Chapter 15: Bevel and Worm Gears

15-1 Bevel Gearing – General

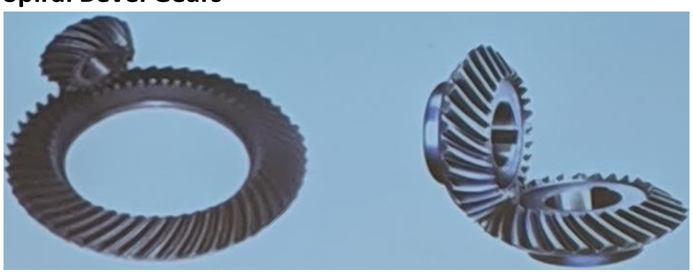
5 Types of Bevel Gearing

- Straight bevel gears
- Spiral bevel gears
- Zerol bevel gears
- Hypoid gears
- Spiroid gears

Straight Bevel Gears

Used to transmit power between two intersecting shafts, at any angle (except 0 and 180 degrees).

Spiral Bevel Gears



Teeth are curved and oblique;
Used to transmit power between two intersecting shafts;
The difference between straight and spiral bevels is similar to the difference between spur and helical gears.

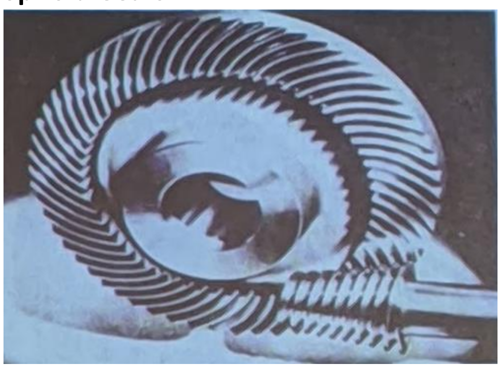
Zerol Gears

Teeth are curved;
Used to transmit power between two intersecting shafts.

Hypoid Gears

Teeth are curved and oblique
Used to transmit power between two offset shafts at any angles;
Used in automotive differentials.

Spiroid Gears

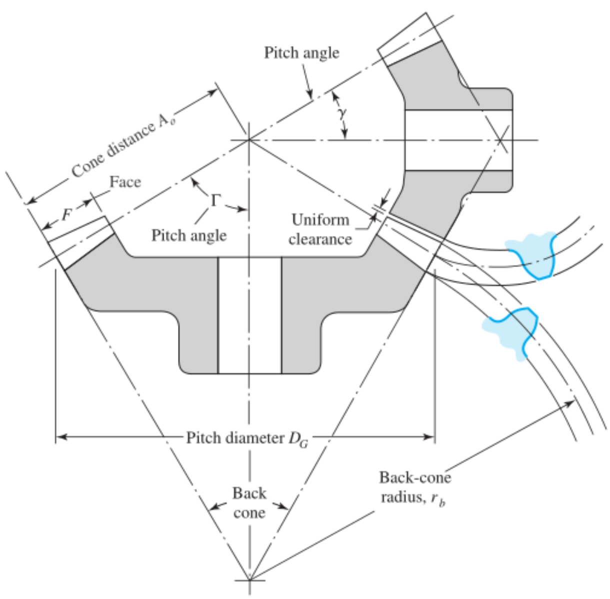


Spiroid gears are seen as the combination of spiral bevel and worm gears;
Used in heavy-duty vehicles.

13.19: Straight Bevel Gears
13-12: Tooth Systems

Figure 13–20

Terminology of bevel gears.



Pitch angles: γ and Γ

Shaft angle $\sum = \gamma + \Gamma$ can be angle, except 0° and 180°;

If $\sum = 90°$, $\tan\gamma = N_P/N_G$ and $\tan\Gamma = N_G/N_P$

$$\tan\gamma = \frac{\sin\sum}{\frac{N_G}{N_P} + \cos\sum}$$
$$\tan\Gamma = \frac{\sin\sum}{\frac{N_P}{N_G} + \cos\sum}$$

For any $\sum > 90°$:
$$\tan\gamma = \frac{\sin(180° - \sum)}{\frac{N_G}{N_P} + \cos(180 - \sum)}$$
$$\tan\Gamma = \frac{\sin(180° - \sum)}{\frac{N_P}{N_G} + \cos(180 - \sum)}$$

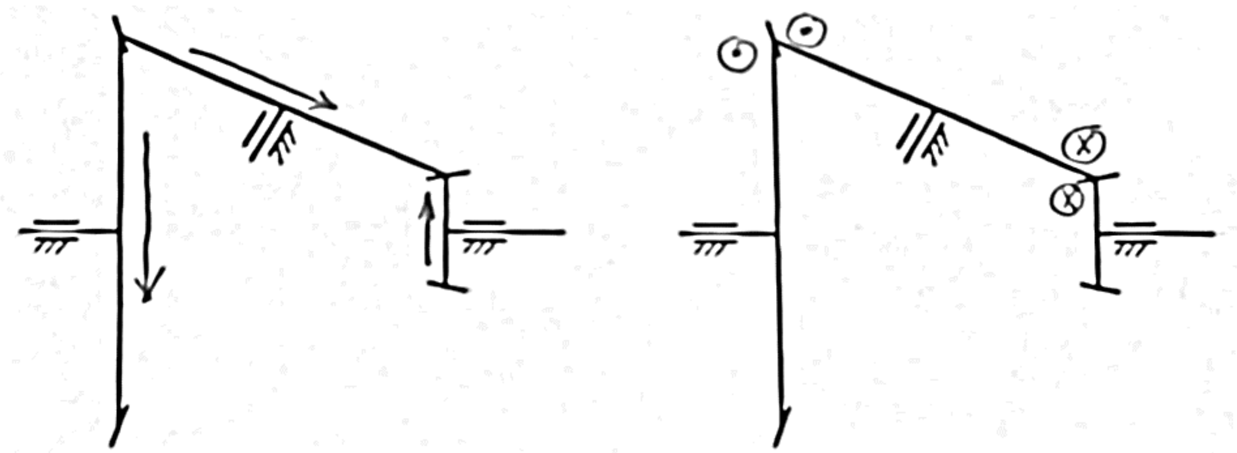
Small end and large end of tooth;
Pitch cone and back cone:
Pitch cone length (distance): A_0, which should be the same for both the pinion and the gear;
Back cone (large end) radi: r_{bP} and r_{bG};
Pitch and pressure angle are defined at the large end; As a result, a bevel gear's size and shape are defined at the large end;
Face width: F, which is the lesser of $0.3A_0$ or $10/P$;
Virtual numbers of teeth: $N_P' = \frac{2\pi r_{bP}}{p}$ and $N_G' = \frac{2\pi r_{bG}}{p}$, where p is the circular pitch measured at the large end of the teeth.

Table 13-3: Tooth proportions, 20° straight tooth bevel gears
Revision is needed for equivalent 90° ratio.

Table 13–3
Tooth Proportions for 20° Straight Bevel-Gear Teeth

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \frac{0.54}{P} + \frac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Sigma = 90°$
	$m_{90} = \sqrt{m_G \frac{\cos\gamma}{\cos\Gamma}}$ when $\Sigma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \frac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13
	Gear: 16, 17, 20, 30

- Velocity ratio: $|VR| = N_P/N_G$ regardless or $\sum$. The sign, however, is not as simple as "-" for external set and "+" for internal set.



Example 1: Determine the dimensions of the bevel gearset ($N_P = 21$, $N_G = 35$, $P = 4$ teeth/in, 20° pressure angle, and straight teeth). Shaft angle is $(1)\sum = 90°$ and $(2)\sum = 75°$.

Solution:
$(1)\sum = 90°$

	Pinion	Gear
Pitch angle, °	30.964	59.036
Gear ratio, m_G	1.667	
Equivalent 90° ratio, m_{90}	1.667	
Addendum, in	0.1764	
Working depth, in	0.5	
Pitch diameter, in	5.25	8.75
Cone distance, in (A_0)	5.102	5.102
Back cone radius, in	3.061	8.503
Virtual number of teeth	24.5	68.0
Face width, in	The lesser of 1.53 or 2.5; so $F = 1.53$	

$(2)\sum = 75°$

	Pinion	Gear
Pitch angle, °	26.641	48.359
Gear ratio, m_G	1.667	
Equivalent 90° ratio, m_{90}	1.497	
Addendum, in	0.1860	
Working depth, in	0.5	
Pitch diameter, in	5.25	8.75
Cone distance, in (A_0)	5.854	5.854
Back cone radius, in	2.937	6.584
Virtual number of teeth	23.5	52.7
Face width, in	The lesser of 1.76 or 2.5; so $F = 1.76$	

Planetary Bevel Gear Train

The tabular method can be used to analyze planetary gear trains consisting of bevel gears, but with modifications.

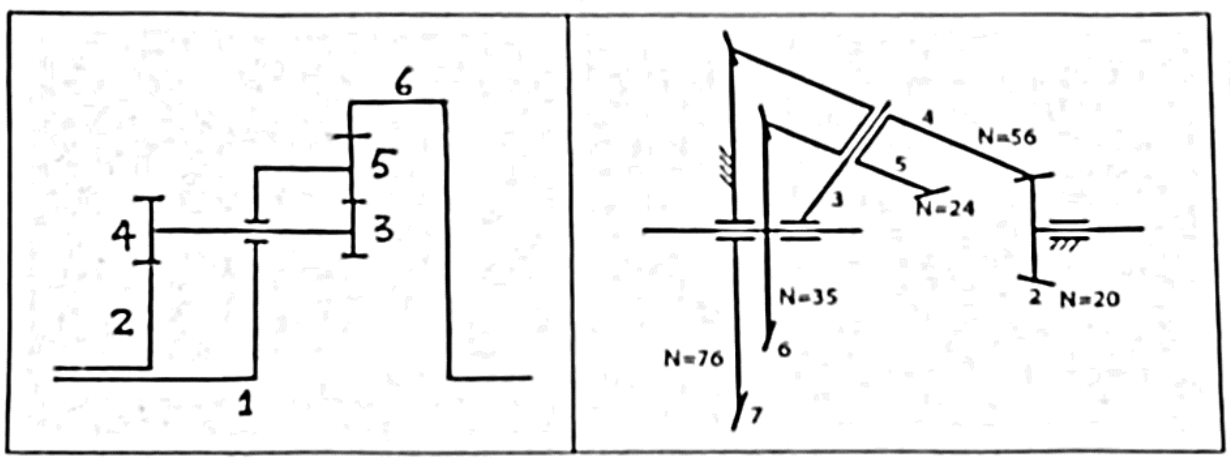
Recalling the following for planetary gear trains involving spur gears (and parallel helical gears):

$$VR = \frac{\left[\omega_{gear/arm}\right]_{driven}}{\left[\omega_{gear/arm}\right]_{driver}} = \pm\frac{N_{driver}}{N_{driven}}$$

Where "+" is used with internal set and "-" with the external set.

The reasons behind the convention for the "+" and the "-" are,

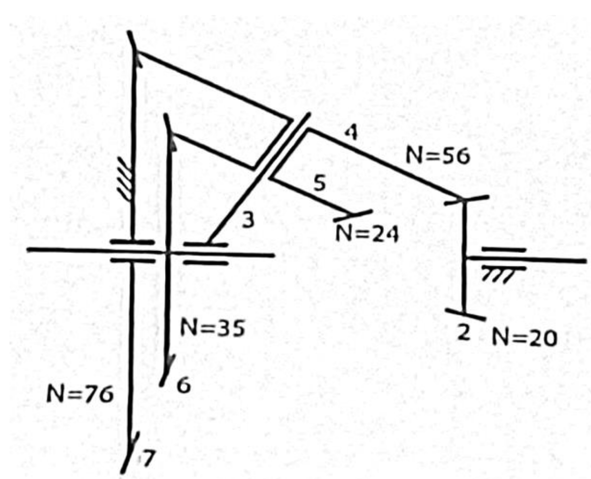
(1) The angular velocities, as vectors, are parallel to each other; and
(2) An internal set of gears rotate with the same sense while an external set rotate with opposite senses.



When bevel gears are involved in a planetary gear train, angular velocities, as vectors, are not always parallel to each other. So the signs must be determined manually, on gear-to-gear basis.

Example 2: The Humpage gear train.

The schematic of the Humpage train is given below. Find the train's velocity ratio.



Solution:
$N_2 = 20$
$N_4 = 56$
$N_5 = 24$
$N_6 = 35$
$N_7 = 76$

Power flows 2 → 4 → 7 and 2 → 4 & 5 → 6
"3" is the arm
Assume the input speed is n_2.

Gear	n_{gear} $=$	n_{arm} $+$	$n_{gear/arm}$	$\lvert VR \rvert$
2	n_2	n_3	$n_2 - n_3$	$\left(\frac{N_2}{N_4}\right)\left(\frac{N_4}{N_7}\right) = 0.2631$
4	(left blank)			
7		n_3	$(n_2 - n_3)(-0.2631)$	

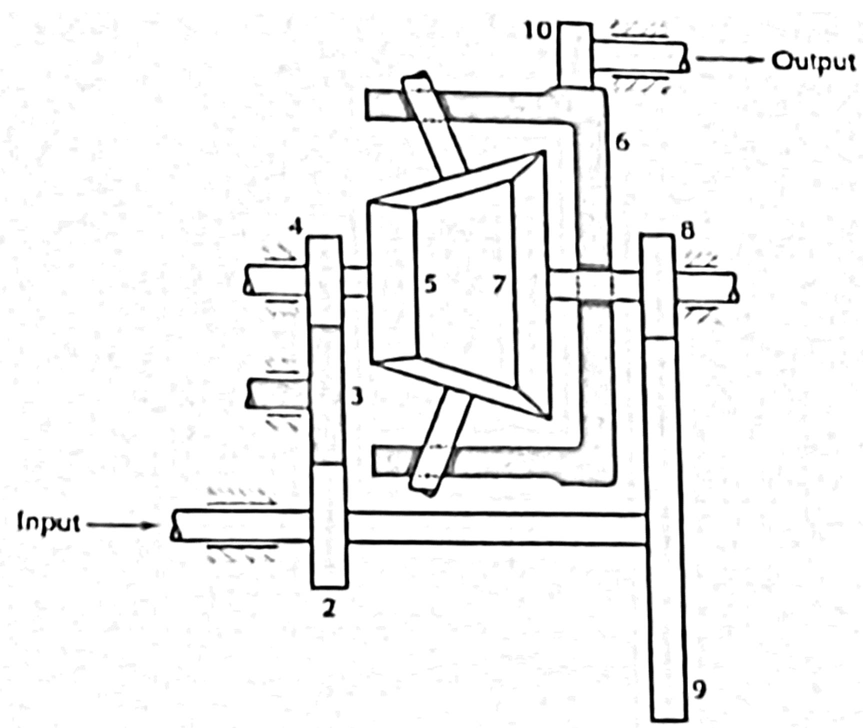
From $n_3 + (n_2 - n_3)(-0.2631) = 0 \rightarrow n_3 = 0.2083n_2$

Gear	n_{gear} $=$	n_{arm} $+$	$n_{gear/arm}$	$\lvert VR \rvert$
2	n_2	n_3	$n_2 - n_3$	$\left(\frac{N_2}{N_4}\right)\left(\frac{N_5}{N_6}\right) = 0.2449$
4	(left blank)			
5	(left blank)			
7	n_6	n_3	$(n_2 - n_3)(-0.2449)$	

From $n_6 = n_3 + (n_2 - n_3)(-0.2449) = 0.1441n_2$

So, $VR = 0.01441$

Example 3: Given $\omega_2 = 100\ rad/s$, $N_2 = 40$, $N_4 = 30$, $N_5 = 25$, $N_6 = 120$, $N_7 = 50$, $N_8 = 20$, $N_9 = 70$, $N_{10} = 20$. Determine ω_{10}.



5 and 7 are the sun gears;
Planetary gears are not labeled, and teeth numbers not given;
6 is the arm.

Solution:
(1) From $\omega_2 = 100\ rad/s$, ω_4 and ω_8 can be determined.
$\omega_4 = \omega_2 N_2/N_4 = 133.3\ rad/s$
$\omega_8 = \omega_2 N_9/N_8 = -350\ rad/s$

(2) The planetary gear train consists of gears 5, the upper planetary gear and 7, with gear 6 being the arm.

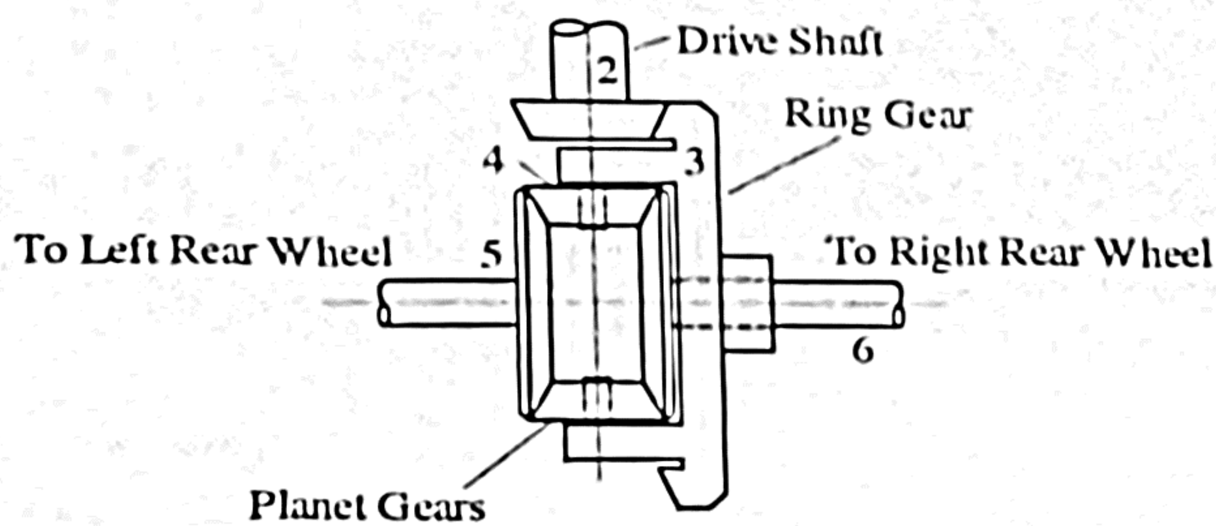
Power flows 4 & 5 → 7 & 8.

Gear	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	$\lvert VR \rvert$
4	133.3	x	$(133.3 - x)$	$\left(\frac{N_5}{N_7}\right) = 0.5$
5				
7	-350	x	$(133.3 - x)(-0.5)$	
8				

Since $-350 = x - (133.3 - x)(0.5)$, solving leads to $x = -188.9\ rad/s$

(3) $\omega_6 = x = -188.9 \frac{rad}{s}$
$\omega_{10} = -\omega_6 N_6/N_{10} = 1133\ rad/s$

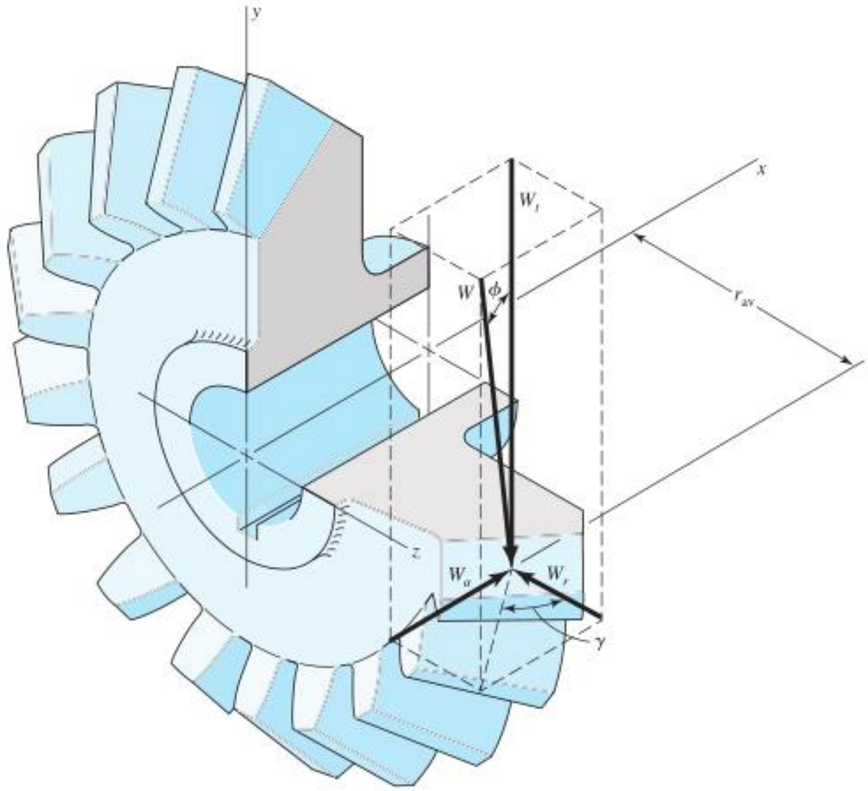
Example 4: The differential



13-15 Force Analysis – Bevel Gearing

Figure 13–35

Bevel-gear tooth forces.



Point of application of W: the actual point of application is somewhere between the midpoint and large end of a tooth, but it is typically assumed that W is applied at the midpoint, with a radius r_{av}

$$(r_{av})_P = r_P - \frac{F}{2} sin\gamma$$
$$(r_{av})_G = r_G - \frac{F}{2} sin\Gamma$$

Transmitted load W_t: determined from known power and pitch line velocity, or from known torque, in the same way as for spur gears, but replacing d (pitch diameter) with $2r_{av}$ average diameter.

Radial load W_r
Axial load W_a

(Eq. 13-37): W_t from torque

$$W_t = \frac{T}{r_{av}} \qquad (13\text{–}37)$$

(Eq. 13-38) W_r and W_a from W_t

$$W_r = W_t \tan\phi \cos\gamma$$
$$W_a = W_t \tan\phi \sin\gamma \qquad (13\text{–}38)$$

The relation $|(W_t)_P| = |(W_t)_G|$ is always true;

But $|(W_r)_P| = |(W_a)_G|$ and $|(W_a)_P| = |(W_r)_G|$ are only true when the shaft angle is $\sum$ is 90°.

Example 5: Determine the force components acting on the bevel pinion and gear, respectively. Then show the components on an isometric drawing of the gears. Given $N_P = 21$, $N_G = 35$, $P = 4$ teeth/in, 20° pressure angle, and straight teeth. The bevel gearset is to transmit 25 hp. Pinion speed is 500 rpm. Shaft angle is 75°.

Solution:

(1) Geometric quantities are, from Example 1

	Pinion	Gear
Pitch angle,	26.641	48.359
Pitch diameter, in	5.25	8.75
Face width, in	$F = 1.76$	
Average radius, in	2.230	3.717

(2) Pinion

$V = \pi 2(r_{av})_P n_p/12 = 583.8\ ft/min$

$W_t = 33{,}000 \cdot H/V = 1413\ lb$

$W_r = W_t \tan\phi \cos\gamma = 459.7\ lb$

$W_a = W_t \tan\phi \sin\gamma = 230.6\ lb$

$W = 1504\ lb$

(3) Gear

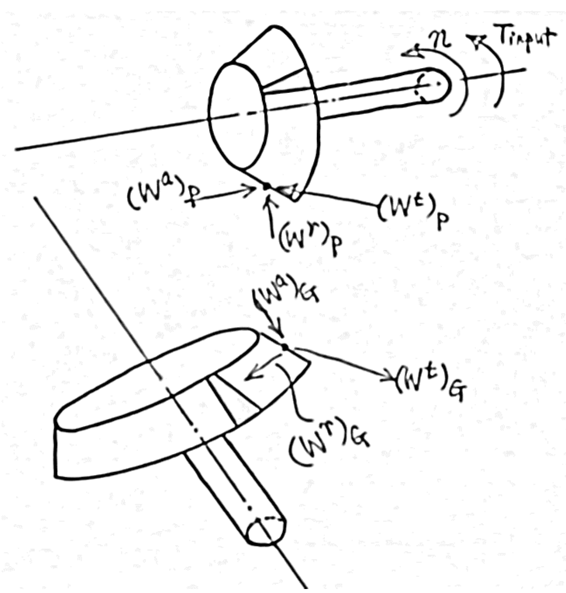
$W_t = 1413\ lb$

$W_r = W_t \tan\phi \cos\Gamma = 341.7\ lb$

$W_a = W_t \tan\phi \sin\Gamma = 384.3\ lb$

$W = 1504\ lb$

(4) Draw and label the forces



NOTE:

Radial force always points towards central axis

Axial force always points towards the larger face

Tangential force is directed to counteract input rotation and torque

15-2 Bevel Gear Stresses and Strengths
Contact stress, (Eq. 15-1)
Allowable contact stress, (Eq. 15-2)
Bending stress, (Eq. 15-3)
Allowable bending stress, (Eq. 15-4)

15-3 AGMA Equation Factors
15-4 Straight Bevel Gear Analysis
Example 15-1

15-5 Design of a Straight-Bevel Gear Mesh
Decisions made beforehand and during design.

Example 6: A gearbox contains a set of bevel gears. It is driven by a single cylinder engine, and to drive a reciprocating compressor. Output shaft rotates at 1500 rpm, with a maximum torque of 550 lb-in. Teeth numbers are 33 and 83, with P = 10 teeth/in, 20° pressure angle, and straight teeth.

Assumptions (1) through (4), and (6) through (7) are the same as the spur gearset example. Assumption (5) becomes that both shafts are straddle mounted.

Solution
1. $d_p = N_P/P = 33/10\ 3.3$, $d_G = N_G/P = 83/10 = 8.3$
Use 20° full depth, straight teeth

Bevel gears of straight teeth are typically somewhat crowned during manufacture; but when assessing factors of safety, S_F it is compared with S_H^2.

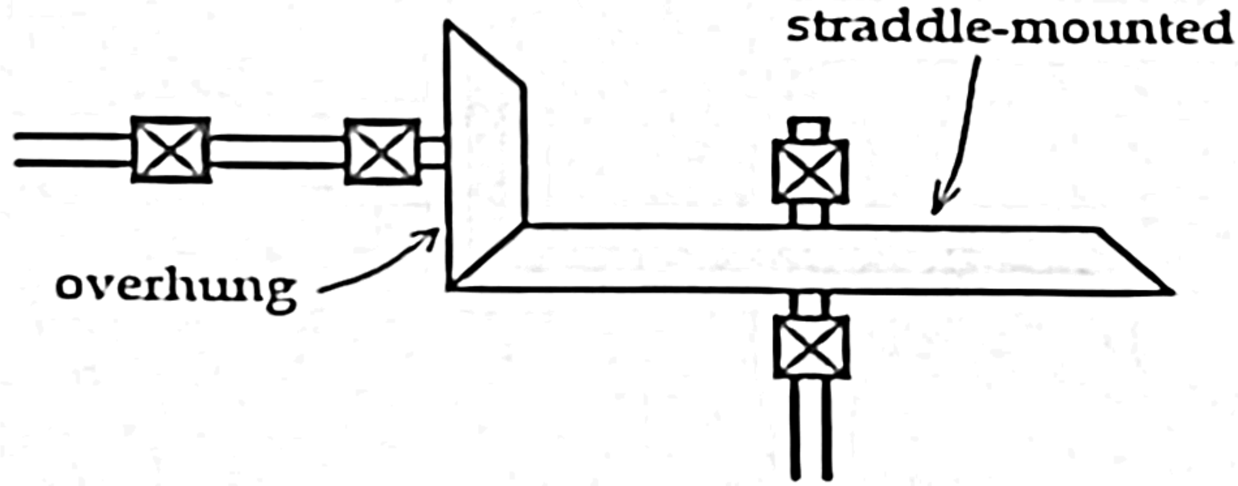
2. Other geometric quantities. (Note that not all listed quantities are needed for applying AGMA equations.)

	Pinion	Gear
Pitch angle, °	21.682	68.318
Addendum, in	0.06127	
Working depth, in	0.2	
Pitch diameter, in	3.3	8.3
Cone distance, in	4.466	4.466
Face width, in	The less of 1.34 and 1; $F = 1$	
Average radius, in	1.465	3.685

3. Transmitted load
$W^t = T_{max}/(r_{av})_G = 550\ /\ 3.685 = 149.3\ lb$
$V = \pi d_G n_G/12 = 3259\ ft/min$ (use pitch diameter here)

4.-17. Factors
- Geometry factors are from charts: $I = 0.1, J_P = 0.295, J_G = 0.255$
- Size factor is from a chart as well;
- Load distribution factor K_m: depends on the mounting of the gears

Mounting can be any combinations of straddle mounted and overhung.



- Hardness-ratio factor C_H
    - As with spur gearsets, $C_H = 1$ for pinion, and C_H for gear is determined.

(Eq. 15-16) or Figure 15-10 is for through-hardened steels
(E1. 15-17) or Figure 15-11 is for surface-hardened steels
(Eq. 15-16), N/n means N_G/N_P.

	Pinion	Gear		Pinion	Gear
$\boldsymbol{W^t}$	149.3		$\boldsymbol{W^t}$	149.3	
$\boldsymbol{P_d = P}$	10		$\boldsymbol{d_P}$	3.3	
$\boldsymbol{F}$	1		$\boldsymbol{F}$	1	
$\boldsymbol{K_o}$	1.75		$\boldsymbol{K_o}$	1.75	
$\boldsymbol{K_v}$	1.229		$\boldsymbol{K_v}$	1.229	
$\boldsymbol{K_m}$	1.0036		$\boldsymbol{K_m}$	1.0036	
$\boldsymbol{K_s}$	0.5082		$\boldsymbol{C_s}$	0.5625	
$\boldsymbol{K_x}$	1		$\boldsymbol{C_{xc}}$	1.5	
J	0.295	0.255	$\boldsymbol{C_p}$	2290	
			$\boldsymbol{I}$	0.1	
$\boldsymbol{\sigma}$	5501	6364	$\boldsymbol{\sigma_c}$	65734	65734
$\boldsymbol{S_{at}}$	17500	14420	$\boldsymbol{S_{ac}}$	142970	119100
$\boldsymbol{K_L}$	0.9066	0.9126	$\boldsymbol{C_L}$	0.8927	0.9355
$\boldsymbol{K_T}$	1		$\boldsymbol{K_T}$	1	
$\boldsymbol{K_R}$	1		$\boldsymbol{K_R}$	1	
			$\boldsymbol{C_H}$	1	1.004
$\boldsymbol{\sigma_{all}}$	$15866/S_F$	$13289/S_F$	$\boldsymbol{\sigma_{c,all}}$	$127629/S_H$	$111864/S_H$
$\boldsymbol{S_F}$	2.88	2.09	$\boldsymbol{S_H}$	1.94	1.70