Chapter 6: Fatigue Failure (Review)

I) **Sec 6-17 Road Maps and Important Equations**

II) **Loading**

Simple loading

- Axial loading

- Torsion

- Bending

Combined Loading

III) **Characterizing Stress (Fig. 6-23d, e, f)**

Completely reversed stress

Repeated stress

Fluctuating stress

IV) **Endurance Limit S'_e (Eq. 6-8)**

Correction Factors: $k_a\ k_b\ k_c\ k_d\ k_e$ (Sec. 6-9)

Corrected Endurance Limit S_e (Sec. 6-18)

V) **Equivalent Stresses**

Theoretical stress concentration factors: $k_t\ k_{ts}$ (A-15)

North sensitivity: $q\ q_{shear}$ (Sec. 6-10)

Fatigue stress concentration factors: $k_f\ k_{fs}$ (Eq. 6-32)

Von Mises Stress for alternating comp. (Eq. 6-55)

Von Mises Stress for mid-range comp. (Eq. 6-56)

VI) $\sigma'_m = 0\ (Sec. 6-8)$

If $\sigma'_a \le S_e$; infinite life and factor of safety is:

$$n = \frac{S_e}{\sigma'_a}$$

Else finite life and number of stress cycles N is by (Eq. 6-16) where σ_{rev} is set to σ'_a

$\sigma'_m > 0\ (Sec. 6-12)$

Decide what to use, Soderberg? Modified Goodman? Gerber? ASME Elliptic?

(Eq. 6-45 to Eq. 6-48) for factor of safety n.

If $n \ge 1$, finite life.

Else finite life and N is by (Eq. 6-16) but σ_{rev} is per Step 4 on pp. 340.

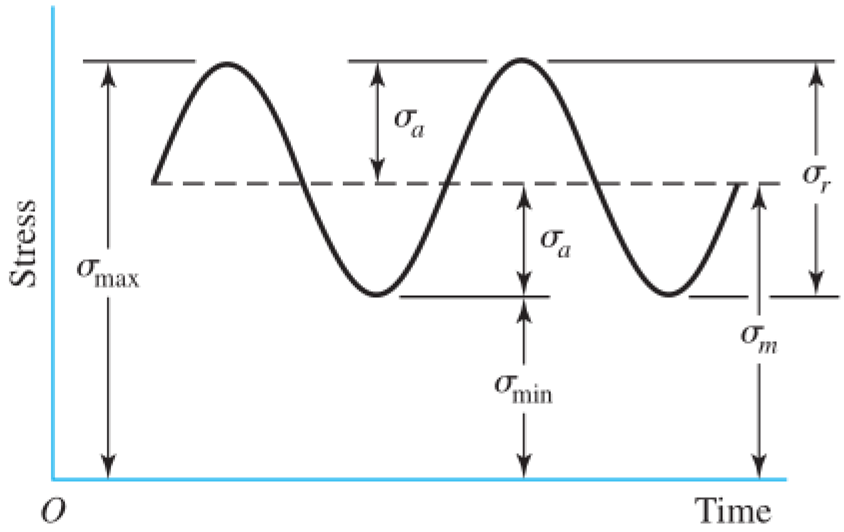

Figure 1: Fluctuating Stress

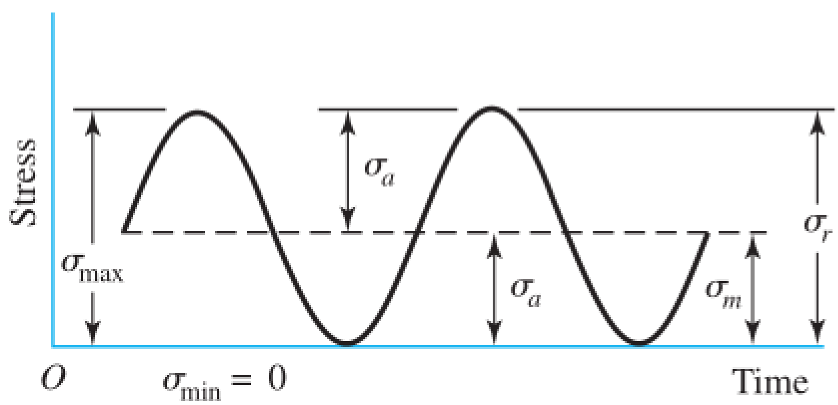

Figure 2: Repeated Stress

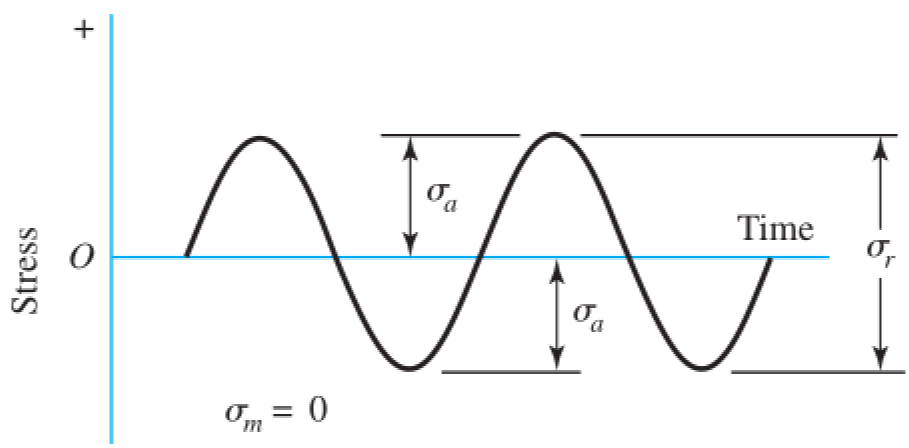

Figure 3: Completely Reversed Stress

Example (1): A low carbon steel stock is lathe-turned to have a diameter of 1" The stock has $S_{ut} = 100\ ksi$, $S_y = 76\ ksi$. Axial load varies $-10 \sim 50\ kips$. Fatigue stress concentration factor is $K_f = 1.3$. Find factor of safety n if infinite life, or number of cycles N if infinite life. Assume room temperature and 99% reliability.

Case: Simple loading, fluctuating stress

$$F_{min} = -10\ kips$$
$$F_{max} = 50\ kips$$

$$\sigma_{min} = \frac{k_f F_{min}}{A} = -16.552\ ksi$$
$$\sigma_{max} = \frac{k_f F_{max}}{A} = 82.761\ ksi$$

$$\sigma_m = 33.105\ ksi$$
$$\sigma_a = 49.657\ ksi$$

$$S_e' = 0.5 S_{ut} = 50\ ksi$$

$$k_a = 0.797$$
$$k_b = 1$$
$$k_c = 0.85$$
$$k_d = 1$$
$$k_e = 0.814$$

$$S_e = 27.572\ ksi$$

Criterion	Equation	n
Soderberg	6-45	0.45
Modified Goodman	6-46	0.47
Gerber	6-47	0.54
ASME-elliptic	6-48	0.54

$\therefore\ finite\ life$

$$f = 0.845$$
$$a = 258.97\ ksi$$
$$b = -0.16213$$

Criterion	σ_{rev}, ksi	N
Soderberg	87.981	779
Modified Goodman	74.231	2223
Gerber	55.769	12974
ASME-elliptic	55.166	13874

(see pp. 314 for procedure)

Chapter 7: Shafts and Shaft Components

(7-1): Introduction

Shaft Loading

- Power transmission shafting is to transmit power/motion from an input source (e.g. motors, engines) to an output work site.
- Shafts are supported by bearings, and loaded torsionally, transversely, and/or axially as the machine operates.
- Shafts can be solid or hollow, and are often stepped.
- They are widely required by virtually all types of machinery and mechanical systems.

New Shaft Design Procedure

- Conceptual sketch for shaft <u>layout</u>, based on functional spec. and system config (7-3)
- Shaft materials (7-2)
- An appropriate <u>design factor</u>
- Support reactions, bending moment diagrams (in tow planes, as well as resultant/combined), and torque diagram, critical cross-sections.
- Shaft <u>diameters</u> based on strength requirement (7-4)
- <u>Slopes and Deflections</u> at locations of interest in order to select bearings, couplings, etc.: or to ensure proper functioning of bearings, couplings, gears, etc. (7-5)
- <u>Critical Speeds</u> and other vibration characteristics (7-6)

Calculations Needed

- Shaft diameters based on strength requirement > by ANSI/ASME standard B106-1M-1985 “Design for Transmission Shafting”, or by other practices.
- Slopes and deflections > simplified / approximate geometry, numerical, graphical, FEA;
- Critical speed and other vibration characteristics > specific deflections of shaft.

(7-2): Shaft Materials

- Requiring generally/typically high strength and high modulus of elasticity
- Typical selection: low carbon steel (cold-drawn or hot-rolled) such as ANSI 1020-1050 steels
- If higher strength is required, alloy steels plus heat treatment such as ANSI 1340-50, 4140, 4340, 5140, 8650.

- Cold-drawn steel is used for diameters under 3 inches; machining is not needed where there is no fitting with other components.
- Hot-rolled steels should be machined all over.
- Stainless steels when environment is corrosive, for example.

Equations for the Fatigue Failure Criteria
(6-40) Soderbeg
(6-41) Modified Goodman
(6-42) Gerber
(6-43) ASME-Elliptic

For factor of safety:
Replace S_a with $n\sigma_a$
Replace S_m with $n\sigma_m$
Resulting in Equations $(6-45) \sim (6-48)$

For σ_{rev} (which is needed for number of cycles N):
Replace S_a with σ_a
Replace S_m with σ_m
Replace S_e with σ_{rev}
and solving for σ_{rev}

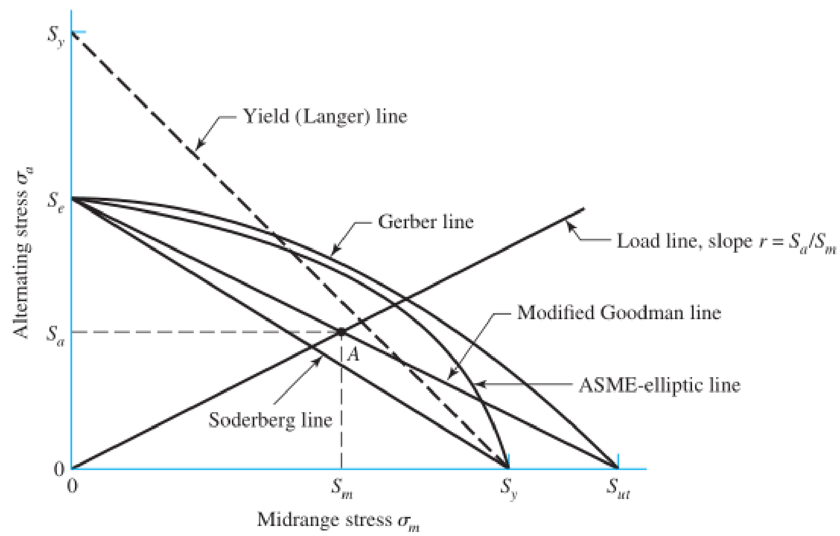

Figure 6–27

Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a.

Modified-Goodman line – too dangerous, goes directly to S_{ut}
Soderberg – too conservative

Most of the time, we'll be using ASME-elliptic line, not

Example (2): A non-rotating shaft is lathe-turned to have a 1"-diameter. The shaft is subject to a torque that varies $0 \sim T_{max} (in\ lb - in)$. Determine T_{max} such that the shaft will have an infinite life with a factor of safety of 1.8. The shaft's material has $S_{ut} = 100\ ksi$ and $S_y = 76\ ksi$. Assume room temperature and 99% reliability. Fatigue stress concentration factor is $k_{fs} = 1.6$.

Answer: Based on simple loading and ASME-elliptic criterion, $T_{max} \approx 2480\ lb - in$

Design Factor vs. Factor of Safety (not from the text)

- Design factor is to indicate the level of overload that the part/component is required/intended to withstand
- Safety factor indicated how much overload the designed part will actually be able to withstand.
- Design factor is chosen, generally in advance and often set by regulatory code or an industry's general practice.
- Safety factor is obtained from design calculations.

The following are some recommended values of design factor based on strength considerations. They are valid for general applications.

- 1.25 - 1.5: for reliable materials under controlled conditions subjected to loads and stresses known with certainty;
- 1.5 – 2: for well-known materials under reasonably constant environmental conditions subjected to known loads and stresses;
- 2 – 2.5: for average materials subjected to known loads and stresses;
- 2.5 – 3: for less well-known materials under average conditions of load, stress and environment;
- 3 – 4: for untried materials under average conditions of load, stress and environment;
- 3 – 4: for well-known materials under uncertain conditions of load, stress and environment.

Courtesy of Mechanical Design, 2nd Edition, P. Childs, Elsevier Ltd. (p. 95)

(7-4): Shaft Design for Stress

Loads on a Shaft

- The primary function of a shaft is to transmit torque, typically through only a portion of the shaft;
- Due to the means of torque transmission, shafts are subject to transverse loads in two planes such that the shear and bending moment diagrams are needed in two planes.

Shaft Stress from Fatigue Perspective

- In 1985, ASME published ANSI/ASME Standard B106.1M-1985 "Design for Transmission Shafting". However, it was withdrawn in 1994.
    - Little or no axial load
    - Fully reversed bending and steady (or constant) torsion
- General case: Fluctuating bending and fluctuating torsion
    - As considered by the text

Fully reversed bending and steady torsion is in fact special cases of the general one. In reality, the general case is not as common as the typical case of fully reversed bending and steady torsion.

Critical Locations

Stresses are evaluated at the critical locations. Look or where:

- Bending moment is large
- There is torque
- There is stress concentration

Factor of Safety or Required Shaft Diameter – General Case
Generally, $\sigma_{bending} = \sigma_m$ and σ_a
And $\tau_{torsion} = \tau_m$ and τ_a

Assuming negligible axial load, due to fluctuating bending and fluctuating torsion, the amplitude and mean stresses are given by (Eq. 7-1) and (Eq. 7-2):

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I} \qquad (7\text{–}1)$$

$$\tau_a = K_{fs} \frac{T_a r}{J} \qquad \tau_m = K_{fs} \frac{T_m r}{J} \qquad (7\text{–}2)$$

For solid shaft, stresses can be written in terms of d, the diameter, see (Eq. 7-3) and (Eq. 7-4):

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32 M_m}{\pi d^3} \qquad (7\text{–}3)$$

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16 T_m}{\pi d^3} \qquad (7\text{–}4)$$

Next, the equivalent (von Mises) amplitude and mean stresses are determined, resulting in (Eq. 7-5) and (Eq. 7-6):

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}5)$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}6)$$

Now, define terms A and B (pp. 361):

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

Finally, the pair of equations determining factor of safety and diameter, are (DE stands for distortion energy):

- For DE-Goodman: (Eq. 7-7), (Eq. 7-8)

DE-Goodman

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs}T_a)^2]^{1/2} + \frac{1}{S_{ut}}[4(K_f M_m)^2 + 3(K_{fs}T_m)^2]^{1/2}\right\} \quad (7\text{–}7)$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs}T_a)^2]^{1/2} + \frac{1}{S_{ut}}[4(K_f M_m)^2 + 3(K_{fs}T_m)^2]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}8)$$

- For DE-Gerber: (Eq. 7-9), (Eq-7-10)

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \quad (7\text{–}9)$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}10)$$

- For DE-ASME-Elliptic: (Eq. 7-11), (Eq. 7-12)

DE-ASME Elliptic

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs}T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs}T_m}{S_y}\right)^2\right]^{1/2} \quad (7\text{–}11)$$

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs}T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs}T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3} \quad (7\text{–}12)$$

- For DE-Soderberg: (Eq. 7-13), (Eq. 7-14)

DE-Soderberg

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs}T_a)^2]^{1/2} + \frac{1}{S_y}[4(K_f M_m)^2 + 3(K_{fs}T_m)^2]^{1/2}\right\} \quad (7\text{–}13)$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs}T_a)^2]^{1/2} + \frac{1}{S_y}[4(K_f M_m)^2 + 3(K_{fs}T_m)^2]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}14)$$

After the above fatigue-based calculations, it is customary to check against static failure.

- The factor of safety against yielding in the first loading cycle is (Eq. 7-16):

$$n_y = \frac{S_y}{\sigma'_{\max}} \quad (7\text{–}16)$$

Where the equivalent maximum stress is σ'_{max} by (Eq. 7-15)

$$\sigma'_{\max} = [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2} = \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \quad (7\text{–}15)$$

- A quick but conservative check against yielding in the first loading cycle is, see the paragraph following (Eq. 7-16):

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m}$$

Factor of Safety or Required Shaft Diameter – Typical Cases
The typical case is defined as, fully reversed bending and steady (or constant) torsion. Therefore, setting $M_m = 0$ and $T_a = 0$ in above equations will result in what are needed.

ASME-Elliptic:

Setting $M_m = 0$ and $T_a = 0$, (Eq. 7-5) and (Eq. 7-6) and the A and B terms become:

$$\sigma'_a = \sqrt{\sigma_a^2} = \sqrt{\left(K_f \frac{32M_a}{nd^3}\right)^2} = \frac{K_f 32M_a}{nd^3}$$

$$\sigma'_m = \frac{\sqrt{3}K_{fs}16T_m}{nd^3}$$

$$A = 2K_f M_a$$

$$B = \sqrt{3}K_{fs}T_m$$

(Eq. 7-11) and (Eq. 7-12) become,

$$n = \frac{1}{\sqrt{\left(\frac{\sigma'_a}{S_e}\right)^2 + \left(\frac{\sigma'_m}{S_y}\right)^2}} = \frac{nd^3}{16}\frac{1}{\sqrt{\left(\frac{A}{S_e}\right)^2 + \left(\frac{B}{S_y}\right)^2}}$$

$$d = \sqrt[3]{\frac{16}{nd^3}\sqrt{\left(\frac{A}{S_e}\right)^2 + \left(\frac{B}{S_y}\right)^2}}$$

And (Eq. 7-15) simplified to:

$$\sigma'_{max} = \frac{16}{nd^3}\sqrt{A^2 + B^2}$$

Estimating S_e

- Shaft design equations involve S_e which in turn involves five modification factors.
- Therefore, it requires knowing the material, its surface condition, the size and geometry (stress raisers), and the level of reliability.
- Material and surface condition can be decided before the analysis.
- The sizes and geometry are however unknown in the preliminary stage of design.
- For size factor, a diameter may be determined from safety against yielding, or use 0.9 as the estimate of size factor.
- Reliability is typically set at 90%.
- Stress concentration factors K_t and K_{ts} for first iteration are given in (Table 7-1, pp. 365)

Table 7–1

First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts}.

Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

	Bending	Torsional	Axial
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2	3.0
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5	1.9
End-mill keyseat ($r/d = 0.02$)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Missing values in the table are not readily available.

After that (For values not in table - second round), we would have to use (A-15)