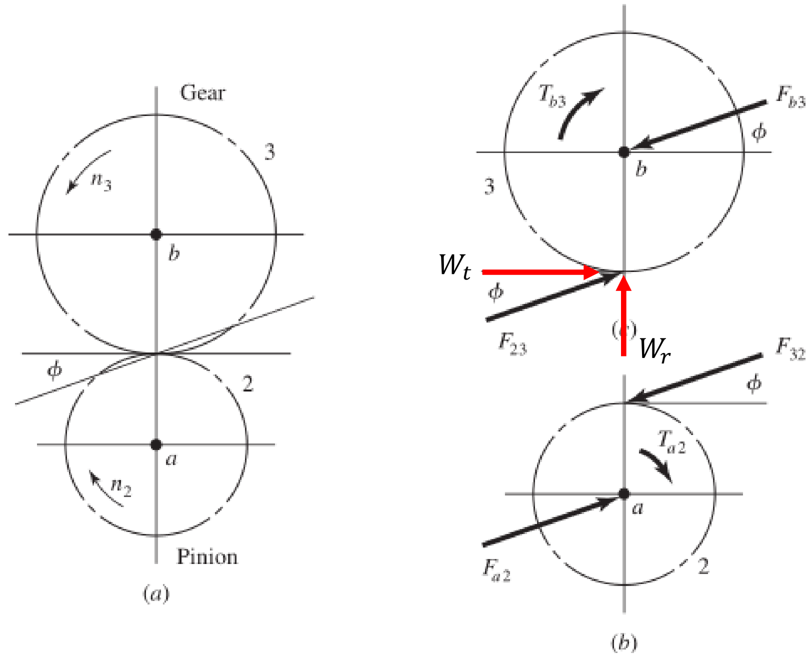
Spur Gears

13-14 Force Analysis – Spur Gearing

Figure 13–32

Free-body diagrams of the forces and moments acting upon two gears of a simple gear train.



F_{23} and F_{32} are action and reaction forces;
Transmitted load: $W_t = F_{23}cos\phi = F_{32}cos\phi$;
Radial load (separating force): $W_r = F_{23}sin\phi = F_{32}sin\phi$

Determination of Transmitted Load

US-customary units

Pitch line velocity

$$V = \pi dn/12 \qquad (13\text{–}34)$$

Where d and n are pinion pitch diameter (in in) and pinion speed (in rpm), or gear pitch diameter (in in) and gear speed (in rpm). V is pitch line velocity in ft/min.

Transmitted Load W_t (or W^t):

$$W_t = 33\,000\frac{H}{V} \qquad (13\text{–}35)$$

Where H is power in hp; V is pitch-line speed in ft/min; and W_t is in lb.

SI Units

$$W_t = \frac{60\,000H}{\pi dn} \qquad (13\text{–}36)$$

Where H is power in kW; d is pitch diameter in mm (of pinion or gear); n is speed (of pinion or gear) in rpm; and W_t is in kN.

Gear Materials

Information from the 6th Ed.

- **Through-hardening:** (by annealing, normalizing and annealing, and quench and temper)
  1040, 1060, 1335, 3135, 4037, 4140, 4340, 5150, 8640, and 8740, with 4140 and 4340 being the most commonly used.
- **Case hardening**
  By carburization (up to 600 HB):
  4118, 4320, 4620, 4720, 4820, 5120, and 8620
  By nitrization:
  4140, 4340

Information from AGMA

- AGMA recommends to specify the following for gear materials:
  - Material designator or stress grade
  - Material cleanliness
  - Surface and core hardness
  - AGMA quality level
- Gear materials are given a stress grade $0 \sim 3$.
  0: Ordinary quality. No gross defects <u>but</u> no close control of quality;
  1: Good quality. Modest control of most important quality items; used in typical industrial applications;
  2: Premium quality. Close control of all critical quality items; improved quality/performance but increased material cost;
  3: Superior quality. Absolute control of all critical quality items; ultimate performance but high material cost.

- Material *cleanliness*
  This is by AMS, Aerospace Material Specification.
  Grade 3 materials: call for AMS 2300 (Premium Aircraft-Quality Steel Cleanliness, Magnetic Particle Inspection).
  Grade 2 materials: call for AMS 2301 (Cleanliness, Aircraft Quality Steel Magnetic Particle Inspection Procedure).
  Grade 1 materials: not required to adhere to any AMS specification.
- AGMA *quality* (or accuracy) *level:*

Q_V	Descriptive	Manufacture	Applications
14-15	Highest level	Trade secret	Highest load & reliability; high speed
12-13	High level	Grinding, shaving	Aerospace turbo-machinery
10-11	Relatively high	Grinding, shaving	Mass production; automotive vehicles; automotive
8-9	Good	Hobbing, shaping	Automotive; electric motor; industrial
6-7	Nominal	Hobbing, shaping (by older machines)	Low speed gears
4-5	Minimal	Casting, molding	Slow speed gears; toys, gadgets

Chapter 14

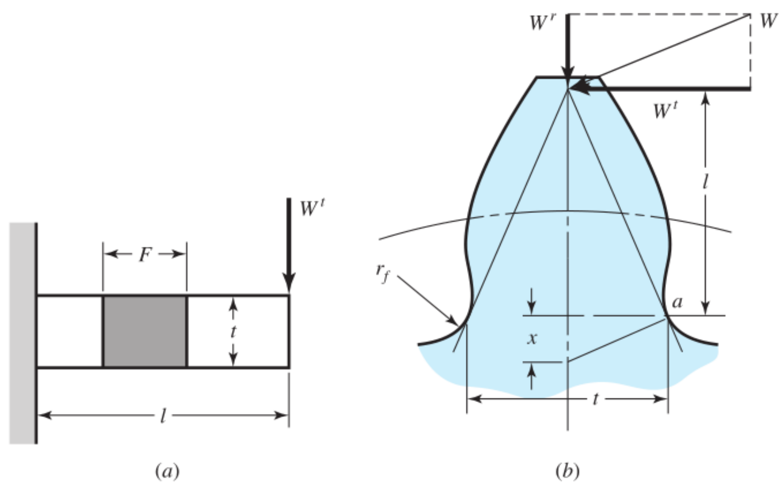
14-1: The Lewis Bending Equation
14-2: Surface Durability
Main failure modes of gearing

- Bending fatigue → Breakage of the tooth
- Contact fatigue → Pitting and spalling
- Wear (due to adhesion, abrasion, corrosion, scoring, scuffing …)

Bending Fatigue is caused by excessive dynamic bending stress at the base of the tooth;

Surface Fatigue is caused by repeated applications of loads on the surface.

Figure 14-1 and the Lewis Bending Equation



A tooth is considered a cantilever beam;

Transmitted and radial loads are labelled W^t and W^r;

Span of beams depends on tooth geometry;

Bending stress at "fixed end": by (Eq. 14-2) or (Eq. 14-3);

$$\sigma = \frac{W^t P}{FY} \tag{14–2}$$

$$Y = \frac{2xP}{3} \tag{14–3}$$

Dynamic effects: (Eq. 14-7) or (Eq. 14-8);

$$\sigma = \frac{K_v W^t P}{FY} \tag{14–7}$$

$$\sigma = \frac{K_v W^t}{FmY} \tag{14–8}$$

Example 14-1: Applying above equations to determine required horsepower;
Example 14-2: Fatigue due to bending stress for infinite life;

Drawbacks: compressive stress due to W^r is not considered.

Surface Durability
At the contact point, each tooth is considered part of a cylindrical surface;

Hertz contact theory: see Sec. 3-19;

Contract stress: (Eq. 14-14);

$$\sigma_C = -C_p \left[\frac{K_v W^t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \tag{14–14}$$

Example 14-3: Applying the above equation;

Drawbacks: fatigue due to contact stress is not considered.

14-3: AGMA Stress Equations
Stress Numbers

- In AGMA terminology, a stress caused by an applied load is a stress number.
- There are two stress numbers: bending and contact.

Bending Stress Number (for spur and helical gears)

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \tag{14–15}$$

Contact Stress Number (for spur and helical gears)

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

14-4 AGMA Strength Equations

Allowable Stress Numbers

- In AGMA terminology, an allowable stress is an allowable stress number.
- There are two allowable stress numbers: allowable bending and allowable contact.

Allowable Bending Stress Number (for spur and helical gears)

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–17}$$

(Where S_F is factor of safety)

Allowable Contact Stress (for spur and helical gears)

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–18}$$

(Where S_H is factor of safety)

14-18 Analysis

Figures 14-17 and 14-18

Summary of above formulas (US-customary units only), including where to find the factors.

Overload Factors K_o

Table of Overload Factors, K_o

	Driven Machine		
Power source	**Uniform**	**Moderate shock**	**Heavy shock**
Uniform	**1.00**	**1.25**	**1.75**
Light shock	**1.25**	**1.50**	**2.00**
Medium shock	**1.50**	**1.75**	**2.25**

Examples of power sources and driven machines in each “shock” category (courtesy Mechanical Design of Machine Elements and Machines, A Failure Prevention Perspective, J.A. Collins, Wiley & Sons, 2003).

Power sources:

- Uniform: electric motors, steam turbines, gas turbines;
- Light shock: multi-cylinder engines;
- Medium Shock: single-cylinder engines.

Driven machines:

- Uniform: generators; uniformly loaded conveyors;
- Medium Shock: centrifugal pumps, reciprocating pumps and compressors, heavy-duty conveyors, main drives of machine tools;
- Heavy Shock: punch press, crushers, shears, power shovels.

Example 14-4: Spur gears
Example 14-5: Helical gears

14-19 Design of a Gear Mesh
Initial Steps

- Choose a diametral pitch; $(P = 8, or\ 10\ teeth/in)$
- Select face width and material;
- Decide on core and surface hardness for pinion and gear, and other details such as reliability, etc.

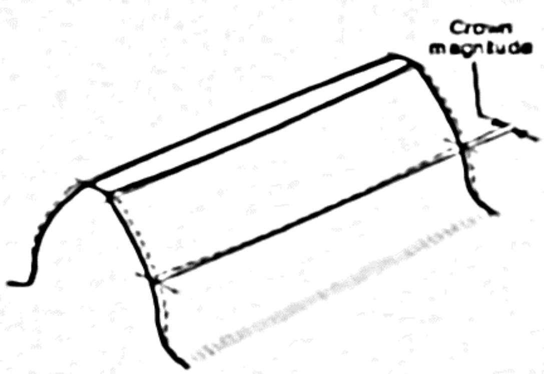
With the second and third steps above, iteration back to the first step may be necessary.

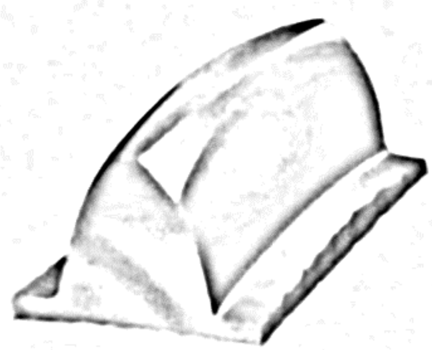
Detailed Calculations

- Referring to page 768;
- It shows all factors, and four factors of safety. They are, one for pinion bending, one for gear bending, one for pinion contact, and one for gear contact.

Example:
A gearbox contains a set of spur ears. IT is driven by a single cylinder engine, and to drive a reciprocating compressor. Output shaft rotates at 1500 rpm, with a maximum torque of 550 lb-in. Gear ratio is 2.5:5 to 1. Pitch diameter of pinion Is expected to be around 3.25”. Assume:

(1) AGMA grade 1 steel through hardened to $H_B = 350$ for pinion and 280 for gear;
(2) 20° full depth and uncrowned teeth;
(3) $Q_V = 10$;
(4) 10-year life of 8-hour shift continuous operation;
(5) Gears are located in the mid-span of their respective shafts;
(6) 99% reliability.
(7) Oil temperature is less than $250°F$





Solution

1. Choose $P = 10\,\frac{teeth}{in}$

   $N_P = P \cdot d_p = (10)(3.25) = 32.5$ - Select $N_p = 33$

   $N_G = (2.5)(33) = 82.5$ - Select $N_G = 83$.

   So $N_P = 33, d_p = 3.3, N_G = 83$, $d_G = 8.3$, 20° full depth and uncrowned teeth.

   Contact ratio is:

$$m_c = \frac{0.81145 + 1.6896 - 1.9837}{\left(\frac{\pi}{10}\right)\cos(20°)} = 1.7525$$

2. Face width $F = (3{\sim}5)p = 4p = 1.26"$; Choose $F = 1.25"$.

3. Transmitted load

$$W^t = \frac{T_{max}}{r_G} = \frac{500}{\left(\frac{8.3}{2}\right)} = 132.5\ lb$$

4. AGMA allowable stress numbers

   Tables 14-3 and 14-6 indicate what to use, based on gear materials.

   Pinion: $H_B = 350, S_t = 39{,}855\ psi, S_c = 141{,}800\ psi$

   Gear: $H_B = 280, S_t = 34{,}444\ psi, S_c = 119{,}260\ psi$

5. Geometry factors

   For bending: $J_P = 0.4, J_G = 0.445$

   For contact: use (Eq. 14-23) where m_N is the load sharing ratio, and m_G is the speed ratio.

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

   That is, $m_N = 1$ (for spur gears), and $m_G = \frac{83}{33} = 2.515$.

   So, $I = 0.115$

6. Elastic coefficient

   $C_P = 2300\sqrt{psi}$

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ $(\sqrt{\text{MPa}})$ *Source: AGMA 218.01*

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

7. Dynamic factor
   $Q_V = 10$; So $B = 0.39685, A = 83.776$
   Pitch line velocity $V = \pi d_G n_G/12 = 3259\ ft/min$
   (Eq. 14-29): max. pitch line velocity $V_{max} = 8240\ ft/min$

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \qquad (14\text{–}29)$$

   (Eq. 14-27): $K_V = 1.229$

$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \qquad (14\text{–}27)$$

8. Overload factor
   $K_o = 1.75$

9. Surface-condition factor
   $C_r = 1$ (currently as a place holder)

10. Size factor

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$

   $F = 1.25"$, $P = 10\ teeth/in$, Y is the Lewis form factor from Table 14-2. By linear interpolation, $Y_P = 0.368$, $Y_G = 0.439$.
   So, $K_{sP} = 1.038$, $K_{sG} = 1.043$
   Note: if calculated value is less than 1, set $K_s = 1$.

11. Load distribution factor

Sec. 14-11 lists conditions under which to use (Eq. 14-30 through (Eq. 14-35).

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

then $C_{mc} = 1$

$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

then $C_{pf} = 0.01600$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases} \qquad (14\text{–}33)$$

then $C_{pm} = 1$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

then $C_{ma} = 0.1466\ (commercial, enclosured\ units)$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

then $C_e = 1$

(Eq. 14-30): $K_m = 1.163$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad (14\text{–}30)$$

12. Hardness-ratio factor

$\dfrac{H_{BP}}{H_{BG}} = \dfrac{350}{280} = 1.25$

$A' = 0.002935$

(Eq. 14-36): $C_{Hg} = 1.004$; but $C_{HP} = 1$.

13. Stress-cycle factors

For pinion, $n_p = \left(\frac{83}{33}\right) \cdot n_G = 3772.7\ rpm$

Pinion's life $N = (10)(365)(8)(60)(3772.7) = 6.610(10^9)$ revs

Figure 14-14: $Y_{NP} = 1.6831 \cdot N^{-0.0323} = 0.8108$

Figure 14-15: $Z_{NP} = 2.466 \cdot N^{-0.056} = 0.6951$

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

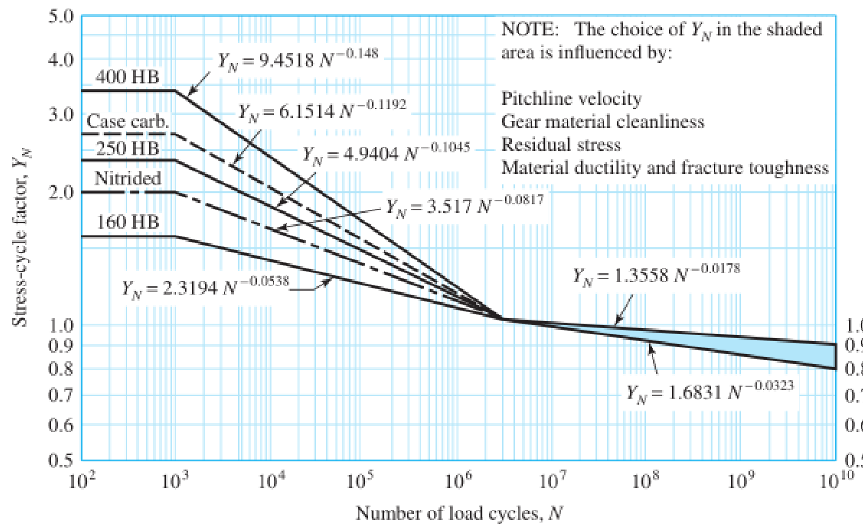
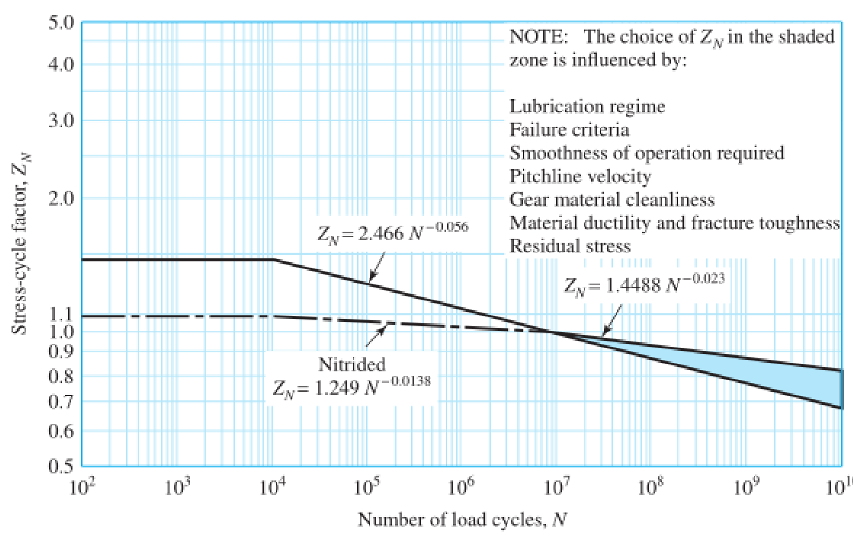


Figure 14–15

Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*



Gears life $N = (10)(365)(8)(60)(1500) = 2.628(10^9)$ revs

Figure 14-14: $Y_{NG} = 1.6831 \cdot N^{-0.0323} = 0.8353$

Figure 14-15: $Z_{NG} = 2.466 \cdot N^{-0.056} = 0.7320$

14. Reliability factor
    Table 14-10 or (Eq. 14-38):
    $K_R = 1$

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \leq R \leq 0.9999 \end{cases} \qquad (14\text{–}38)$$

15. Temperature factor
    Just use $K_T = 1$ (Valid for processes up to $250°F$)

16. Rim-thickness factor
    (Eq. 14-40): $K_B = 1$ (but need to ensure $m_B \geq 1.2$)

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \qquad (14\text{–}40)$$

17. Safety factor S_F and S_H
    They are to be determined from AGMA equations.
    $\sigma_P = 6{,}880\ psi$
    $\sigma_G = 6{,}214$ psi

$$\sigma_{all,P} = \frac{32{,}314}{S_{F,P}}$$
$$\sigma_{all,G} = \frac{28{,}771}{S_{F,G}}$$

$S_{F,P} = 4.70$
$S_{F,G} = 4.63$

Similarly,
$S_{H,P} = 1.49$
$S_{H,G} = 1.41$

SPUR GEAR BENDING

Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

W^t: from the equation above

K_o: Table below

K_v: Eq. (14–27); p. 748

K_s: 1 [or Eq. (a), Sec. 14–10]; p. 751

K_m: Eq. (14–30); p. 751

K_B: Eq. (14–40); p. 756

J: Fig. 14–6; p. 745

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

S_t: ${}_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 740, 741

Y_N: Fig. 14–14; p. 755

K_T: 1 if $T < 250°$F

K_R: Table 14–10, Eq. (14–38); pp. 756, 755

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

SPUR GEAR WEAR

Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 751

Eq. (14–30); p. 751

1

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

Eq. (14–23); p. 747

Eq. (14–27); p. 748

Eq. (14–13), Table 14–8; pp. 736, 749

Table below

$_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 743, 744

Fig. 14–15; p. 755

Section 14–12, gear only; pp. 753, 754

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\mathrm{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 756, 755

1 if $T < 250°\mathrm{F}$

Gear only

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25