Assessments

- Refer to pp. 768-769, and p. 772
- Ideally, the four factors of safety should be at the same level, between 1 and 3;
- Factors of safety $S_{F,P}$ and $S_{F,G}$ are directly proportional to transmitted load W^t;
- Factors of safety $S_{H,P}$ *and* $S_{H,G}$ are proportional to the square root or cubic root of the transmitted load W^t;
- So S_F should be compared with S_H^2, or S_H^3 if teeth are crowned;
- For the spur gearset example, calculated factors of safety are:

	Pinion	Gear		Pinion	Gear
S_F	4.70	4.63	S_H	1.59	1.41

Comparison should be done as:

	Pinion	Gear		Pinion	Gear
S_F	4.70	4.63	S_H^2	2.53	1.99

- For steels with HB < 500, good balance between bending and contact can be achieved by going for higher P (or finer pitch);
- For steels with HB ≥ 500, it is suggested to start with P = 8 $(teeth/in)$
- Remember that a broken tooth is more dangerous than a worn tooth;

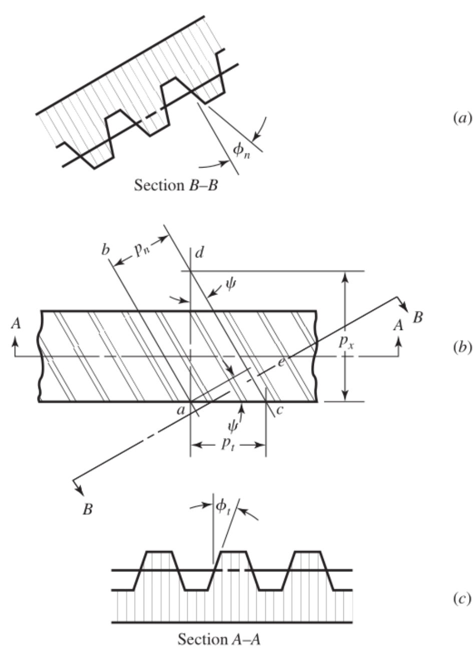
13-10: Parallel Helical Gears

13-12 Tooth Systems

- Shaping and gobbing are the two commonly used gear manufacturing methods.
- Depending on the manufacturing method, there are two tooth systems for helical gears.
- Shaped helical hears follow the transverse tooth system, but hobbed helical gears follow the normal tooth system.
- **Figure 13-22:** Transverse plane A-A and normal plane B-B

Figure 13–22

Nomenclature of helical gears.



- Transverse tooth system vs. normal tooth system

  They refer to the fact that tooth proportion (addendum and dedendum) is determined in the transverse plane for shaped gears, and in the normal plane for hobbed gears.

- **Table 13-4:** standard tooth proportions for hobbed US-customary helical gears.

  Helix angle ψ, not standardized but $< 30°$; a value between $15°$ and $30°$ is common.

  Standard values are applied to:
  - Normal pitch P_n; and
  - Normal pressure angle φ_n

  Transverse circular pitch p_t ↔ Normal circular pitch p_n: (Eq. 13-16)

$$p_n = p_t \cos \psi \qquad (13\text{–}16)$$

  Transverse pitch P_t ↔ Normal pitch P_n: (Eq. 13-18)

$$P_n = \frac{P_t}{\cos \psi} \qquad (13\text{–}18)$$

  Transverse pressure angle φ_t ↔ Normal pressure angle φ_n: (Eq. 13-19)

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t} \qquad (13\text{–}19)$$

  Axial (circular pitch) p_x: (Eq. 13-17)

$$p_x = \frac{p_t}{\tan \psi} \qquad (13\text{–}17)$$

  Face width ≥ 2x axial pitch
  Example $13-2$ on applying the above.

Table 13–4

Standard Tooth Proportions for Helical Gears

Quantity*	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$	External gears:	
Dedendum	$\frac{1.25}{P_n}$	Standard center distance	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos\psi}$	Gear outside diameter	$D + 2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos\psi}$	Pinion outside diameter	$d + 2a$
Normal arc tooth thickness†	$\frac{\pi}{P_n} - \frac{B_n}{2}$	Gear root diameter	$D - 2b$
Pinion base diameter	$d \cos\phi_t$	Pinion root diameter	$d - 2b$
		Internal gears:	
Gear base diameter	$D \cos\phi_t$	Center distance	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan\psi \cos\phi_t)$	Inside diameter	$D - 2a$
		Root diameter	$D + 2b$

*All dimensions are in inches, and angles are in degrees.

†B_n is the normal backlash.

Example 1: Determine the pitch diameter, radius of addendum circle, radius of base circle, and face width of a helical gear (18-teeth, 20°-pressure angle, 22.5°-helical angle, and full-depth tooth profile) when It is cut by a hob with $P_n = 8\ teeth/in$.

Solution:

	Transverse Plane	Normal Plane
Addendum, in		0.125
Dedendum, in		0.1563
Transverse pitch, $teeth/in$	7.391	
Transverse pressure angle, °	21.50	
Pitch diameter, in	2.435	
Radius of addendum circle, in	1.343	
Radius of base circle, in	1.133	
Transverse circular pitch, in	0.4251	
Transverse base circular pitch, in	0.3955	
Axial pitch, in	1.026	
Face width, in	≥ 2.052	

- **Configuration of helical gears**

  For a set of helical gears to mesh, they must have the same diametral pitch and pressure angle.

  If the helical gears have helix angle ψ_1 and ψ_2, respectively, the angle between the shafts is $\sum = \psi_1 + \psi_2$.

  When $\sum \neq 0$ (that is, $\psi_2 \neq -\psi_1$), the helical hears form the so-called cross helical gears.

Crossed helical gears are in point contact, and may have low efficiency due to sliding motion between the teeth in contact. The load carrying capacity is limited to $400\ N$ or so.

Parallel helical gears: this is when $\psi_2 = -\psi_1$ and $\sum = 0$. That is, the gears have the same helical angle, but the helices are of opposite hands.

Left-handed, right-handed, standard configuration

Crossed configuration

- **Contact ratios**
  A helical gearset has transverse, normal and axial contact ratios.

  Transverse contact ratio, m_P, is determined in the same way as the m_c for a pair of spur gears.

  Face or axial contact ratio is $m_F = F/p_x$.

  Total contact ratio is $m_P + m_F$.

Example 2: Determine the total contact ratio of a set of hobbed helical gears ($P_n = 8\ teeth/in$, 20° pressure angle, 22.5° helical angle, and full-depth tooth profile). Given $N_p = 18$, $N_G = 35$, and face width is 2.5".

Solution:

	Pinion	Gear
Addendum, in	0.125	
Dedendum, in	0.1563	
Transverse pitch, $teeth/in$	7.391	
Transverse pressure angle, °	21.50	

Pitch diameter, in	2.435	4.732
Radius of addendum circle, in	1.343	2.491
Radius of base circle, in	1.133	2.201
Transverse circular pitch, in	0.3955	
Transverse base circular pitch, in	0.4251	
Axial pitch, in	1.026	
Face width, in	2.5	
Centre-to-centre distance, in	3.584	

$$L_{ab} = 0.721 + 1.166 - 1.314 = 0.5730"$$

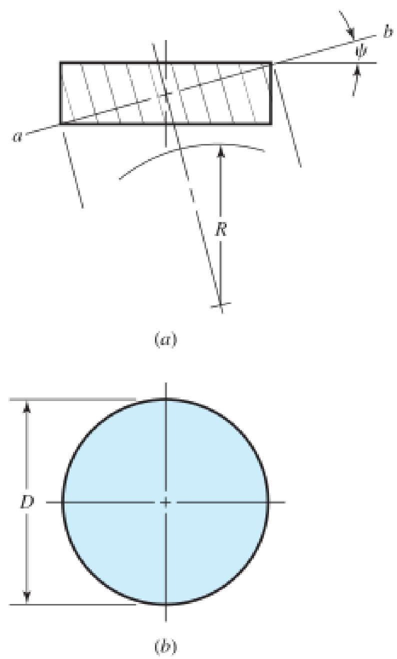
$$m_p = \frac{0.5730}{0.3955} = 1.449$$

$$m_F = \frac{2.5}{1.026} = 2.437$$

Total contact ratio = 3.866

Figure 13–23

A cylinder cut by an oblique plane.



ab is the normal plane. ab "cuts" an ellipse.

Radius of curvature of the ellipse at contact point is R.

$$R = \frac{D}{2\cos^2\psi} \geq \frac{D}{2}$$

Imagine a pitch cylinder of radius R and placing teeth on it.

Virtual number of teeth N' is the number of teeth that can be placed on a pitch cylinder of radius R.

$$N' = \frac{N}{\cos^3\psi}$$

N' is used to determine the Lewis form factor Y.

And the minimum number of teeth to avoid interference applied to N' for helical gears.

For example, $N_{min} = 17$ for hobbed spur gears; for helical gears, with $\psi = 30°$, $N' = 17$, and $N = N' \cos^3\psi = 11.04$

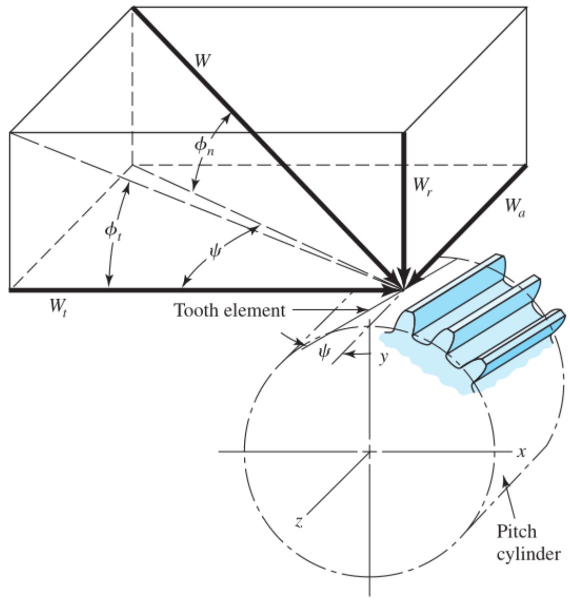
- Advantages/Disadvantages of helical gears over spur gears
  - Smoother and quieter operations (due to large total contact ratio)
  - Higher load-carrying capacity
  - Allowing for higher pitch line velocity

- Smaller minimum number of actual teeth to avoid interference
- Presence of axial thrust
- Double helical cancels out thrust but increases cost

13-16 Force Analysis – Helical Gearing

Figure 13–37

Tooth forces acting on a right-hand helical gear.



Transmitted load W_t: determined from given power or torque in the same way as for spur gears
Radial load W_r
Axial load (or separating force) W_a

$$\begin{aligned} W_r &= W \sin \phi_n \\ W_t &= W \cos \phi_n \cos \psi \\ W_a &= W \cos \phi_n \sin \psi \end{aligned} \tag{13–39}$$

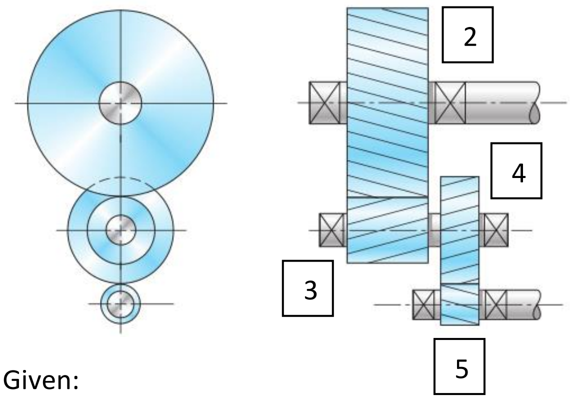
W_t, W_r, W_a in terms of W, the total force

$$\begin{aligned} W_r &= W_t \tan \phi_t \\ W_a &= W_t \tan \psi \\ W &= \frac{W_t}{\cos \phi_n \cos \psi} \end{aligned} \tag{13–40}$$

W_r, W_a and W in terms of W_t

LH versus RH, and Force Components and Directions

Example 3 (Figure 13-28 for diagram)



Given:
$N_2 = 96$
$N_3 = 16$
$N_4 = 80$
$N_5 = 15$
$P_n = 8\frac{teeth}{in}$
$\varphi = 20°$ for all gears
Helix angle is 25° for N_2 and N_3
Helix angle is 15° for N_4 and N_5

Determine, draw and label the W^t, W^r and W^a at each of the contact points, in terms of input torque T_2. Gear 2 rotates CCW. The unit of T_2 is in $lb \cdot in$.

<u>Solution:</u>
(1) Transverse diametral pitch and pitch diameter
$P_{t2,3} = 7.250\ teeth/in$
$P_{t4,5} = 7.727\ teeth/in$
$d_2 = 13.241''$
$d_3 = 2.207''$
$d_4 = 10.353''$
$d_5 = 2.071''$

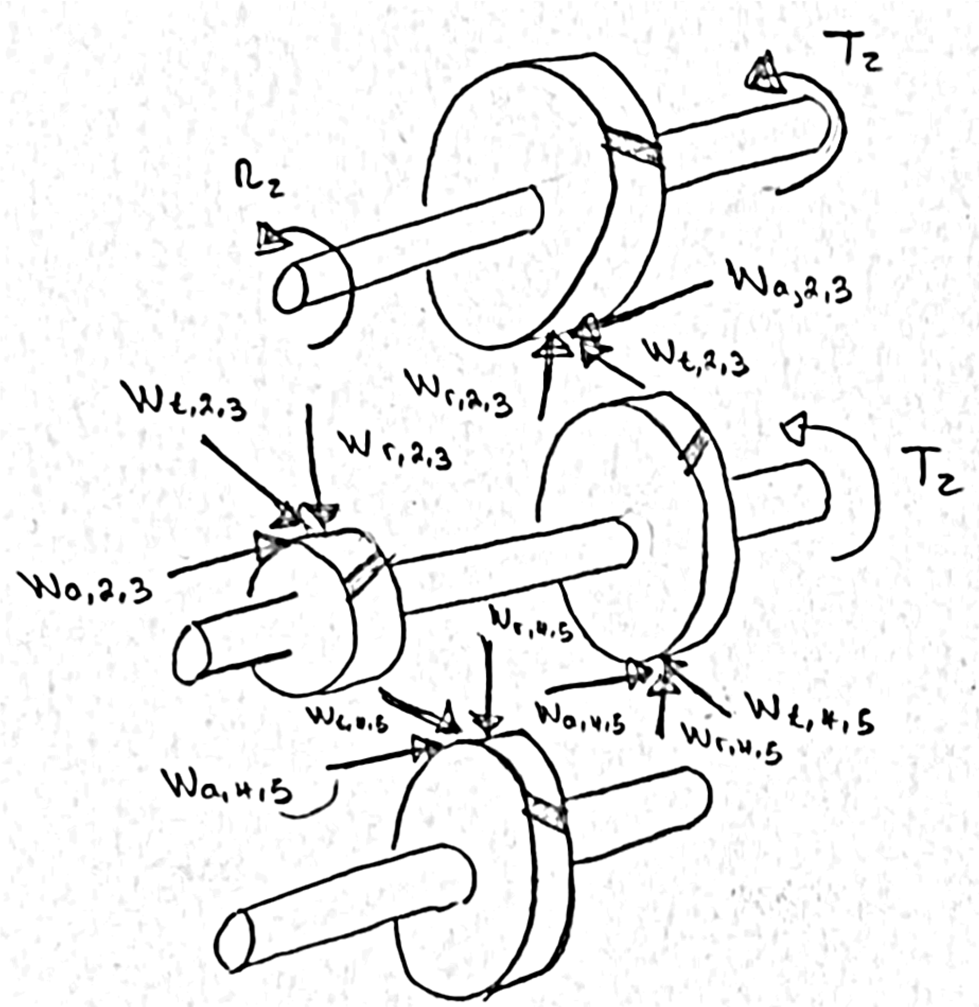
(2) Transverse pressure angle
$\varphi_{t2,3} = 21.880°$
$\varphi_{t4,5} = 20.647°$

(3) Forces between Gears 2 and 3, per (Eq. 13-40)
$W_{2,3}^t = T_2/r_2 = 0.1510\ T_2$
$W_{2,3}^r = W_{2,3}^t tan\varphi_{t2,3} = 0.06064\ T_2$
$W_{2,3}^a = W_{2,3}^t tan\psi_{2,3} = 0.07041\ T_2$

(4) Forces between Gears 4 and 5, per (Eq 13-40)
Torque balance requires $W_{2,3}^t r_3 = W_{4,5}^t r_4$
$W_{4,5}^t = W_{2,3}^t r_3/r_4 = 0.03219\ T_2$
$W_{4,5}^r = W_{4,5}^t tan\varphi_{t4,5} = 0.01213\ T_2$

$W_{4,5}^{a} = W_{2,3}^{t} tan\psi_{4,5} = \ 0.008625\ T_2$

(5) Draw and label the forces



14-18 Analysis
14-19 Design of a Gear Mesh

Example 14-5 (helical gears)

Example 4
A gearbox contains a set of hobbed helical gears. It is driven by a single cylinder engine, and to drive a reciprocating compressor. Output shaft rotates at $1500\ rpm$, with a maximum torque of $550\ lb - in$. Gear ratio is 2.5 to 1. Pitch diameter of pinion must be 3.25 ($\pm$ 1%), and $N_p = 31$. The seven assumptions are the same as the spur gearset example.

Solution:

1. Choose $P_n = 10\ teeth$/in.

$$d_p = \frac{N_p}{P_t} = \frac{N_p}{(P_n\ cos\psi)};$$

$$3.25 = \frac{31}{(10\ cos\psi)}$$

Solving for helix angle $\psi = 17.475°$.

$N_G = (2.5)(31) = 77.5$. Select 78.

So $\psi = 17.475°, N_P = 31, d_P = 3.250"$ (within 3.25" $\pm$ 0.0325"), $N_G = 78$, $d_G = 8.177"$, and use 20° full-depth and uncrowned teeth.

2. Other geometric quantities, including contact ratio

	Pinion	Gear
Addendum, in	0.1	
Dedendum, in	0.125	
Pitch diameter, in	3.250	8.177
Radius of addendum circle, in	1.725	4.189
Radius of base circle, in	1.519	3.820
Transverse base circular pitch, in	0.3077	
Axial pitch, in	1.041	
Face width, in	2.1	
Center-to-center distance, in	5.7135	
Virtual number of teeth	35.7	89.9

$L_{ab} = 0.8175 + 1.7191 - 2.0373 = 0.4993"$

$m_p = 0.4993/0.3077 = 1.623$

$m_F = 2.1/1.041 = 2.017$

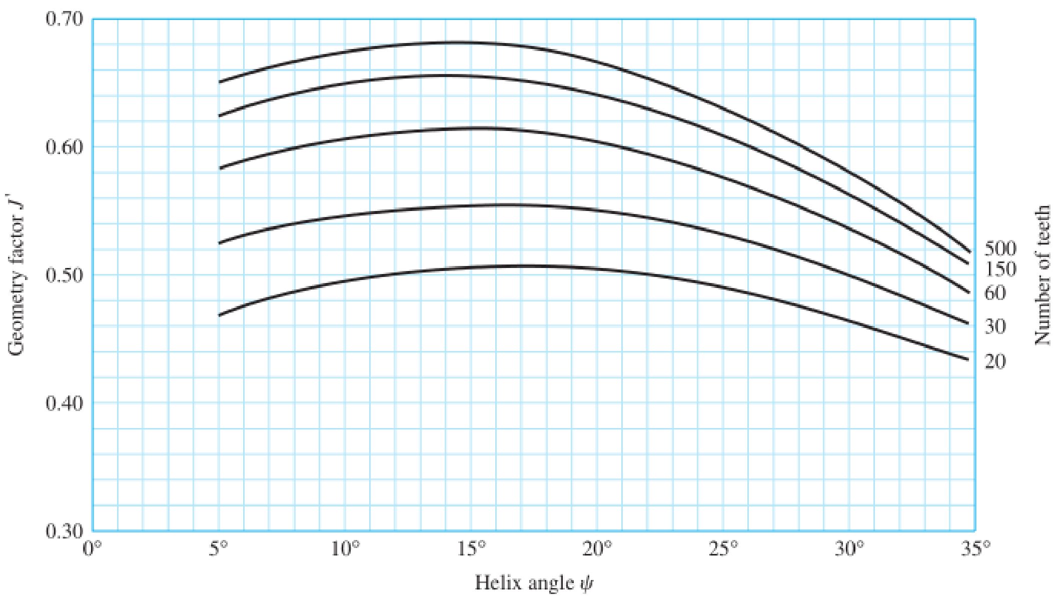
Total contact ratio $= 3.631$

3. Transmitted load

$$W^t = \frac{T_{max}}{r_G} = \frac{550}{\left(\frac{8.177}{2}\right)} = 134.5\ lb$$

5. Geometry factors

<u>For bending:</u> Figures 14-7 and 14-8 (instead of Figure 14-6)



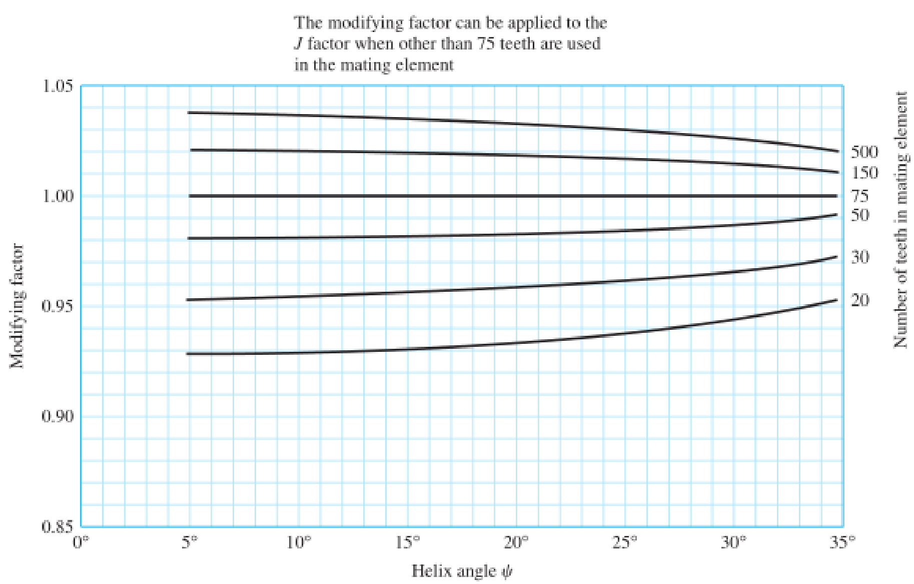
(*b*)

Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Figure 14–8

J'-factor multipliers for use with Fig. 14–7 to find J. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



Limitations are, $\varphi_n = 20°$ and $m_F \geq 2$

$J_P = (0.56)(0.96) = 0.54$

$J_G = (0.62)(1.00) = 0.62$

For contact: (Eq 14-21)

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

$$\left(m_N = \frac{p_N}{0.95\, L_{ab}}\right)$$

Where p_N is the normal base circular length pitch, $p_N = (\pi/P_n)\cos(\varphi_n)$, and $Z = L_{ab}$ is the length of action. p_N and Z are given by (Eq. 14-24) and (Eq. 14-25)

$$p_N = p_n \cos\phi_n \qquad (14\text{–}24)$$

$$Z = [(r_P + a)^2 - r_{bP}^2]^{1/2} + [(r_G + a)^2 - r_{bG}^2]^{1/2} - (r_P + r_G)\sin\phi_t \qquad (14\text{–}25)$$

Therefore, $p_N = \left(\frac{\pi}{10}\right)\cos(20°) = 0.2952"$, $L_{ab} = 0.4993"$, and $m_N = 0.6223$.

Geometry factor I is by (Eq. 14-23) where $m_N = 0.6223$, $m_G = \frac{78}{31} = 2.516$, $\varphi = 20.89°$. So, I $= 0.1915$

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

10. Size factor

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P_n}\right)^{0.0535}$$

$F = 2.1"$, $P_n = 10$ teeth/in, Y is the Lewis form factor from Table 14-2 where "Number of Teeth" means N'. By linear interpolation, $Y_p = 0.376, Y_G = 0.442$

So, $K_{sP} = 1.068$, $K_{sG} = 1.073$

Note: If calculated value is less than 1, set $K_s = 1$.

17. Safety factors S_F and S_H

Bending: $\sigma_P = 3.156\ psi$;
$\sigma_G = 2{,}771\ psi$;
$S_{F,P} = 10.2$
$S_{F,G} = 10.4$

Surface Contact:
$\sigma_{C,P} = 38{,}120\ psi$
$\sigma_{C,G} = 38{,}209\ psi$
$S_{H,P} = 2.6$
$S_{H,G} = 2.3$

18. Assessment

If T_{max} is increased to, say, 3.5 times of the current level, the resulting factors of safety are, assuming all else being the same

$S_{F,P} = 2.9$
$S_{F,G} = 3.0$

$S_{H,P} = 1.4$
$S_{H,G} = 1.2$