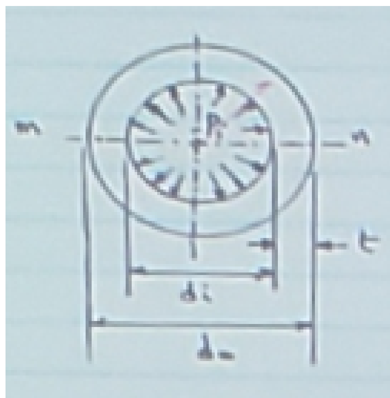
Lecture (Feb. 12th, 2019)

$$\pi d_i t \sigma_2 = \frac{\pi d_i^2 p_i}{4}$$

Where:
σ_1 = circumferential (hoop) stress, tangential stress
σ_2 = longitudinal stress
d_i = internal diameter
t = wall thickness
p = internal pressure

Thick-walled Cylinders with Internal and External Pressures



$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_o^2 r_i^2}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_o^2 r_i^2}{r^2 (r_o^2 - r_i^2)}$$

Thick Walled Cylinders with Internal Pressure only

Tangential Stress:

$$\sigma_t = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Radial Stress:

$$\sigma_r = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

Axial Stress:

$$\sigma_t = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

$$\sigma_{t,max} = \frac{p_i\left(r_o^2 - r_i^2\right)}{r_o^2 - r_i^2} \quad (At\, r = r_i)$$

$$\sigma_{r,max} = -p_i \quad (At\, r = r_i)$$

23.2 Metal Fits

- Basic size is the exact theoretical size. Limiting variations begin from the basic dimension.

- The nominal size of a part is the designation used for the purpose of general identification.

- Limits are the stated maximum and minimum permissible dimensions.

- Tolerance is the total permissible variation in size. (- The difference between the two limits)

Example: A $1.500 \pm 0.010\ in$ shaft is a shaft that has a basic size of $1 - 1/2\ \ in$, (in this case the basic size is also the nominal size), in diameter and a tolerance of $0.020\ in$.

- Unilateral tolerance is when one of the limits is the basic size

Example: $1.500^{+0.000}_{-0.010}$

Unilateral tolerances are usually used in specifying fits for interchangeable parts.

- Bilateral tolerance is when variation is permitted in both directions from the basic size.

Example: $1.500\ \pm 0.010$

- Natural tolerance is equal to plus and minus three standard deviations from the mean. For normal distributions, 99.73% of production is within natural tolerance limits.
- Clearance is used when the internal member of two mating parts is smaller than the external member.
  a – diametral clearance is the measured difference in the two diameters.
  b – radial clearance is the difference in the two radii.
- Interference is when the internal member is larger than the external member.

23.3 Force Fits and Shrink Fit

In a force-fit assembly, the pressure between the parts depends on the amount of interference.

If the radial interference is δ, the contact pressure at the interference radius R is:

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + v_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - v_i\right)\right]}$$

If the members are of the same material, then:

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right]$$

If the mating parts are of the same material and $r_i = 0$ (hub and solid shaft):

$$p = \frac{E\delta}{2R}\left[1 - \frac{R^2}{r_o^2}\right]$$

The maximum tangential and radial stresses at the inside surface of the external member are:

$$(\sigma_t)_{max} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = \frac{p\left(1 + \frac{R^2}{r_o^2}\right)}{\left(1 - \frac{R^2}{r_o^2}\right)} \quad ; \quad (\sigma_p)_{max} = -p$$

Substituting for p:

$$(\sigma_t)_{max} = \frac{E\delta}{2R}\left[1 + \frac{R^2}{r_o^2}\right]$$

$$(\sigma_r)_{max} = \frac{-E\delta}{2R}\left[1 - \frac{R^2}{r_o^2}\right]$$

The maximum shearing stress is:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_t - \sigma_r}{2}\right)^2} = \frac{E\delta}{2R}$$

For brittle material, the maximum normal stress should not exceed the ultimate tensile strength of the material.

$$\frac{S_{ult}}{n_d} = \frac{E\delta}{2R}\left(1 + \frac{R^2}{r_o^2}\right)$$

For ductile material, based on the maximum shear theory.

$$\frac{S_{yp}}{n_d} = \frac{E\delta}{R}$$

Where
n_d = design factor
S_{ult} = ultimate tensile strength, psi
S_{yp} =yield strength, psi

$$(\sigma_r)_{max} = -\frac{E\delta}{2R}\left(1 - \frac{R^2}{r_o^2}\right)$$

The maximum shearing stress is:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_t - \sigma_r}{2}\right)^2} = \frac{E\delta}{2R}$$

For brittle material, the maximum normal stress should not exceed the ultimate tensile strength of the material.

$$\frac{S_{ult}}{n_d} = \frac{E\delta}{}$$

23-4 Force Fits – Steel Shaft & Cast-iron Hub

$$p = \frac{E_c\delta\left[1 - \left(\frac{d_i^2}{d_o^2}\right)\right]}{d_i\left[1.53 + 0.47\left(\frac{d_i^2}{d_o^2}\right)\right]}$$

Where, E_c = modulus of elasticity of cast iron. And:

$$\frac{S_{ult}}{f_s} = \frac{E_c\delta\left[1 - \left(\frac{d_i^2}{d_o^2}\right)\right]}{d_i\left[1.53 + 0.47\left(\frac{d_i^2}{d_o^2}\right)\right]}$$

23-5 Holding ability of Force and Shrink Fits

$$T = \frac{fp\pi d_i^2 L}{2}$$

Where:
$T =$ Transmitted torque, $lb - in$
$p =$ contact pressure, psi
$d_i =$ Diameter, in
$L =$ length of hub, in
$f =$ coefficient of friction (usually from 0.1 to 0.05)
$\delta =$ diametral interference, in

-Thermal-stresses and strains

When the temperature of an unrestrained body is uniformly increased, the body expands, and the normal strain is:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha(\Delta T)$$

Where:
$\alpha =$ coefficient of thermal expansion (Table 3.3)
$\Delta T =$ temperature change in degrees

If a straight bar is restrained at the ends, the compressive stress is:

$$\sigma = \varepsilon E = \alpha(\Delta T)E$$

If a uniform plate is restrained at the edges

$$\sigma = \frac{\alpha(\Delta T)E}{1 - v}$$

Although referred to as thermal stresses, the above are not thermal stresses, but arise from the edge restrains. A thermal stress is one which arises because of the existence of a temperature gradient in a body.

23-6 Assembly of Shrink Fits

- The minimum change in temperature for assembly is:

$$\Delta T = \frac{\delta}{\alpha d_i}$$

Where:
$\delta =$ diametral interference, in
$\alpha =$ coefficient of expansion, $in\ per\ in\ per\ °F$
$\Delta T =$ exchange in temperature, $°F$

- The force required to press the parts together is:

$$F = 2\pi r_i Lpf$$

$$\pi d_i Lpf$$

Lecture (Feb. 14th, 2019)

Curved Beams in Bending

The neutral axis and the centroidal axis of a curbed beam do not coincide, and the stress distribution is not linear.

The location of the neutral axis with respect to the center of curvature is given by:

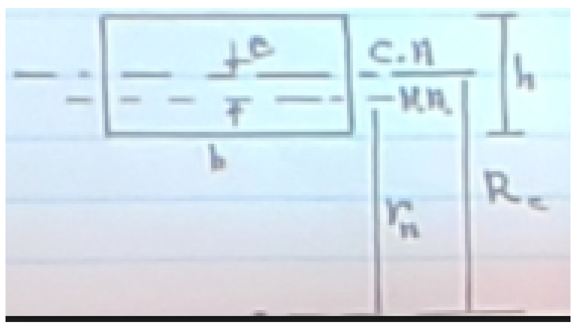
$$r_n = \frac{A}{\int \frac{dA}{r}}$$

For rectangular section:

$$R_c = r_i + \frac{h}{2}$$

And:

$$r_n = \frac{A}{\int \frac{dA}{r}} = \frac{bh}{\int_{r_i}^{r_o} \frac{b}{r}\, dr} = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)}$$



For solid round section:

$$R_c = r_i + \frac{d}{2}$$

And:

$$r_n = \frac{d^2}{4\left(2R_c - \sqrt{4R_c^2 - d^2}\right)}$$

The stress distribution is given by:

$$\sigma = \frac{My}{Ae(r_n - y)}$$

(Where e is the distance between the neutral axis and the centroidal axis.)

At the inner fiber:

$$\sigma_i = \frac{Mc_i}{Aer_i}$$

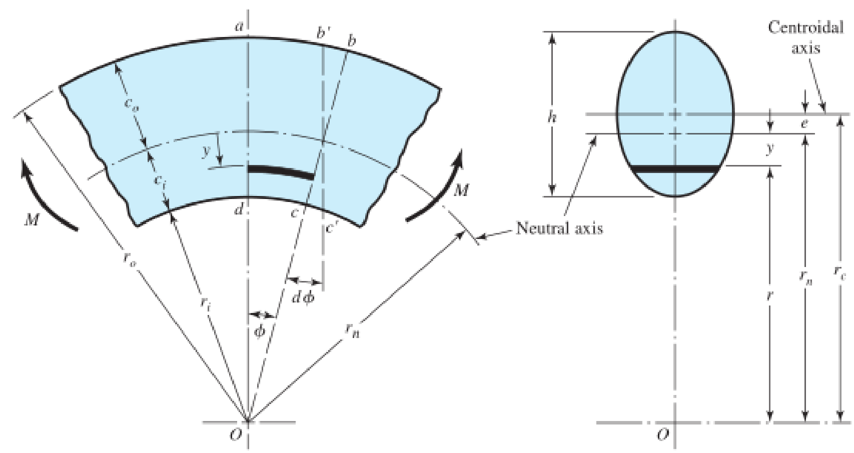
At the outer fiber:

$$\sigma_o = \frac{Mc_o}{Aer_o}$$

(For other cross-section shapes refer to Table 3.4)

Figure 3–34

Note that y is positive in the direction toward the center of curvature, point O.



r_o = radius of outer fiber

r_i = radius of inner fiber

h = depth of section

c_o = distance from neutral axis to outer fiber

c_i = distance from neutral axis to inner fiber

r_n = radius of neutral axis

r_c = radius of centroidal axis

$e = r_c - r_n$, distance from centroidal axis to neutral axis

M = bending moment; positive M decreases curvature

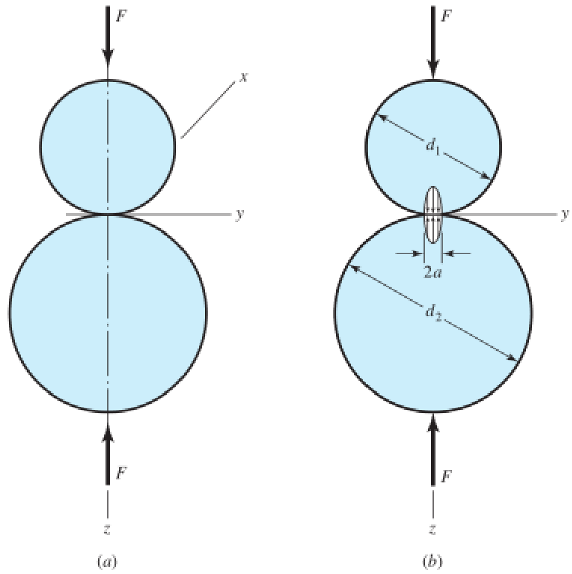


Figure 3–36

(*a*) Two spheres held in contact by force *F*; (*b*) contact stress has a hemispherical distribution across contact zone of diameter 2*a*.

Hertz Contact Stresses

When two solid spheres are passed together with a force F, the radius of the circular contact area is:

$$a = \sqrt[3]{\frac{3F}{8}\frac{[(1-v_1^2)/E] + [(1-v_2^2)/E_2]}{(1/d_1) + (1/d_2)}}$$

Where:

a = radius of the circular area of contact
d_1 = diameter of sphere 1
d_2 = diameter of sphere 2
E_1 = modulus of elasticity of sphere 1
E_2 = modulus of elasticity of sphere 2
v_1 = Poisson's radio of sphere 1
v_2 = Poisson's ratio of sphere 2
F = applied force

The maximum pressure at the centre of the contact area is:

$$p_{max} = \frac{3F}{2\pi a^2}$$

The above equations are also valid for the case of a sphere and a plane surface on a sphere and an internal spherical surface. For a plane surface use $d = \infty$, and for internal surfaces the diameter is expressed as a negative quantity.

Plane: $d = \infty$

Internal Spherical Surface: $d < 0$

The maximum stress occur on the $z - axis$ which is the axis of application of external force: (These are principal stresses)

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \left|\frac{z}{a}\right|\tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{1+\frac{z^2}{a^2}}$$

In using the above equations, the value of Poisson's ratio used must be that of the sphere under consideration.

Also:

$$\tau_{xz} = \tau_{yz} = \frac{\sigma_x - \sigma_z}{2} = \frac{\sigma_y - \sigma_z}{2} = \tau_{max}$$

Since:

$$\sigma_x = \sigma_y = \tau_{xy} = 0$$

In the case of two contacting cylinders of length l and diameter, d_1 and d_2, the area of contact is a rectangle of width $2b$ where:

$$b = \sqrt{\frac{2F}{\pi l}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

And the maximum pressure is:

$$p_{\max} = \frac{2F}{\pi b l}$$

The above equations are also applicable for a cylinder and a plane surface as well as for a cylinder and an internal cylindrical surface where:

For a plane surface: $d = \infty$

For cylindrical surface: $d < 0$

The stress state on the $z - axis$ given by the following:

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1 + \frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right)$$

$$\sigma_y = -p_{\max}\left(\frac{1 + 2\frac{z^2}{b^2}}{\sqrt{1 + \frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right)$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{\sqrt{1 + z^2/b^2}}$$

Note that:

For $0 \leq z \leq 0.436b$: $\sigma_1 = \sigma_x$; $\tau_{max} = (\sigma_1 - \sigma_3)/2$

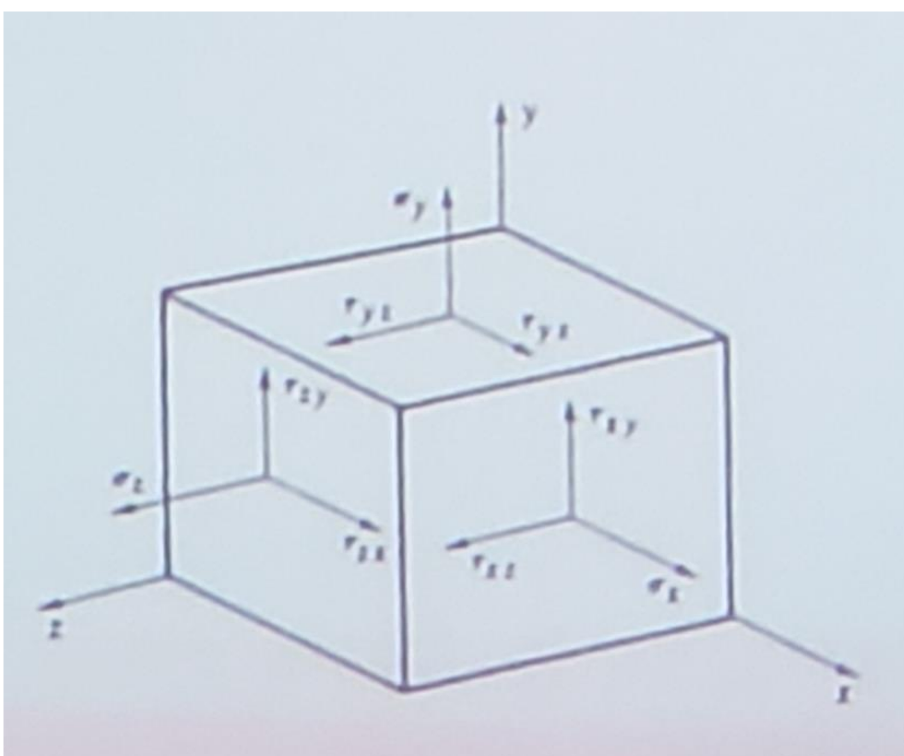
And:

For $z \geq 0.436b$: $\sigma_1 = \sigma_y$; $\tau_{max} = (\sigma_1 - \sigma_3)/2$

Also note that τ_{xy} here is not the largest of the three shears for all values of z/b, but is max for $z/b = 0.786$ and is the larger at this point.

<u>6.2 - Determination of Principal Stresses</u>

Whatever the aspect of the stress at a joint may be, it can always be expressed in terms of normal stresses and shear stresses.



Where:

$\sigma_x, \sigma_y, \sigma_z$; are normal stresses

$\tau_{yx} = \tau_{xy}$
$\tau_{yz} = \tau_{zy}$
$\tau_{zx} = \tau_{xz}$; are shear stresses

Two-dimensional Stress

Consider a section of this element:

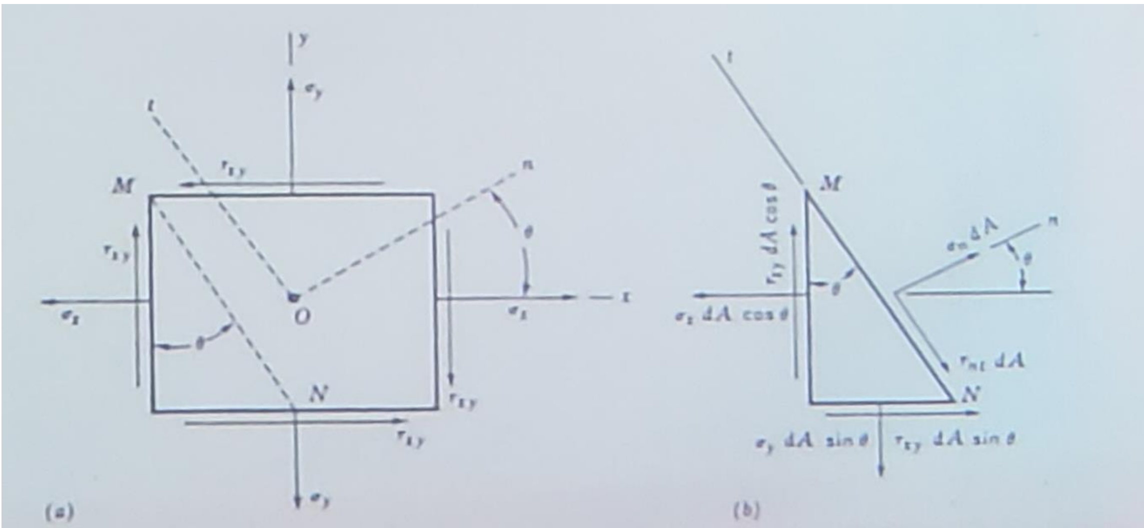


Fig. 1-13 Stress and force components on an element.

$$\sum F_n = 0$$
$$\sigma_n\, dA - \sigma_x\, cos\theta\, dA - \sigma_y\, sin\theta\, dA\, sin\theta + \tau_{xy}\, cos\theta\, dA\, sin\theta + \tau_{xy}\, sin\theta\, dA\, cos\theta = 0$$
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - 2\tau_{xy} sin\theta cos\theta$$
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - \tau_{xy} 2sin\theta$$
$$\{2sin\theta cos\theta = sin2\theta\}$$
$$Replacing \cos^2\theta = \left(\frac{1}{2}\right)(1 + cos2\theta)$$

$\sum F_t = 0$ leads to

$$\tau_{nt} = (\sigma_x - \sigma_y) sin\theta cos\theta + \tau_{xy}(\cos^2\theta + \sin^2\theta)$$
$$\tau_{nt} = \frac{\sigma_x - \sigma_y}{2} sin2\theta + \tau_{xy} cos2\theta$$

The direction of the principal stresses (maximum and minimum values) is found by differentiating σ_n with respect to θ, setting the values to zero and solving for θ. The result is:

$$tan2\theta_{1,2} = -\frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

Substituting in the expression of σ_n to find:

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{1,2} = 0$$

<u>6.3 - Mohr's Circle</u>

The above results can be represented graphically by a diagram known as "Mohr's Circle" as shown: