Sep. 3/19

Engineering vibration - 4th

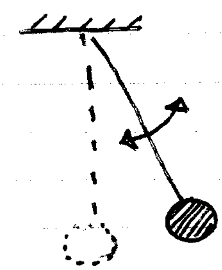
Office hours: Tues / Thurs → (2:30 ~ 3:30)

Tutorials to have questions marked in class (worth 20%)
↳ groups of 2/3

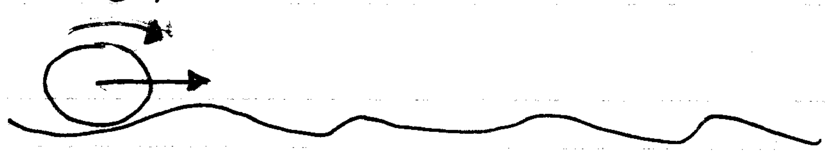
Midterm: October 10TH/19 during lecture time (75 min)
↳ Formula Sheet provided

Chapter 1 - Introduction to Vibration and Free Response
Fundamentals of Vibration
Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point.

The oscillations of the vibrations may be periodic (like a pendulum):



or random (the movement of a tire on gravel road):

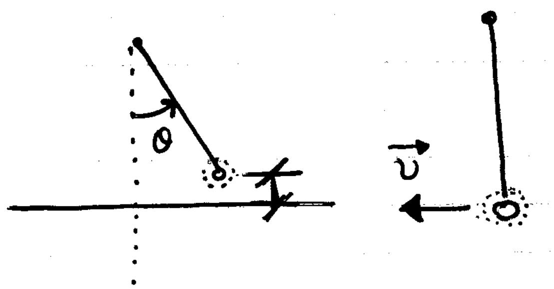


Avoid vibrations
* Cyclic motion implies cyclic forces:
  - aircraft frame and wings
  - imbalances in rotating parts
* even modest levels of vibration can cause discomfort
  - automobiles
* Vibrations generally lead to a loss of precision in controlling machinery

"Tacoma narrows bridge"
- opened July 1, 1940
- collapsed Nov. 7, 1940

"<u>Vehicle suspension systems</u>"

Good uses of vibrations:
- music, guitar, speakers
- structural analysis (ultrasonic), detecting cracks



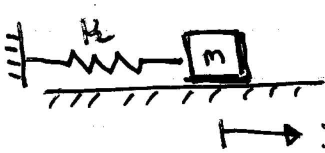
For any vibration system

1°: means for storing potential energy
- spring / elasticity

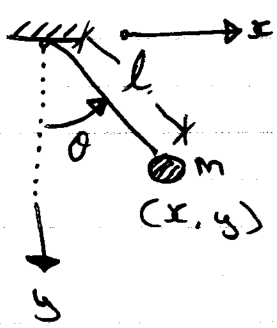
2°: means for storing kinetic energy
- mass / inertia

3° means by which energy is gradually lost
- damper

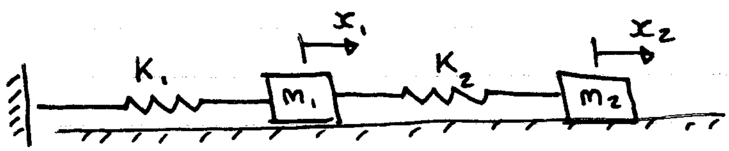
<u>Degree of freedom</u>:
The DOF of a system is defined as the minimum numbers of independent coordinates required to determine completely the positions of all parts of the system at any instant of time.



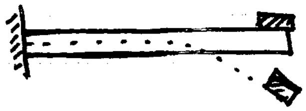
→ 1 DOF



→ $x^2 + y^2 = \ell^2$
two coordinates, but not independent, so still 1 DOF.



2 DOF



Infinite number of DOF

Discrete and continuous systems

- Systems with finite number of DOF is called discrete or <u>lumped</u> parameters system
- Systems with infinite number of DOF is called <u>continuous</u> system

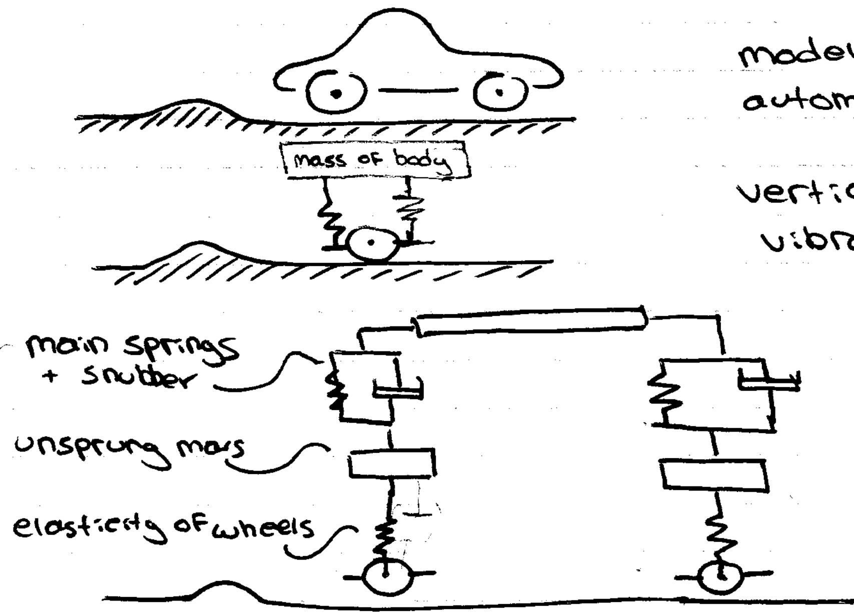
Vibration:

- Free vibration: the system, after an initial disturbance, is left to vibrate on its own.
- Forced vibration: the system is subjected to an external force.
- undamped vibration: no energy lost
- damped vibration: energy lost
- linear vibration: if all the basic components of a system is linear, the principle of superposition holds, and the differential equation is linear.
- (beyond scope of class) Non-linear vibration
- Deterministic vibration: the values of the exciting forces is known at all times
- Random vibration

- Vibration analysis

The vibration analysis of an engineering system involves the following four steps;

1- mathematical modeling: the mathematical model is simprified, keeping in view the purpose of the analysis



modeling of automobile

vertical vibration

(Expanded look...)

2 - Governing Equation

3 - Solution

(*) concentration of course

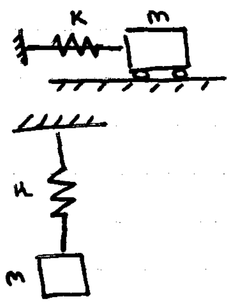
4 - Interpretation of Results

Spring element 

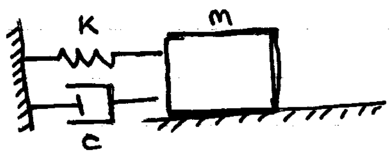
mass element 

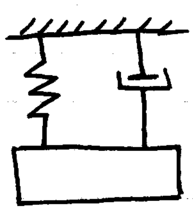
damper 

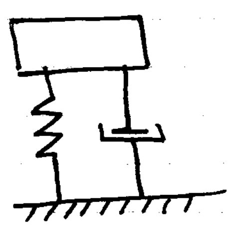
Example: spring-mass



Example: spring-mass-damper

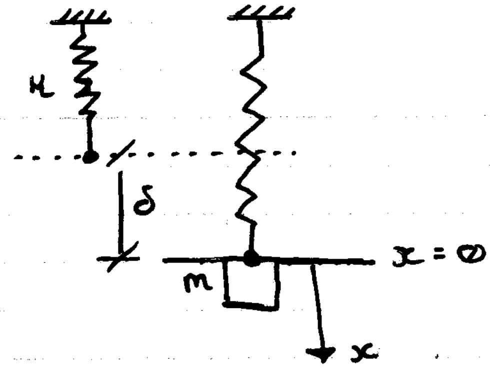


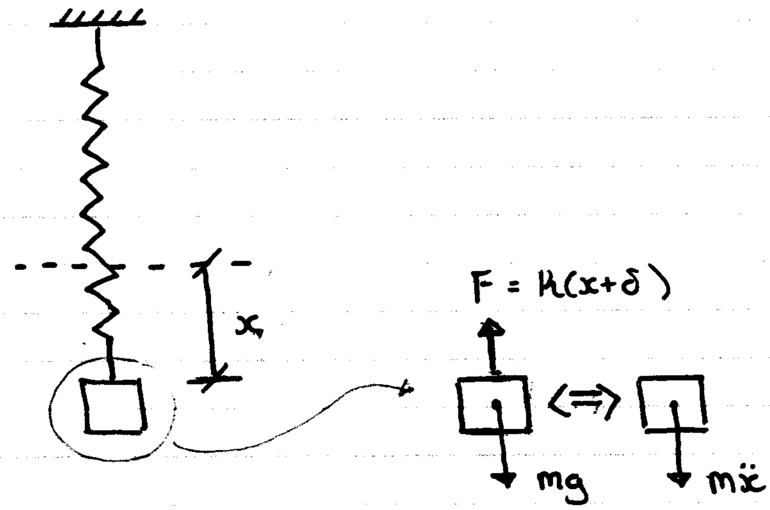




All represent 1 DOF

Modeling a single DOF





$$\Sigma F = ma$$

$$mg - k(x+\delta) = m\ddot{x}$$

Since $mg = k\delta$ (Hooke's Law)

$$\boxed{m\ddot{x} + kx = \varnothing}$$

The solution:

$$x(t) = A \sin(\omega_n t + \phi) \quad (*)$$

(in radians)

Since

$$\dot{x}(t) = \omega_n A \cos(\omega_n t + \phi)$$

Then

$$\ddot{x}(t) = -A\omega_n^2 \sin(\omega_n t + \phi) = -\omega_n^2 x(t) \quad (*)$$

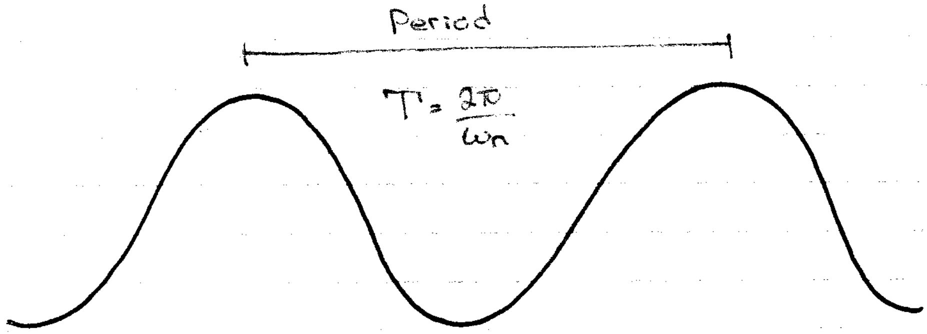
$$\Rightarrow m(-\omega_n^2 x) + kx = \varnothing$$

$$\Rightarrow (-m\omega_n^2 + k)x = \varnothing$$

$$\Rightarrow -m\omega_n^2 + k = \varnothing$$

The "natural frequency" $\omega_n = \sqrt{\frac{k}{m}}$ → unit of ω_n : rad/s

$$x(t) = A \sin(\omega t + \phi)$$



Period : $\omega_n T = 2\pi$

$$T = \frac{2\pi}{\omega_n}$$

Frequency (f_n) : $f_n = \frac{1}{T} = \frac{\omega_n}{2\pi}$ (measured in Hz)

$$\omega_n = 2\pi f_n$$

Given initial distance x_o and initial velocity v_o :

$$\begin{cases} x_o = x(t)\big|_{t=0} = A \sin\phi \\ v_o = \dot{x}(t)\big|_{t=0} = A\omega_n \cos\phi \end{cases}$$

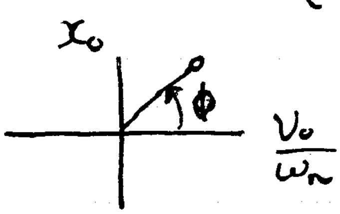
$$\frac{v_o}{\omega_n} = A\cos\phi$$

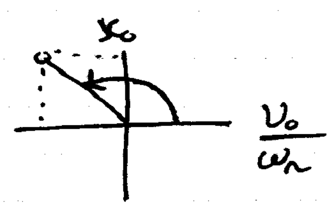
$$x_o^2 + \left(\frac{v_o^2}{\omega_n}\right) = A^2 \sin^2\phi + A^2\cos^2\phi = A^2$$

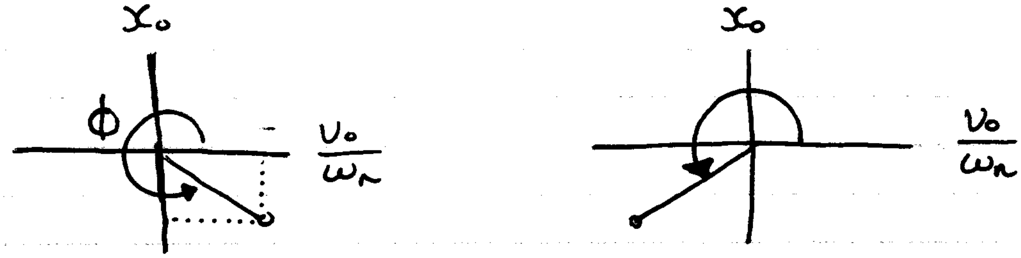
$$\rightarrow \quad A = \sqrt{x_o^2 + \left(\frac{v_o}{\omega_n}\right)^2} = \frac{1}{\omega_n}\sqrt{\omega_n^2 x_o^2 + v_o^2}$$

$$\frac{\omega_n x_o}{v_o} = \tan\phi$$

$$\phi = \tan^{-1}\left(\frac{\omega_n x_o}{v_o}\right)$$

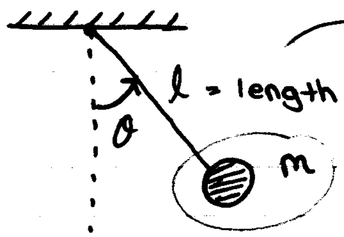






$$\boxed{x(t) = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} \sin\left(\omega_n t + \tan^{-1} \frac{\omega_n x_0}{v_0}\right)}$$

Pendulum

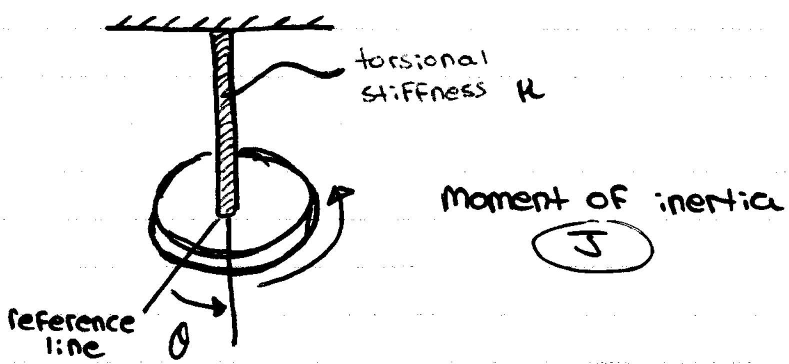


consider mass of bar (compared to mass of weight) as zero.

$$\ddot{\theta} + g/l\,\theta = 0$$

$$\omega_n = \sqrt{\frac{g}{l}}$$

Shaft and disk



$$J\ddot{\theta} + K\theta = 0$$

$$\omega_n = \sqrt{\frac{K}{J}}$$

Example: The total mass of the model is $m = 30$ kg, the frequency of the model is $f_n = 10$ Hz, what is the k?

Solution: Since $\omega_n = \sqrt{k/m}$

$$\Rightarrow \quad k = m\omega_n^2$$
$$= (30)(2\pi f_n)^2$$
$$= (30)(2\pi(10))^2$$
$$= 1.184 \times 10^5 \text{ N/m}$$

standard unit for spring constant

Example: $m = 2$ kg
$k = 200$ N/m

For the following initial conditions
a) $x_0 = 2$mm, $v_0 = 1$ mm/s
b) $x_0 = -2$mm, $v_0 = 1$ mm/s
c) $x_0 = 2$mm, $v_0 = -1$ mm/s

Find the response of the system.

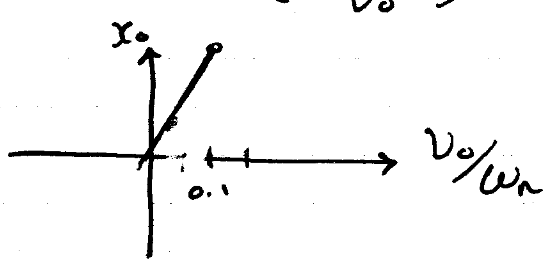
Solution: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{200}{2}} = 10$

The amplitude:

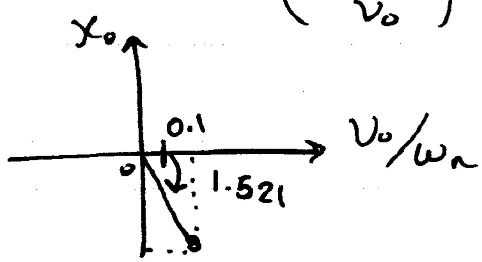
$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} = \frac{\sqrt{10^2(\pm 2)^2 + (\pm 1)^2}}{10} = 2.0025 \text{ mm}$$

Phase:

a) $\phi = \tan^{-1}\left(\dfrac{\omega_n x_0}{v_0}\right) = \tan^{-1}\left(\dfrac{(10)(2)}{(1)}\right) = 1.521$ rad $(87.147°)$

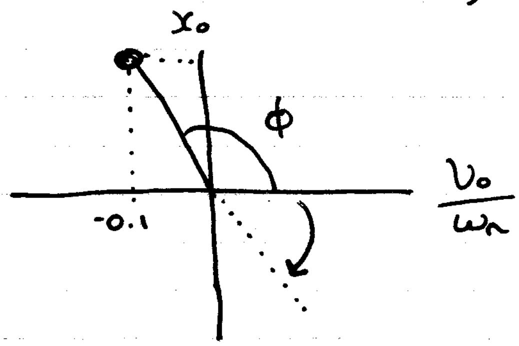


b) $\phi = \tan^{-1}\left(\dfrac{\omega_n x_0}{v_0}\right) = \tan^{-1}\left(\dfrac{(10)(-2)}{1}\right) = -1.521$ rad



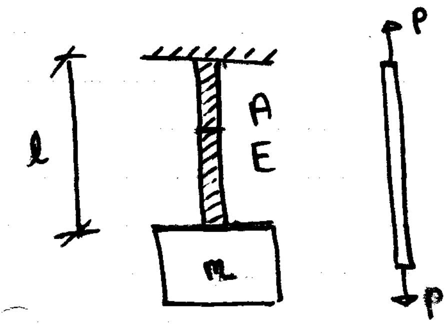
c) $\phi = \tan^{-1}\left(\frac{\omega_n x_o}{v_o}\right) = \tan^{-1}\left(\frac{(10)(2)}{(-1)}\right) = -1.521 + \pi \text{ rad}$

$= 1.621 \text{ rad}$



(2nd + 4th quadrant, add π to get correct value.)

More on springs and stiffness

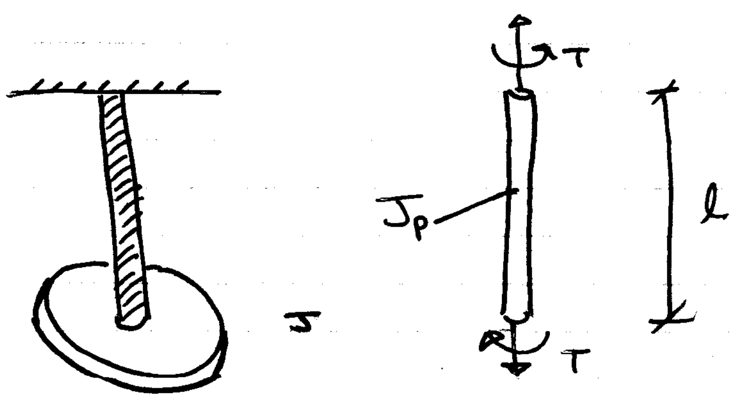


The change in length

$$\Delta u = \frac{Pl}{AE}$$

$$P = \frac{AE}{l}\Delta u$$

$$\therefore k = \frac{AE}{l}$$



$$\theta = \frac{Tl}{GJ}$$

$$T = \frac{GJ_p}{l}\theta$$

$$k = \frac{GJ_p}{l}$$