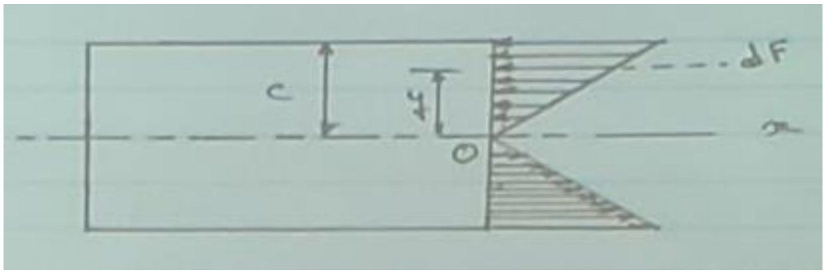
Lecture (Feb. 5th, 2019)

Midterm - ATAC-1003, 1:00pm on Saturday
Will cover material from Weeks 1-4 only

Normal Stress in Bending

It can be shown that the neutral axis and centroidal axis in straight beams are the same.



Moment of dF about O is:

$$dM = dF\, y$$

And:

$$M = \int^{A} y\, \sigma\, dA$$

But:

$$\frac{\sigma}{y} = \frac{\sigma_c}{c} \quad ; \quad \sigma = \frac{\sigma_c}{c}$$

And:

$$M = \int^{A} y^2\, \frac{\sigma_c}{c}\, dA$$

$$= \frac{\sigma_c}{c} \int^{A} y^2\, dA$$

$$= \frac{\sigma}{y} \int^{A} y^2\, dA$$

But:

$$\int^{A} y^2 \cdot dA = I$$

And:

$$M = \frac{\sigma}{y} \cdot I$$

And:

$$\sigma = \frac{My}{I}$$

$\sigma =$ Stress at distance y from the neutral axis
$I =$ Moment of inertia of the cross-section about the neutral axis
$M =$ Applied bending moment

$$\sigma_{t,max} = \frac{Mc_t}{I} \quad ; \quad \sigma_{c,max} = \frac{Mc_c}{I}$$

For symmetric beam sections:

$$c_t = c_c = c$$

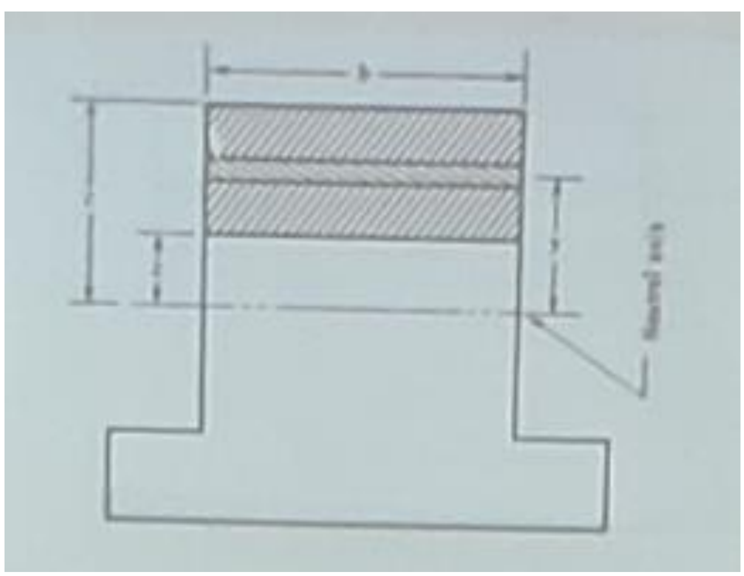
$$|\sigma_t| = |\sigma_c| = \frac{Mc}{I}$$

<u>Shear Stress Due to Bending</u>

In addition to normal stresses induced by bending of a beam, transverse shearing stresses are induced between he elements or fibers, provided the bending moment varies along the length of the beam. According to the strength-of-materials methods.

$$\tau = \frac{V}{Ib}\int_z^c y\, dA = \frac{VQ}{Ib}$$

$\tau =$ Shear Stress
$I =$ Moment of inertia of the cross-section
$b =$ beam width at the section
$Q = \int_z^c y\, dA =$ moment of area of the element about the neutral axis
$V =$ shearing force at the section
$z =$ location where shear stress is of interest



For rectangular cross-section:

$$\tau_{max} = \frac{3V}{A}$$

For solid circular cross section:

$$\tau_{max} = \frac{4V}{3A}$$

For thin-walled circular tube:

$$\tau_{max} = \frac{2V}{A}$$

$A =$ cross sectional area

<u>Two-Plane Bending</u>

When bending occurs in both xy and xz planes of cross sections with one or two planes of symmetry, the bending stresses are given by:

$$\sigma_x = \frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

The maximum bending stress for a solid circular section in this case is:

$$\sigma_{max} = \frac{M\, c}{I} = \frac{\left(M_y^2 + M_z^2\right)^{\frac{1}{2}} \left(\frac{d}{2}\right)}{\pi \frac{d^4}{64}}$$

$$\sigma_{max} = \frac{32}{\pi d^3} \left(M_y^2 + M_z^2\right)^{\frac{1}{2}}$$

<u>Torsion of Circular Shafts</u>

Torsional moments induce shear stresses on cross-sections normal to the axis of bars and shaft.

For circular shafts:

$$\tau = \frac{T\, r}{J}$$

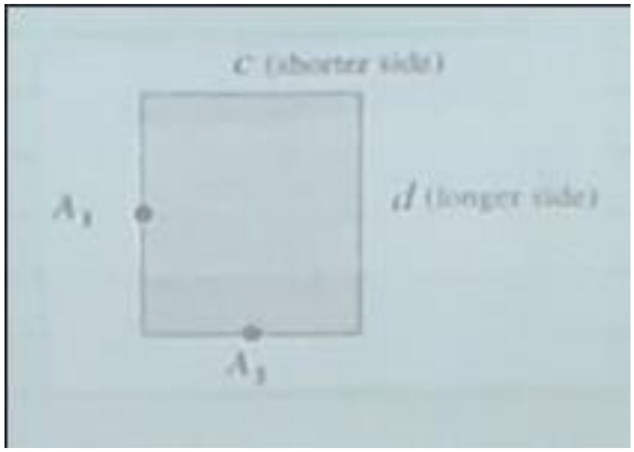
$\tau =$ Induced shear stress
$r =$ distance from the center of the shaft to the point of stress
$J =$ polar moment of inertia

For a solid circular shaft:

$$\tau_{max} = \frac{T\, d}{2\, J} = \frac{T\, d}{2\, \pi \frac{d^4}{32}} = \frac{16\, T}{\pi\, d^3}$$

Torsion of Rectangular Bars

The general equations for stress and deformation in rectangular bars may be written in the following form.



For point A_1:

$$\tau = \frac{T}{\propto_1 \; b\, c^2}$$

For point A_2:

$$\tau = \frac{T}{\propto_2 \; b\, c^2}$$

For angular deformation (radians per inch of length):

$$\theta_1 = \frac{T}{\beta\, G\, b\, c^3}$$

$T =$ Torque
$b = d =$ breadth of section (width)
$t = c =$ thickness of section
$\propto_i =$ coefficient from the table below
$\beta =$ coefficient from the table below
$G =$ shear modulus

The maximum shear stress on the cross-section occurs at the center A_1 of the long side and is found by using $\propto_1$

TABLE 3-3 CONSTANT FOR TORSION OF RECTANGULAR BARS

b/c	1.00	1.20	1.50	1.75	2.00	2.50	3.00	4.00	5.00	6.00	8.00	10.00	∞
α_1	0.208	0.219	0.231	0.239	0.246	0.258	0.267	0.282	0.291	0.299	0.307	0.312	0.333
α_2	0.208	0.235	0.269	0.291	0.309	0.336	0.355	0.378	0.392	0.402	0.414	0.421	...
β	0.1406	0.166	0.196	0.214	0.229	0.249	0.263	0.281	0.291	0.299	0.307	0.312	0.333

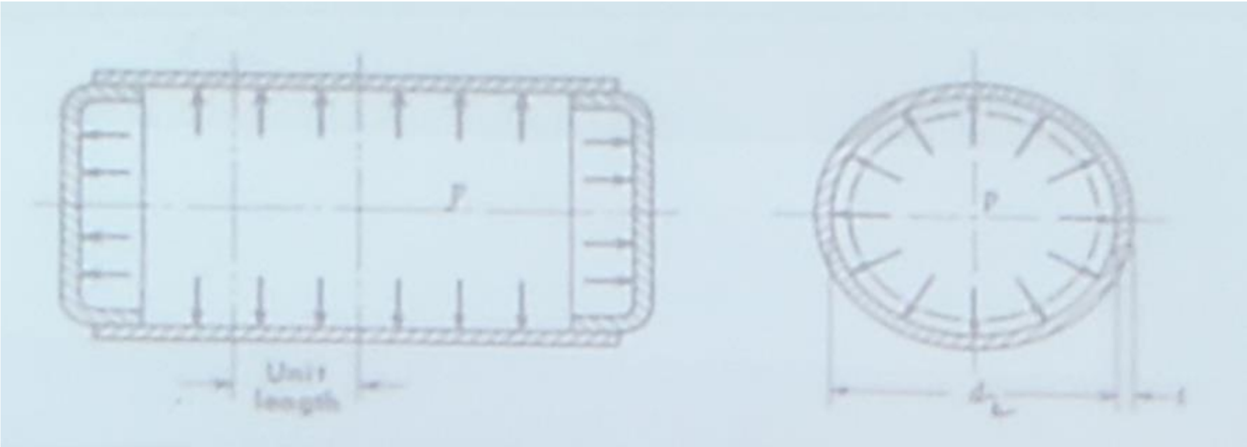
The following approximate formula for the maximum torsional stress in a rectangular section was given by Timoshenko and McCullough

$$\tau_{max} = \frac{T}{b\,t^2}\left(3 + 1.8\frac{t}{b}\right)$$

Pressure Cylinder

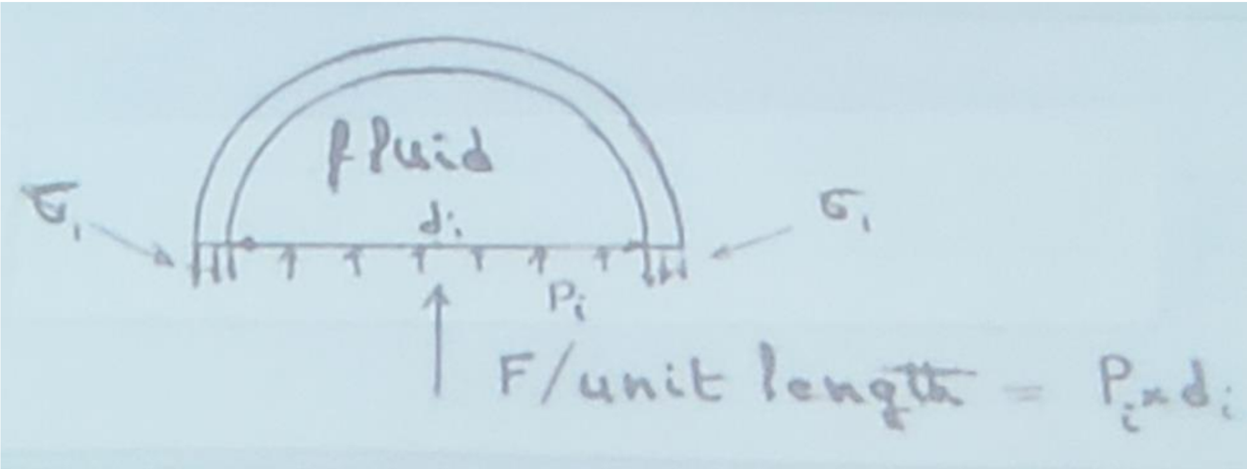
Thin walled cylinders $d/t \gg 10$:

Neglecting the effects of curvature of the cylinder wall, and assuming tensile stresses are uniformly distributed over the section of the wall.



Cylinder with internal pressure.

$2\,\sigma_1\,t = P_i\,d_i$



The average tangential stress is:

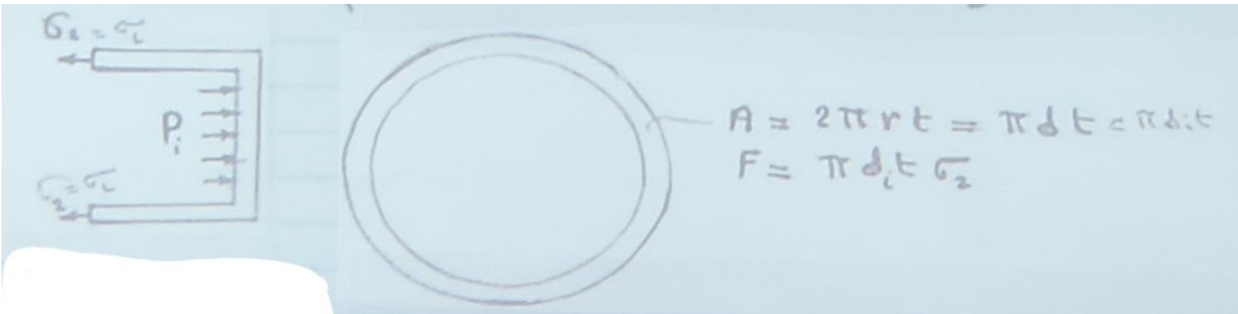
$$\sigma_1 = \sigma_t = \frac{P_i d_i}{2t}$$

$\sigma_1 = \sigma_t =$ tangential stress or hoop stress

And an approximation to the maximum tangential stress is:

$$\sigma_{t,max} = \frac{P_i(d_i + t)}{2t}$$

The longitudinal tensile stress or σ_z is:



$$F = p_i A \quad ; \quad A = \frac{\pi d_i^2}{4}$$

$$F = p_i \frac{\pi d_i^2}{4}$$

$$\therefore\ \pi\, d_i\, t\, \sigma_2 = \frac{\pi\, d_i^2\, p_i}{4}$$

$$\sigma_2 = \sigma_L = \frac{p_i\, d_i}{4\, t}$$

σ_1 = circumferential, or hoop, or tangential stress
σ_2 = longitudinal stress
d_i = internal diameter
t = wall thickness
p = internal pressure