transmission
  Keys
  Splines
  Setscrews
  Pins
  Press/shrink fits
  Tapered fits
  etc.
- Means to provide for axial location – large axial load shoulders
  Shoulders
  Retaining rings
  sleeves
  Collars
  etc.
- Means to provide for axial location – small axial load
  Press/shrink fits
  Setscrews
  etc.
- Locating rolling element bearings
  See Ch. 11

**7-7 Miscellaneous Shaft Components**

Includes:

- Setscrews
- Keys and pins
- Retaining rings

Focus:

Keys

Example 7-6

**7-8 Limits and Fits**

- Fits (clearance, transition, and interference) are to ensure that a shaft and its components/attachments will function as intended.
- Preferred fits are listed in Table 7-9
- Medium drive fit and force fit will give rise to the press/shrink fits.
- Press-fit is typically for small hubs; Shrink fit (or expansion fit) it used with larger hubs
- How much diameter interference to have?
  0.001” for up to 1” of diameter
  0.002” for diameter 1” to 4”
- Press/shrink fits can be designed to transfer torque and axial load.
- Press/shrink fits are known to be associated with fretting corrosion (loss of material from the interface)
- (Section 3-14) for stress distributions in thick-walled cylinder under pressures.
- (Section 3-16) for stresses developed in the shaft and hub due to pressure induced by a press/shrink fit; or (Eq 7-39) to (Eq. 7-47)
- Axial load and torque capacities: (Eq. 7-48) and (Eq. 7-49)
- Radial interference versus diametral interference (not necessarily the same thing).

# Chapter 11: Rolling-Contact Bearings

Part 1: (Introduction)
11-1 Bearing Types

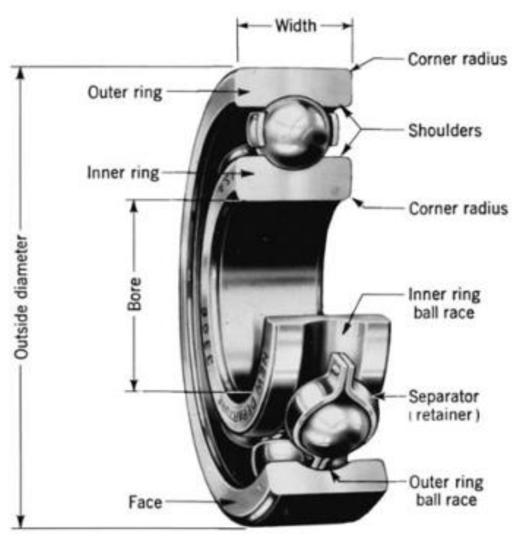
Part 2: (The Basics)
11-2 Bearing Life
11-3 Bearing Life at Rated Reliability
11- 4Reliability versus Life – The Weibull Distribution
11-5 Relating Bearing Load, Life and Reliability

Part 3: (Selection of Bearing)
11-6 Combined Radial and Thrust Bearing
11-8 Selection of Ball and Roller Bearings
11-7 Variable Loading
11-10 Design Assessment

Part 4: Others
11-12 Mounting and Enclosure
11-11 Lubrication
11-9 Selection of Tapered Roller Bearings

**11-1 Bearing Types**
**Nomenclature**
See Figure 11-1

**Figure 11–1**
Nomenclature of a ball bearing. *(General Motors Corp. Used with permission, GM Media Archives.)*




**Classifications**

- By shape of rolling elements (sphere, cylinder, tapered, etc.)
- By type of loads taken (radial only, axial only, combination)
- By permissible slope (Self-aligning, non-self-aligning)
- Sealed? Shielded?
- Figures 11-2, 11-3

**Figure 11–2**

Various types of ball bearings.

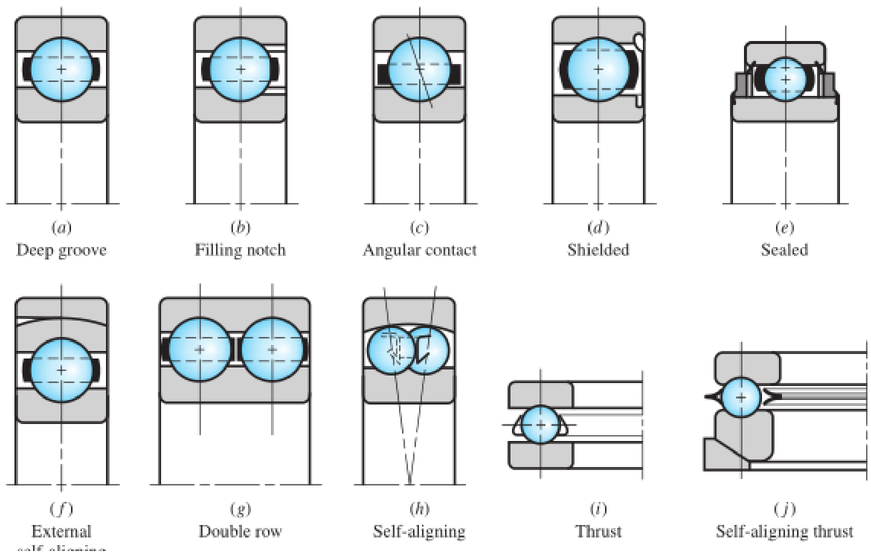
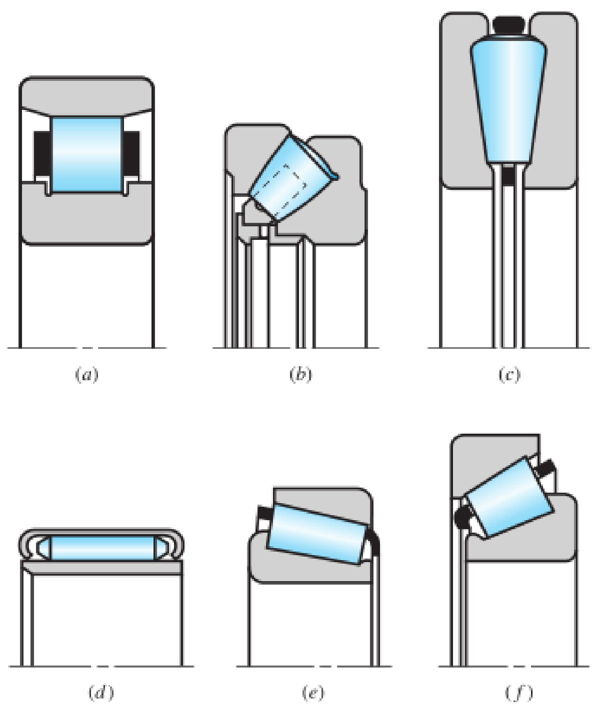



**Figure 11–3**

Types of roller bearings: (*a*) straight roller; (*b*) spherical roller, thrust; (*c*) tapered roller, thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. *(Courtesy of The Timken Company.)*




**11-2 Bearing Life**

**Why Bearing Life?**

- Bearings are under cyclic contact stresses (compressive as well as shear). As a result, they may experience crack, putting, spalling, fretting, excessive noise, and vibration.
- Common life measures are:
  - Number of revolutions of the inner ring (outer ring stationary) until the first evidence of failure; and
  - Number of hours of use at standard angular speed until the first evidence of fatigue
- Number of revolutions is more common

**Rating Life (**or **Rated Life)**

- This is the terminology used by ABMA (American Bearing Manufacturers Association) and most bearing manufacturers.
- It is defined as, of a group of nominally identical bearings, the number of revolutions that 90% of the bearings in the group will achieve or exceed, before failure occurs.
- It is denoted as $L_{10}$ or $B_{10}$ life.
- The typical value for $L_{10}$ or $B_{10}$ is 1 million.
- However, a manufacturer can choose its own specific rating life.
- For example, Timken uses 90 million for tapered roller bearings, but 1 million for its other bearings.
- Refer to bearings catalog for value(s) of $L_{10}$.

**11-3 Bearing Life at Rated Reliability**

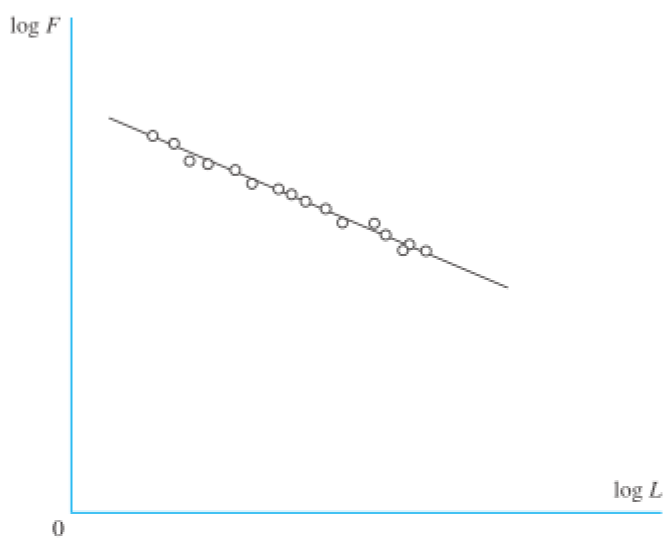
- At rated reliability of 90%, bearing life $L$ relates to bearing's radial load $F$ by:

$$FL^{1/a} = constant$$

  Where $a = 3$ for ball bearings and $a = 10/3$ or roller bearings.
  Figure 11-4 shows the meaning of (Eq. 11-1)

  **Figure 11–4**

  Typical bearing load-life log-log curve.




  - It's a straight line on log-log scales;
  - Points on the line will have the same reliability;
  - The line corresponding to 90% reliability is called the rated line.
- To determine the constant on the RHS of (Eq. 11-1), $L$ is set to $L_{10}$. The corresponding $F$ is designated as $C_{10}$.
  $C_{10}$ is called the **Basic Dynamic Load Rating**, or the **Basic Dynamic Rated Load** It is defined as the radial load that causes 10% of the group of nominally identical bearings to fail at or before $L_{10}$ (1 million, or 90 million, or revs as chosen by a manufacturer).
- (Equation 11-1) becomes:

$$F_D L_D^{1/a} = C_{10} L_{10}^{1/a} \qquad (11-3)^*$$

$$C_{10} = F_R = F_D\left(\frac{L_D}{L_R}\right)^{1/a} = F_D\left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} \qquad (11\text{–}3)$$

Where the subscript $D$ means design. (Equation 11-3)* is essentially (Eq. 11-3) of the text. In (Eq. 11-3), the subscript $R$ means rated.

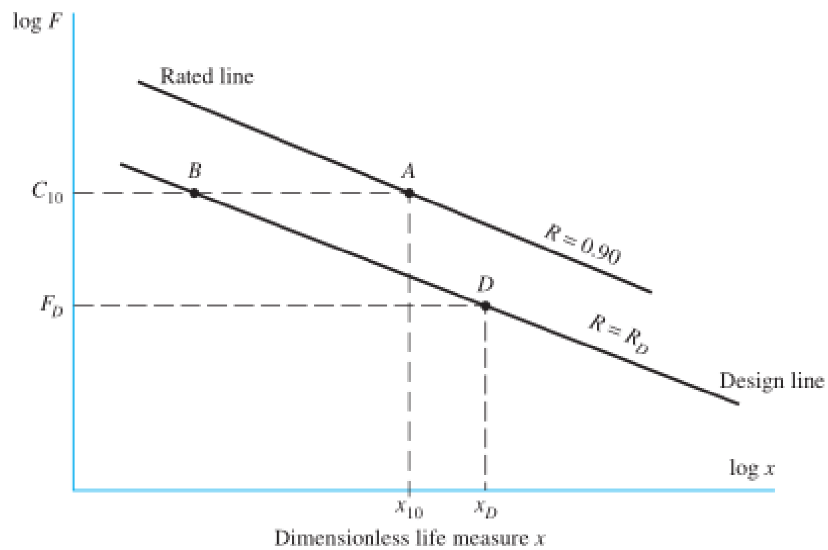
- Example 11-1: find $C_{10}$ from known $L_D$, $F_D$ and $L_{10}$

**11-4 Reliability versus Life – The Weibull Distribution**
**11-5 Relating Load, Life and Reliability**

- At 90% reliability, (Eq. 11-3*) forms the basis for selecting a bearing.
- What if the reliability is not 90%?
  Figure 11-5 shows the process of going from the rated line to a different line.
  $A \to D$, with $B$ being the intermediary.

**Figure 11–5**

Constant reliability contours. Point $A$ represents the catalog rating $C_{10}$ at $x = L/L_{10} = 1$. Point $B$ is on the target reliability design line $R_D$, with a load of $C_{10}$. Point $D$ is a point on the desired reliability contour exhibiting the design life $x_D = L_D/L_{10}$ at the design load $F_D$.




- $A \to B$: Load is constant, and life measure (which is a random variable) follows a three-parameter Weibull distribution
- $B \to D$: Reliability is constant, and (Eq. 11-3*) is valid; The mathematics is given in (Sec. 11-4) and (Sec. 11-5).
- Another approach is to use a reliability factor $a_1$, the value of which depends on $R$, the reliability. Values of $a_1$ are available from a number of references.

| **R, %** | **Reliability Factor, $a_1$** |
|---|---|
| 90 | 1.00 |
| 95 | 0.62 |
| 96 | 0.53 |
| 97 | 0.44 |
| 98 | 0.33 |
| 99 | 0.21 |

- The factor $a_1$ can be determined by (courtesy of, for example SKF catalog)

$$a_1 = 4.48\left(\ln\frac{100}{R}\right)^{2/3}$$

Where $R$ is in %, e.g., $R = 92.5$. Note: $R \le 99$.

- Timken recommends the following formula for $a_1$

$$a_1 = 4.26\left(\ln\frac{100}{R}\right)^{2/3} + 0.05$$

Where $R \leq 99.9$

**The Basic Bearing Equation**

The basic bearing equation can now be obtained by, in (Eq. 11-3), introducing the reliability factor $a_1$ and a load-application factor $k_a$. That is,

$$F_D L_D^{1/a} = C_{10} L_{10}^{1/a}$$

Becomes,

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

$k_a$ is given in (Table 11-5)

**Table 11-5**

Load-Application Factors

| Type of Application | Load Factor |
|---|---|
| Precision gearing | 1.0–1.1 |
| Commercial gearing | 1.1–1.3 |
| Applications with poor bearing seals | 1.2 |
| Machinery with no impact | 1.0–1.2 |
| Machinery with light impact | 1.2–1.5 |
| Machinery with moderate impact | 1.5–3.0 |

This basic equation can be used for bearing selection and for assessment after selection.

**Example 1**

A SKF deep-groove ball bearing is subjected to a radial load of 495 lb. The shaft rotates at 300 rpm. The bearing is expected to last 30,000 hours (continuous operation). Catalog shows a $C_{10} = 19.5\ kN$ on the basis of $10^6$ revs. (1) is the bearing suitable for 90% reliability? (2) Also assess the bearing's reliability. Set $k_a = 1$.

Solution:

The basic bearing equation is:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

(Where $a$ and $L_{10}$ are generally set by the manufacturer, $k$ is a variable we can change.)

Where:

$C_{10} = 19.5\ kN = 4387.5\ lb$; $L_{10} = 10^6\ revs$; $k_a = 1$; $a = 3$

Also $F_D = 495\ lb$; and $L_D = (30{,}000)(60)(300) = (540)(10^6)\ revs$.

Assume 90% reliability, then $a_1 = 1$. From the basic bearing equation,

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting values, $LHS = 8.92$, $RHS = 8.14$. Therefore, $(L_D, F_D)$ is not on the rated line.

There are a number of ways to seek the answer.
90% reliability, $a_1 = 1$;

<u>The first</u>: *similar to the typical calculations done for selecting a bearing*
Set $F_D = 495\ lb$; $L_D = (540)(10^6)\ revs$; and $L_{10} = 10^6\ revs$; find $C_{10}$ and check if it is less than the $4387.5\ lb$ that the bearing is capable of providing.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{C_{10}}{(1)(495)} = \left(\frac{(540)(10^6)}{(1)(10^6)}\right)^{1/3}$$

Resulting in $C_{10} = 4031\ lb$
Since it's less than the catalog's $C_{10}$, or $4387.5\ lb$, the selected bearing is suitable for 90% reliability.

<u>The second:</u> *can be used to select a bearing (pre-selecting a bearing, then checking to make sure it is suitable)*
Set $L_D = (540)(10^6)\ revs$, find $F_D$, and check is $F_D \geq 495\ lb$.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)F_D} = \left(\frac{(540)(10^6)}{(1)(10^6)}\right)^{1/3}$$

Solving gives $F_D = 538.8\ lb$
∴ with 90% reliability and a life of $(540)(10^6)\ revs$, the bearing can take on a maximum radial load of $538.8\ lb$. Since the applied radial load is only $495\ lb$, the bearing will have better than 90% reliability.

<u>The third:</u> *similar to post-selection calculation to evaluate the life of the bearing.*
Set $F_D = 495\ lb$, find $L_D$, and check if $L_D \geq (540)(10^6)\ revs$.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)(495)} = \left(\frac{L_D}{(1)(10^6)}\right)^{1/3}$$

Solving gives $L_D = (696)(10^6)\ revs$
With a radial load at $495\ lb$, the bearing has 90% chance proabability to survive at least $(696)(10^6)\ revs$. The chance of surviving only $(540)(10^6)\ revs$ is better than 90%.

(2) Set $F_D = 495\ lb$; $L_D = (540)(10^6)\ revs$
To assess reliability means to evaluate $a_1$. This is typically done after selection.

From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)(495)} = \left(\frac{(540)(10^6)}{a_1(10^6)}\right)^{1/3}$$

Solving gives $a_1 = 0.775$

Finally, from:

$$a_1 = 4.26\left(\ln\frac{100}{R}\right)^{2/3} + 0.05$$

Solving for $R$ results in $R = 93\%$.
With the radial load at $495\ lb$, there is a $7\%$ of chance that the bearing would fail at or before $540$ millions of revs.
It shows that the bearing is more than suitable for $90\%$ reliability.

**Example 2**
Select bearings A and B for the shaft of Example 7-2. They are to be used for a minimum of 1,000 hours of continuous operation. Shaft rpm is 450. Radial loads are, $F_A = 375\ lb$ and $F_B = 1918\ lb$. Shaft diameter at both locations is $1"$ ($D_1$ and $D_7$ in Figure 7-10). Assume $90\%$ reliability.

Solution:
Table 11-2
Table 11-3
Since there is no thrust load, deep-groove ball bearings may