Lecture (Jan. 17th, 2019)

Reliability and Probability of Failure

The statistical measure of the probability that a mechanical element will not fail in use is called the reliability of the element and is related to the probability of failure, P_f

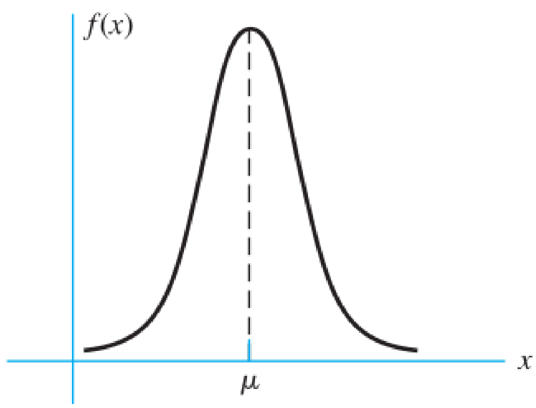
Probability of Failure, P_f

The probability density function (PDF) represents the distribution of events within a given range of values. The Gaussian (normal) distribution and the Weibull distribution are the most important continuous probability distributions for engineering use. The Weibull distribution is used in rolling contact bearing design and will be covered with this topic.

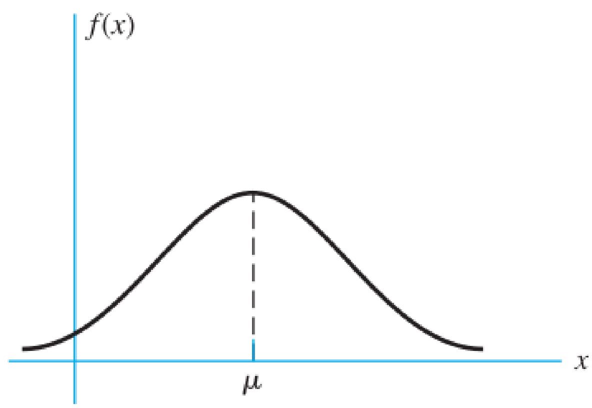
The probability density function (PDF) of the Gaussian (normal) distribution is expressed in terms of its mean, μ_x and its standard deviation $\widehat{\sigma_x}$ as:

$$f(x) = \frac{1}{\widehat{\sigma_x}\ \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{(x-\mu_x)}{\widehat{\sigma_x}}\right)^2\right]$$

And shown graphically below.



a) Small $\widehat{\sigma_x}$

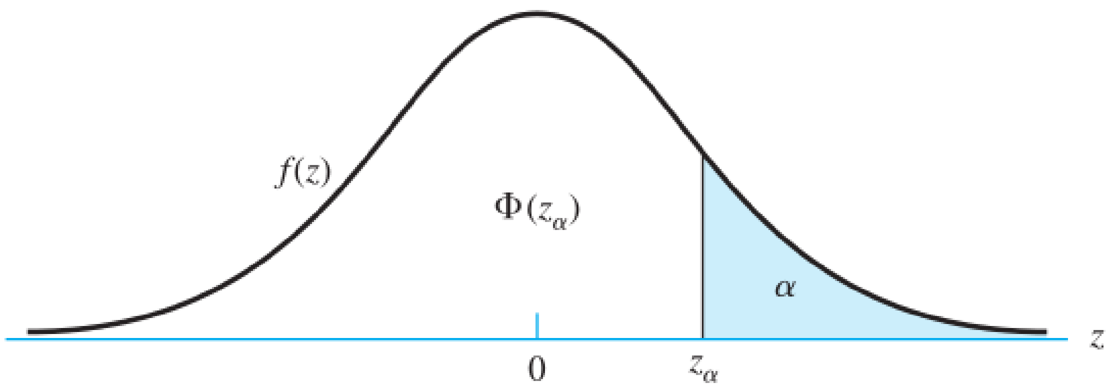


b) Large $\widehat{\sigma_x}$

The obtain the values of p_f the above equation must be integrated. The integrand in tabulated in Table 1-10 where x is placed in dimensionless form using the transform.

$$z = \frac{(x-\mu_x)}{\widehat{\sigma_x}}$$

And ∝ is defined as shown in the figure below.



The variant z has a mean value of zero and a standard deviation of one. In Table A-10, the probability of an observation less than z is $\Phi_{(z)}$ for negative values of z and $1 - \Phi_{(z)}$ for positive values of z.

Example:
The lives of parts are often expressed as the number of cycles of operation that a specified percentage of a population will exceed before experiencing failure. The symbol L is used to designate this definition of life. Thus, we can speak of L_{10} life as the number of cycles to failure exceeded by 90 percent of a population of parts. Given a normal distribution model, with a mean of $\bar{L} = 122.9$ kilocycles and standard deviation of $S_L = 30.3$ kilocycles, estimate the corresponding L_{10} life.

Solution:

$\bar{L} = 122.9\ Kcycles\ ;\ S_L = 30.3\ Kcycles$

$$z_{10} = \frac{(x - \mu_x)}{\widehat{\sigma_x}} = \frac{x_{10} - 122.9}{30.3}$$

$or, \quad x_{10} = 122.9 + 30.3\, z_{10} = L_{10}$

$From\ Table\ A - 10,\ \ for\ 10\%\ of\ failure,\ \ z_{10} = -1.282$

$\therefore L_{10} = 122.9 + 30.3(-1.282) = 84.1\ Kcycles$

Discrete distributions may be approximated by continuous distributions. In an N samples of events, let x_i be the value of an event $(1, 2, \ldots, k)$ and f_i the number of times the event x_i occurs within the frequency range. The discrete mean $\bar{x}$ and the standard deviation S_x are thus defined as:

$$\bar{x} = \frac{1}{\mathrm{N}} \sum_{i=1}^{k} f_i x_i$$

$$S_x = \sqrt{\frac{\left(\sum_{i=1}^{k} f_i x_i^2 - N\bar{x}^2\right)}{N - 1}}$$

Example – Problem 1-14

Determination of the ultimate tensile strength of stainless-steel sheet (17-7 PH, condition TH1050), in sizes from 0.016 to 0.062 in, in 197 tests combined into seven classes were

$S_{ut}\ (kpsi)$	174	182	190	198	206	214	222
f	6	9	44	67	53	12	6

Where S_{ut} is the class mid point and f is the class frequency estimate the mean and the standard deviation.

x	f	fx	fx^2
174	6	1044	181656
182	9	1638	298116
190	44	8360	1588400
198	67	13266	2626668
206	53	10918	2249108
214	12	2568	549552
222	6	1332	295704
Σ	197	39126	7789204

Reliability

$$\bar{x} = \frac{1}{\mathrm{N}}\sum_{i=1}^{k} f_i x_i = \frac{39126}{197} = 198.61\ Kpsi$$

$$S_x = \sqrt{\frac{\left(\sum_{i=1}^{k} f_i x_i^2 - N\bar{x}^2\right)}{N-1}}$$

$$S_x = \left[\frac{7789204 - 197(198.61)^2}{197-1}\right]^{\frac{1}{2}} = 9.68\ Kpsi$$

Reliability Mathematics

Definition: Reliability is defined as the probability that equipment will perform its intended function satisfactorily for the intended time in the intended environment.

If no units are started with, and N_f failure are experienced in a given time t, the reliability R for time t is defined mathematically as (1):

$$R = \frac{N_o - N_f}{N_o} = \frac{N_s}{N_o} = 1 - \frac{N_f}{N_o}$$
$$= 1 - p_f \ \ with \ \ 0 \le R \ \le 1$$

The rate of failure is the time rate of change of reliability or (2):

$$r = \frac{dR}{dt}$$

Substitution (1) in (2) and differentiating:

$$\frac{dR}{dt} = \frac{d}{dt}\left(1 - \frac{N_f}{N_o}\right) = -\frac{1}{N_o}\frac{dN_f}{dt}$$

And the rate at which units fail and/or survive is:

$$\frac{dN_f}{dt} = -N_o\frac{dR}{dt} = \frac{d}{dt}(N_o - N_s) = -\frac{dN_s}{dt}$$

The instantaneous probability of failure per hour, p, can be found by dividing the above rate by the number of units surviving at that instant (3):

$$\rho = \frac{1}{N_s}\frac{dN_f}{dt} = -\frac{N_o}{N_s}\frac{dR}{dt} = -\frac{1}{R}\frac{dR}{dt}$$

$$\therefore \rho\, dt = -\frac{dR}{R}$$

$$and\ \ln R = -\int_0^t \rho\, dt$$

$$But\ at\ t = 0, R = 1\ ;\ and\ R(t) = e^{-\int_0^t \rho\, dt}$$

Combining (2) and (3) to get:

$$r = \frac{dR}{dt} = -\rho R$$