Worm Gears

13-11: Worm Gears
13-12: Tooth Systems

- **Features of Worm Gearing**

It is used to transmit power between two non-intersecting shafts;

Power is typically transmitted from worm to worm gear;

A set of worm and worm gear can be designed to self-lock; that is, the worm gear can't back-drive the worm;

Being able to have large speed and torque ratios, but low efficiency; the ratios can be as high as 360, but are commonly up to about 100;

Another key feature is the compact design.

- **Configurations**

Single enveloping
Double enveloping

- **Geometry Figures (13-24 and 13-25)**

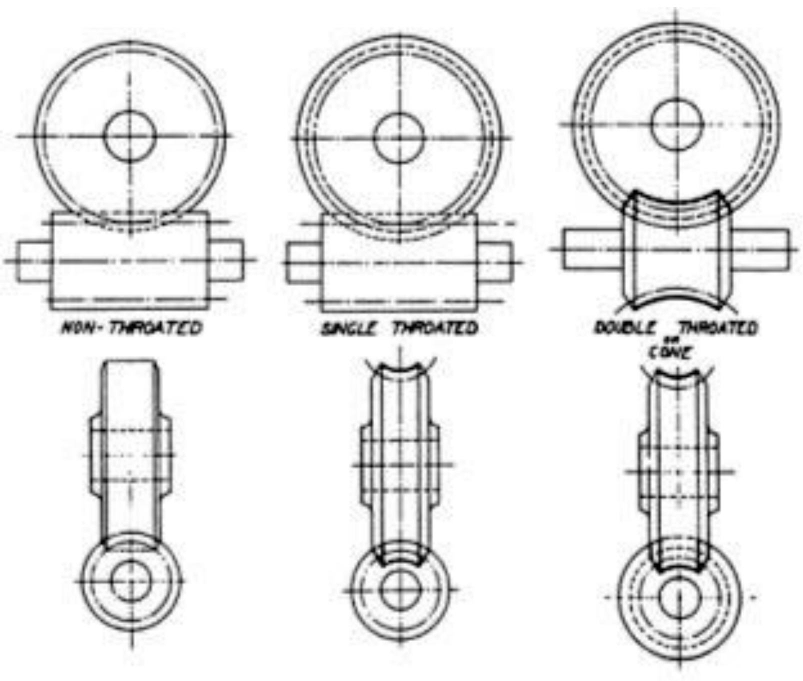


Figure 1: Three Worm Gear Designs

- Left is not used at all,
- Middle is single enveloping
- Right is double enveloping

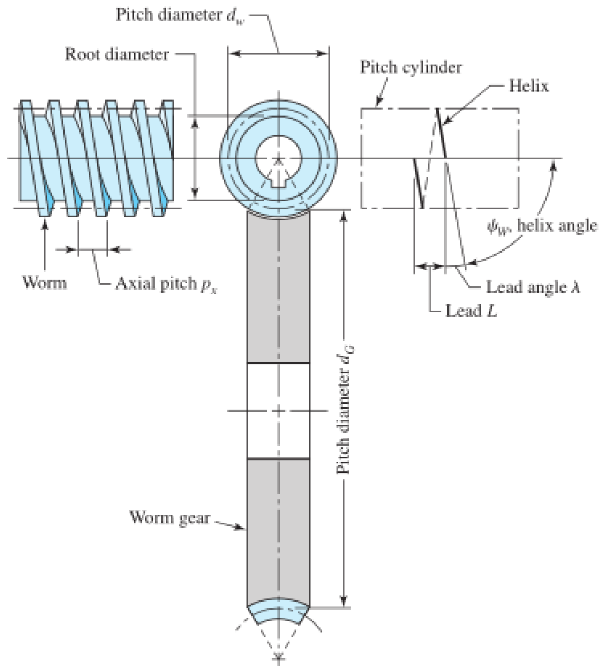


Figure 13–24

Nomenclature of a single-enveloping worm gearset.

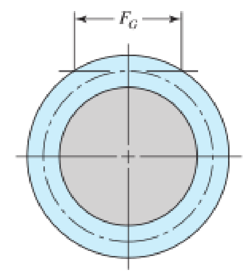


Figure 13–25

A graphical depiction of the face width of the worm of a worm gearset.

In principle, worm and worm gear work like a pair of helical gears in crossed configuration.

The worm and worm gear have helices of the same hand.

The worm has a large helix angle while the worm gear has a small one (e.g., 85° versus 5°).

Shaft angle $\sum$ does not have to be 90°, although 90° is typical, and the scope of Chapters 13 and 15.

Due to the large helix angle of the worm, it can be thought of as a power screw. Terminologies for power screws are adopted for the worm.

Helix angle on worm is ψ_W. If lead angle on worm is λ, then $\lambda = 90° - \psi_W$.

Helix angle on worm gear is ψ_G. For 90° shaft angle, then $\psi_G = \lambda$

Number of teeth of worm, or number of threads (or starts) on worm is, N_W. Typically, $N_W \leq 4$, a recommendation found in many references. However, $N_W \leq 10$ is recommended by Norton (Machine Design – An Integrated Approach, 3rd Ed., Pearson Hall, 2006).

In terms of N_W, and N_G (number of teeth on worm gear), the following is recommended:

- $N_W = 1$ if velocity ratio > 30, $N_W > 1$ if velocity ratio ≤ 30; and $N_G \geq 24$ (recommended by many references);
- $N_G + N_W > 40$. (Machine Design – An Integrated Approach, 3rd Ed., R.L. Norton, Pearson Prentice Hall, 2006).

Axial (circular) pitch p_x for worm; and transverse (circular) pitch for worm gear p_t; Proper meshing requires $p_x = p_t$

Pitch diameter of worm gear:

$$d_G = \frac{N_G p_t}{\pi} \qquad (13\text{–}25)$$

Pitch diameter of worm d_w is determined by (Eq. 13-25) or (Eq. 15-27) which is recommended by AGMA for optimal power transmission capacity.

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7} \qquad (13\text{–}26)$$

$$\frac{C^{0.875}}{3} \le d \le \frac{C^{0.875}}{1.6} \qquad (15\text{–}27)$$

The C in (Eq. 13-25) is the center-to-center distance and

$$C = \frac{d_W + d_G}{2}$$

Velocity ratio

$$\left|\frac{\omega_W}{\omega_G}\right| = \frac{N_G}{N_W} \neq \frac{d_G}{d_W}$$

Face width of the worm gear is F_G

Face width of the worm is F_W

- **Tooth Systems**

Worm and worm gear are not as highly standardized as spur/helical/bevel gears.

Table 13-5: A list of "recommended" normal pressure angle, addendum and dedendum. Addendum and dedendum are in terms of p_X.

Table 13–5

Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

But really, **Table 15-5** should be used instead:

Table 15-5: a list of addendum and dedendum for single-enveloping worm and worm gear.

Table 15–5

Allowable Contact Stress Number for Iron Gears, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material	Material Designation ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) lbf/in^2 (N/mm^2)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	50 000 (345)
	Class 40	Grade 300	As cast	200 HB	65 000 (450)
Ductile (nodular) iron	ASTM A536	ISO/DIS 1083			
	Grade 80-55-06	Grade 600-370-03	Quenched and tempered	180 HB	94 000 (650)
	Grade 120-90-02	Grade 800-480-02		300 HB	135 000 (930)

More dimensions calculations are given within **15-6 AGMA Equations**

The uploaded formulas may help.

Example 1

A worm and worm gear set is used as a speed reducer to drive a winch lift. Velocity ratio is 75. Center-to-center distance is to be around 5.5”. Normal pressure angle is 20°. Determine the dimensions of the worm and worm gear.

Solution:

(1) Choose $N_W = 1$ since velocity ratio is greater than 30. Then $N_G = 75$.

(2) Based on (Eq. 13-26) or (Eq. 15-27), d_w is in the range of $C^{0.875}/3$ and $C^{0.875}/1.6$, or $1.48" \leq d_w \leq 2.78"$ s

So, choose $d_w = 2"$. Also $d_G = 2C - d_w = 9"$

(3) But $d_G = \frac{N_G p_t}{\pi}$, so $p_t = \frac{\pi(9)}{75} = 0.377$”. So choose $p_t = p_x = 3/8" = 0.375"$

(4) As a result, $d_G = (75)(0.375)/\pi = 8.95"$, and $C = (2 + 8.95)/2 = 5.475"$

(5) Dimensions are tabulated as follow.

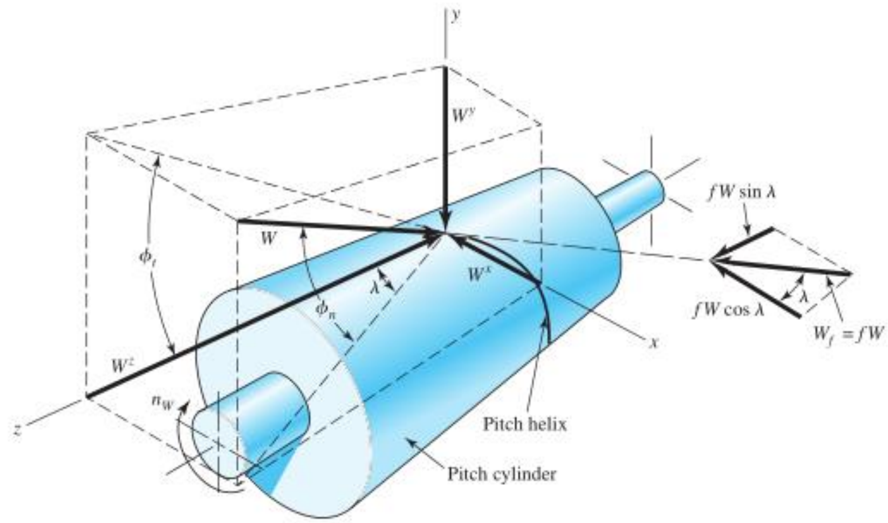
	Worm	Worm-gear
Number of teeth	1	75
Pitch diameter, in	2	8.95
Lead, in	0.375	
Lead angle, °	3.416	
Helix angle, °	86.584	3.416
Addendum, in	0.1194	
Dedendum, in	0.1381	
Clearance, in	0.0187	0.0187
Outside diameter, in	2.239	
Throat diameter, in		9.189
Root diameter, in	1.729	8.674
Mean diameter, in	2	8.95
Face width, in	≤ 2.924	1.333

13-17 Force Analysis – Worm Gearing

(*) Worm/worm gear is for sure on final exam

Figure 13–40

Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.



There's friction components now (on the right)

Friction must be taken into consideration;

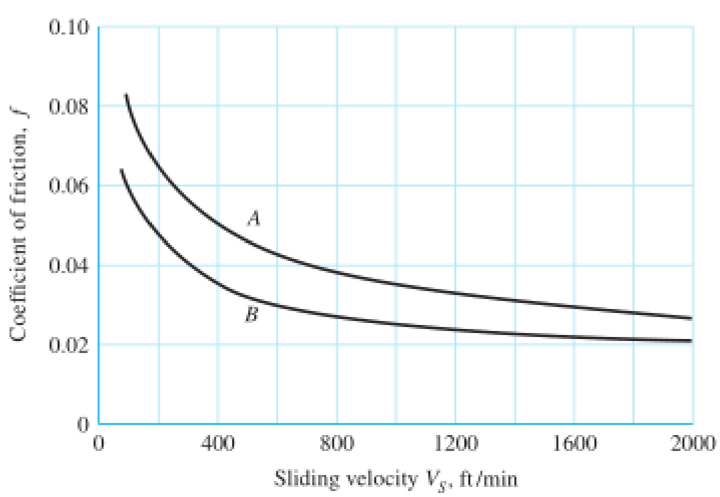
Transmitted load on worm W_w^t: determined based on the torque on the worm, or the horsepower and pitch line velocity of the worm. See Example 13-10;

Efficiency η: by (Eq. 13-46) where the coefficient of friction f is by Figure 13-42 or (Eq. 15-43a)

$$\eta = \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda} \qquad (13\text{–}46)$$

Figure 13–42

Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve *B* for high-quality materials, such as a case-hardened steel worm mating with a phosphor-bronze gear. Use curve *A* when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear.



AGMA reports the coefficient of friction f as

$$f = \begin{cases} 0.15 & V_s = 0 \\ 0.124\exp(-0.074V_s^{0.645}) & 0 < V_s \le 10 \text{ ft/min} \\ 0.103\exp(-0.110V_s^{0.450}) + 0.012 & V_s > 10 \text{ ft/min} \end{cases} \qquad (15\text{–}38)$$

Total force W is (reworked Eq. 13-43a):

$$W = \frac{W_W^t}{\cos\phi_n\, sin\lambda + f cos\lambda}$$

Transmitted load on worm gear W_G^t = axial load on worm W_W^a by (Eq.13-43c)

$$W^z = W(\cos\phi_n \cos\lambda - f\sin\lambda)$$

Radial load on worm W_W^r = radial load on worm gear W_G^r by by (Eq.13-43b)

$$W^y = W\sin\phi_n \qquad (13\text{–}43)$$

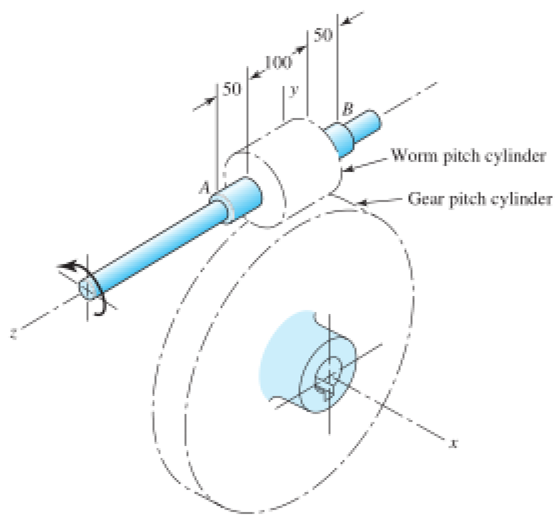
Axial load on worm gear W_G^a = transmitted load on worm W_W^t

Friction $W_f = f \cdot W$;

Note: the above discussions deal with the magnitudes only

Example 2

The worm in Example 1 is left-hand, and runs at 1750 rpm (see Figure for Prob. 13-51 as a reference). The winch requires a torque of 8000 $lb - in$. Determine and visualize the forces acting on the worm and worm gear. What is the efficiency of the set? What is the efficiency of the set? What is the friction?



Solution:

From Example 1,

	Worm	Worm-gear
Number of teeth	1	75
Pitch diameter, in	2	8.95
Lead angle, °	3.416	

Pitch line velocity of the worm:
$V_w = \pi d_w n_w/12 = \pi(2)(1750)/12 = 916.3\ ft/min$

Sliding velocity, by (Eq. 13-47):
$V_s = V_W/\cos\lambda = 916.3/\cos 3.416° = 917.9\ ft/min$

$f = 0.0216$ (Eq. 15-38)

$\eta = 0.721$ (Eq. 13-46)

Torque on the worm: $T_W = T_G/75/\eta = 147.9\ lb - in$

$W_W^t = T_W/(d_w/2) = 147.9\ lb;$

$W = 1907\ lb$

$W_f = 41.19\ lb$

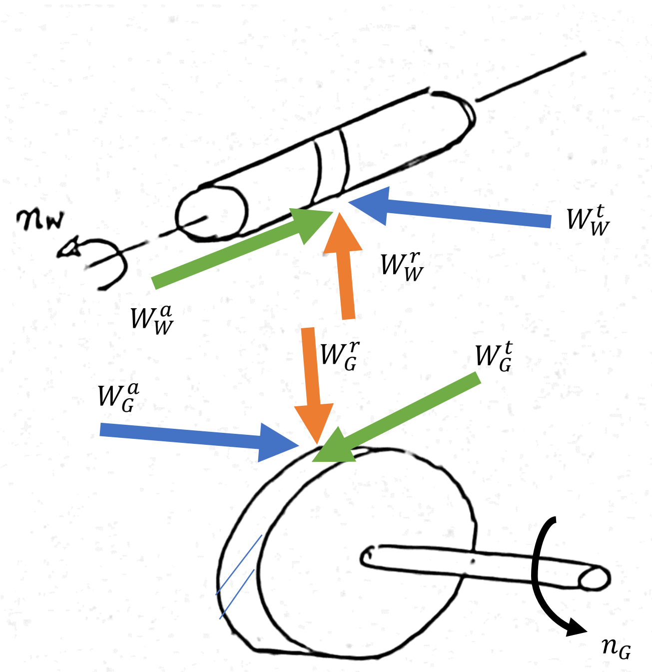
$W_W^r = W_G^r = 652.2\ lb$

$W_W^a = W_G^t = 1789\ lb$

$W_G^a = W_W^t = 147.9\ lb$

To verify, torque on the worm gear $= (1789)(8.95/2) = 8006\ lb - in$ (Compared with the given torque of $8000\ lb - in$)

Visualization:



Materials

Worms:

Low-carbon steels (1020, 1117, 8620 and 4320), case-hardened to HRC58-62.

Medium-carbon steels (4140 and 4150), induction or flame hardened to surface hardness of HRC 58-62.

Grinding or polishing of surfaces may be required.

Note: the above is taken from *Machine Design: An Integrated Approach,* 3rd Ed., R.L. Norton. Pearson Prentice Hall, 2006.

Worm gears:

Bronzes (sand-cast, chill-cast, centrifugal-cast, or forged).

Sec. 15-8 has details.

The first two columns in Table 15-11 show some typical combinations of materials for worm and worm gear.

15-6 Worm Gearing AGMA Equations
15-9 Buckingham Wear Load
Allowable Transmitted Load, (Eq. 15-28)

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v \tag{15–28}$$

Note: if W_W^t or W_G^t is less than $(W^t)_{all}$, it means that the worm and worm gear under consideration will last at least 25,000 hours.

Temperature rise, (Eq. 15-51)

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{\text{CR}} A} = \frac{33\,000(1 - e)(H)_{\text{in}}}{\hbar_{\text{CR}} A} + t_a \tag{15–51}$$

Where H_{loss} is the rate of heat dissipation, in $ft - lb/min$.

$$H_{loss} = 33{,}000(1 - \eta)H_{in}$$

Note: t_s and t_a are the oil sump temperature, and ambient temperature, respectively. It is recommended that $t_s < 160 - 200\ °F$

Other Calculations include Buckingham's equation for dynamic (i.e. fatigue) bending stress, see (Eq. 15-53); and Buckingham's wear load, see (Eq. 15-64) and Sec. 15-9.

$$\sigma_a = \frac{W_G^t}{p_n F_e y} \tag{15–53}$$

$$(W_G^t)_{\text{all}} = K_w d_G F_e \tag{15-64}$$

Buckingham's wear load is considered the predecessor to the AGMA equation.

Only limited data are available for the factor y in (Eq. 15-53)

15-7 Worm Gear Analysis
15-8 Designing a Worm Gear Mesh
Worm and worm gear mesh has lower efficiency due to friction.

Power consumed by friction can be determined by (Eq. 15-63).

$$H_f = \frac{|W_f|V_s}{33\,000}\ \text{hp} \tag{15-63}$$

Cooling, natural or using cooling fans, may be required.

Multi-start worms reduce cooling requirement and reduce d_w as well.

Self-locking means that the worm gear can't drive the worm; self-locking is necessary, even critical, for some applications.

To ensure self-locking, it is required $f_{static} > cos\phi_n tan\lambda$.

Example 15-3
Example 15-4

Example 3
Design the set of worm and worm gear of Example 1. The worm is left-hand, and runs at 1750 rpm. It will be made of carbon steel, case hardened to HRC 58. The material for the worm gear has been chosen to be sand cast bronze. Velocity ratio is 75. Center-to-center distance is to be around 5.5". The winch requires a torque of 8000 lb-in. Self locking is a must-have safety requirement. Temperature rise should not exceed $80°\ F$. Set normal pressure angle to 20°. Assume no cooling fan on the worm shaft.

Solution:
(1) Geometry,
This has been completed in Example 1

(2) Transmitted loads, sliding velocity, friction, efficiency, etc.
They were calculated in Example 2.

(3) Allowable transmitted load is, (Eq. 15-28)

$$(W^t)_{all} = C_s D_m^{0.8} F_e C_m C_v$$

$D_m = 8.95"$, the mean diameter of worm gear

C_s is the materials factor, obtained by one of (Eq. 15-32) through (Eq. 15-35); $C_s = 736.0$

C_m is the ratio correction factor. (Eq. 15-36) gives $C_m = 1.309$

C_v is the velocity factor. Since $V_s = 917.9\ ft/min$, $C_v = 0.2891$, see (Eq. 15-376)

$$C_v = \begin{cases} 0.659 \exp(-0.0011 V_s) & V_s < 700 \text{ ft/min} \\ 13.31\, V_s^{-0.571} & 700 \le V_s < 3000 \text{ ft/min} \\ 65.52\, V_s^{-0.774} & V_s > 3000 \text{ ft/min} \end{cases} \quad (15\text{–}37)$$

F_e is the effective face width of the worm gear. Actual face width is $F_G = 1.3"$. Also, $0.67 d_m = 1.34"$. So, effective face width is $F_e = 1.3$. Also, $0.67 d_m = 1.34"$. So effective face width is $F_e = 1.3$.

Therefore, $(W^t)_{all} = 2091\ lb$

$(W^t)_{all}$ is greater than $W_W^t = 147.9\ lb$, and $W_G^t = 1789\ lb$. So the worm and worm gear will last at least 25,000 hours.

(4) Powers transmitted by the worm and worm gear, and consumed by friction.

(Eq. 15-59) and (Eq. 15-60) give powers transmitted by the worm and worm gear respectively, and in hp. Results are,

$$H_W = \frac{W_W^t V_W}{33\,000} = \frac{\pi d_W n_W W_W^t}{12(33\,000)} \quad (15\text{–}59)$$

$$H_G = \frac{W_G^t V_G}{33\,000} = \frac{\pi d_G n_G W_G^t}{12(33\,000)} \quad (15\text{–}60)$$

$$H_w = 4.11\ hp$$
$$H_G = 2.96\ hp$$

(Eq. 15-63) shows power consumed by friction, in hp. The result is, $H_f = 1.15\ hp$

$$H_f = \frac{|W_f| V_s}{33\,000} \text{ hp} \quad (15\text{–}63)$$

It is seen that $H_G + H_f = H_W$

In general, $H_G + H_f \approx H_W$ due to rounding.

(5) Temperature rise

(Eq. 15-49) shows the rate of heat loss, from the casing/housing, in $ft - lb/min$

$$H_{\text{loss}} = 33\,000(1 - e)H_{\text{in}} \quad (15\text{–}49)$$

H_{in} is the input power in hp. $H_{in} = H_W$.

So, $H_{loss} = 37841\ ft - lb/min$.

Temperature rise is by (Eq. 15-51)

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{\text{CR}} A} = \frac{33\,000(1-e)(H)_{\text{in}}}{\hbar_{\text{CR}} A} + t_a \qquad (15\text{–}51)$$

Where A is the lateral area of the casing/housing, and h_{CR} is the combined convective and radiative coefficient of heat transfer. (NOTE: In our notes e is written as η)

The lateral area of the casing typically includes the external surface area, for example, the 6 rectangular areas of a cube that is the casing/housing.

By (Eq. 15-50), $h_{CR} = 0.3995\ ft - lb/(min \cdot in^2 \cdot °F)$. To limit temperature rise to 80 $°F$, the required lateral area is $A = H_{loss}/(t_S - t_a)/h_{cr} = 1184\ in^2$.

$$\hbar_{\text{CR}} = \begin{cases} \dfrac{n_W}{6494} + 0.13 & \text{no fan on worm shaft} \\[2ex] \dfrac{n_W}{3939} + 0.13 & \text{fan on worm shaft} \end{cases} \qquad (15\text{–}50)$$

Discussions: (Eq. 15-52) shows the AGMA recommended minimum area A_{min}.

$$A_{\min} = 43.20 C^{1.7} \qquad (15\text{–}52)$$

$$A_{min} = 43.20\ C^{1.7} = (43.20)(5.475)^{1.7} = 777.6\ in^2$$

With $A = A_{min}, t_s - t_a = \frac{H_{loss}}{h_{CR}A} = 121.8\ °F$, and $t_s \approx 191.8\ °F$.

If it is desired to keep the surface area at A_{min}, while still limit the temperature rise to 80 $°F$, cooling fans may be installed; cooling fins may be incorporated. Finally, an external heat exchanger may be considered.

(6) Self-locking

Since $cos\phi_n tan\lambda = 0.056$, self locking requires $f_{static} > 0.056$.

The static COF between steel and bronze is, from engineersedge.com, 0.16 when lubricated. Static friction seems enough. However, a brake, for instance, is advised.

Ch. 12 - Lubrication and Journal Bearings

- Charts and their usages
- Interpolation
- What constitutes a Good Design?
    - $h_o \geq h_{min}$, $f > 0.01$, optimal zone, ΔT;
    - Minimum film thickness is greater than shaft deflection across the length of bearing (to prevent binding between shaft and bearing);
    - Suitable materials;
    - Etc.

Ch. 13, 14, 15 – Gearing

- Types
    - Spur, helical, bevel, worm-worm gear
        - Bevel gears: spiral, hypoid, spiroid, etc.
    - Features, from the perspectives of
        - Applications
        - Geometry
        - Analyses
        - …
    - Configurations
        - Helical: parallel, crossed
        - Bevel: shaft angle = 90°, $\neq 90°$
        - Worm-wormgear: single enveloping, double enveloping
    - Geometry, tooth systems, standard values
        - Spur gears: US-customary vs. metric; full-depth vs. stub-profiled; L_{ab}
        - Helical gears: normal plane vs. transverse plane; L_{ab}; m_p m_F
        - Bevel gears: large end, pitch cone, back cone, etc.;
        - Do practise problems posted
    - Planetary gear trains
        - Spur and helical gears: "+" for internal sets "-" for external sets
        - Bevel gears: how to deal with the signs;
    - Force components and visualization
    - Materials
    - AGMA equations

      Spur gears: the basic, and foundation for helical and bevel gears;
      Helical gears: Geometry factors I and J;
      Bevel gears:
      Comparing S_F and S_H;
    - Worms-wormgears

      AGMA equation: allowable transmitted load;
      Friction, efficiency, temperature rise, cooling considerations;
      Self-locking;

Ch. 7 – Shafts and Shaft Components

- Input(s) and power take-off(s) of individual shafts;
- Gears' axial loads → beams FBD;
- Support reactions, shear force diagrams, bending moment diagrams, combined bending moment diagram, torque diagram;
- Static-failure;
- Fatigue-failure;
- Deflection/slope in relation to bearings and gears, and by applying Table A-15;
- Miscellaneous components (keys, pins, etc.)