Lecture (Mar. 19th, 2019)

We have seen how to relate S_e to S_{ut}. The endurance limit of a general mechanical element is obtained from S_e through the use of a variety of modifying factors.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

Where:
S_e = endurance limit
S'_e = endurance limit of test specimen
k_a = surface factor
k_b = size factor
k_c = load factor
k_d = temperature factor
k_e = reliability factor
k_f = miscellaneous effects factor

Surface Factor, k_a

$$k_a = aS_{ut}^b$$

Where:
k_a = modifying surface factor
S_{ut} = minimum tensile strength
a = a factor from Table 6-2
b = exponent from Table 6-2

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

	Factor a		Exponent
Surface Finish	S_{ut}, kpsi	S_{ut}, MPa	b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Size Factor, k_b

The results from 133 rotating circular beam tested in bending and torsion may be written as:

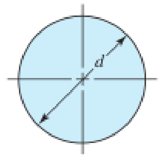
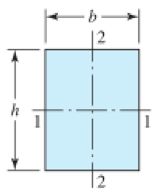
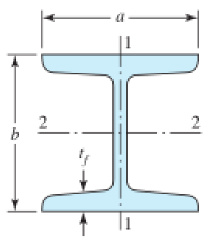
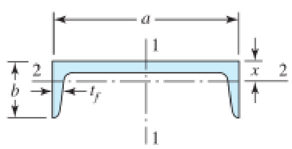
$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

For axial loading, $k_b = 1$

For nonrotating and noncircular cross section an equivalent diameter d_e is used from Table 6-3

Table 6–3

$A_{0.95\sigma}$ Areas of Common Nonrotating Structural Shapes

	$A_{0.95\sigma} = 0.01046d^2$ $d_e = 0.370d$		
	$A_{0.95\sigma} = 0.05hb$ $d_e = 0.808\sqrt{hb}$		
	$A_{0.95\sigma} = \begin{cases} 0.10at_f \\ 0.05ba \end{cases}$	$t_f > 0.025a$	axis 1-1 axis 2-2
	$A_{0.95\sigma} = \begin{cases} 0.05ab \\ 0.052xa + 0.1t_f(b - x) \end{cases}$		axis 1-1 axis 2-2

Loading Factor, k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

Temperature Factor, k_d

Table 6-4 shows the effect of operating temperature on the tensile strength of steel, where S_{RT} is the strength at room temperature.

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

A fourth order polynomial curve fit gives:

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

Where:

$$70 \le T_F \le 1000\ °F$$

Or:

$$k_d = \frac{S_T}{S_{RT}} \quad ; \quad \text{From Table } 6-4$$

Reliability Factor, k_e

$$k_e = 1 - 0.08Z_a$$

Where:

$$Z_a = \frac{x - \mu_x}{\widehat{\sigma_x}} \quad ; \quad And\ can\ be\ found\ in\ Table\ A-10$$

Table 6-5 list values for k_e for some standard reliabilities.

Table 6–5

Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Miscellaneous-Effects Factor, k_f

Actual values of k_f are not always available. However, its presence is a reminder that other effects such as corrosion and others must be considered.

Stress Concentration and Notch Sensitivity

Some materials are not fully sensitive to the presence of discontinuities, such as holes, grooves, or notches. For these, a reduced value of the stress concentration factor k_t or k_{ts} can be used.

The reduced value k_f or k_{fs}, which is called the fatigue stress-concentration factor, is given as:

$$k_f = 1 + q(k_t - 1)$$

Or:

$$k_{fs} = 1 + q_{shear}(k_{ts} - 1)$$

Where q is known as the notch sensitivity. Some known values of q and q_{shear} are shown in Figures 6.20 and 6.21. The functions in these figures are:

$$k_f = 1 + \frac{k_t - 1}{1 + \sqrt{a/r}} \quad ; \quad q = \frac{1}{1 + \sqrt{a/r}}$$

Where:
$r =$ notch radius
$\sqrt{a} =$ Neuber constant given as

For bending and axial:
$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$

For torsion:
$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

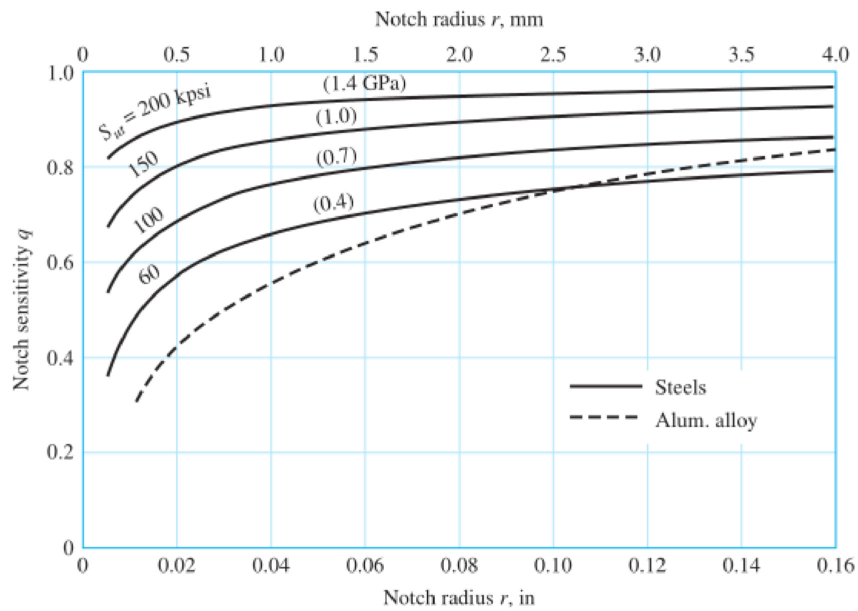
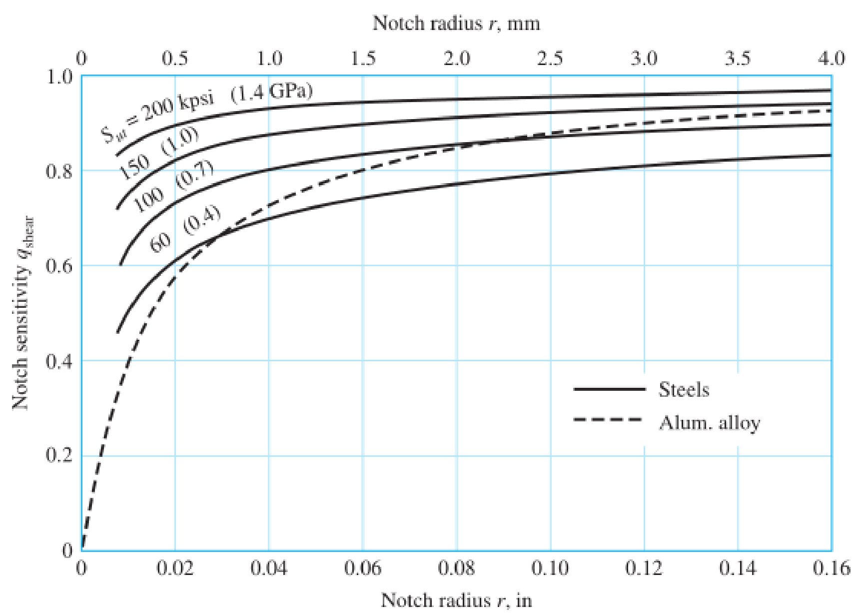


Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to $r = 0.16$ in (4 mm).



The notch sensitivity of cast iron is very low. However, to be on the conservative side, it is recommended that $q = 0.20$ be used for all grades of cast iron.

Reading Assignment:
Exampes 6.6 to 6.9
Section 6.17

Load Line Concept

When Goodman relation is used, the safe-stress line through the working stress point A, as shown in the figure, is constructed parallel to the Goodman line. Note that the sage-stress-line is the locus of all sets of $\sigma_a,\ \sigma_m$ stresses having a factor of safety n and that $S_m = n\sigma_m$ and $S_a = n\sigma_a$.

(TODO – Picture)

Let:
σ_p = the static component of a working stress
σ_m = the mean stress corresponding to $k_f \sigma_a$
S_m = the critical mean stress corresponding to S_a

The line going through the point $(0, \sigma_p)$, $(k_f \sigma_a, \sigma_m)$ and (S_a, S_m) is the load line L of slope m.

Have:

$$S_o = n\sigma_a k_f \quad ; \quad n = \frac{S_o}{\sigma_a k_f} = \frac{S_m - \sigma_p}{\sigma_m - \sigma_p} \quad ; \quad (By\ similar\ triangles)$$

From Goodman relation:

$$S_m = S_{ut}\left(1 - \frac{S_a}{S_e}\right)$$

The load line slope is:

$$m = \frac{\sigma_a k_f}{\sigma_m - \sigma_p} = \frac{S_a}{S_m - \sigma_p}$$

$$\therefore S_a = m(S_m - \sigma_p) = mS_{ut}\left(1 - \frac{S_a}{S_e}\right) - m\sigma_p$$

$$= m(S_{ut} - \sigma_p) - \frac{mS_a S_{ut}}{S_e}$$

Or:

$$S_a + \frac{mS_a S_{ut}}{S_e} = m(S_{ut} - \sigma_p)$$

$$S_a\left(1 + \frac{mS_{ut}}{S_e}\right) = m(S_{ut} - \sigma_p)$$

And:

$$S_a = \frac{m(S_{ut} - \sigma_p)}{1 + \frac{mS_{ut}}{S_e}}$$

Where:

$$m = \frac{k_f \sigma_a}{\sigma_m - \sigma_p}$$

But:

$$(\sigma_m - \sigma_p) = \sigma_a$$

$$\therefore m = k_f$$

(TODO – Picture)

The concept of the load line in conjunction with the failure criteria is used to tabulate the principal intersection in Table 6-6 to 6-8, where $r = S_a/S_m = \sigma_a/\sigma_m$. The first column in each table gives the intersection equations and the second column gives the intersection coordinates.

Reading Assignment:
Examples 6-10 to 6-12

Fatigue Failure of Brittle Materials

(TODO – Picture)

Not enough work has been done on brittle fatigue. Consequently, designed stay in the first and a bit in the second quadrant in the range from:

$$-S_{ut} \leq \sigma_m \leq S_{ut}$$

Lecture (Mar. 21st, 2019)

The first quadrant failure fatigue criteria for many brittle materials follows the Smith-Dolan locus given by:

$$\frac{S_a}{S_e} = \frac{1 - \frac{S_m}{S_{ut}}}{1 + \frac{S_m}{S_{ut}}}$$

Or:

$$\frac{n\sigma_a}{S_e} = \frac{1 - \frac{n\sigma_m}{S_{ut}}}{1 + \frac{n\sigma_m}{S_{ut}}}$$

If r is the load line slope $r = S_a/S_m$, substituting S_a/r for S_m and solving for S_a we get,

$$S_a = \frac{rS_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4rS_{ut}S_e}{(rS_{ut} + S_e)^2}}\right]$$

The portion of the second quadrant is represented by a straight line between the two points $(-S_{ut}, S_{ut})$ and $(0, S_e)$, which is represented by,

$$S_a = S_e + \left(\frac{S_e}{S_{ut}} - 1\right)S_m \quad ; \quad -S_{ut} \le S_m \le 0$$

Properties of gray cast iron are found in Table A-24, where the endurance limit stated include k_a and k_b

Table A-24

Mechanical Properties of Three Non-Steel Metals
(a) Typical Properties of Gray Cast Iron
[The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 20 kpsi. Note particularly that the tabulations are *typical* of several heats.]

ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi Tension†	Modulus of Elasticity, Mpsi Torsion	Endurance Limit* S_e, kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

*Polished or machined specimens.

†The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

Reading Assignment:
Example 6.13
Section 6-13

Combination of Loading Modes

As we have seen, the load factor k_c depends on the type of loading. There may also be stress-concentration factors, which may depend on the type of loading. The question is therefore, 'How do we proceed when the loading is a mixture of axial, bending, and torsional loads?"

To answer this question, we first generate the two stress elements σ_a and σ_m and apply the appropriate fatigue stress-concentration factors to them.

Second, we calculate the equivalent von Mises stress, for each of these two stress elements, σ_a' and σ_m'

Finally, select a fatigue failure criterion to complete the fatigue analysis.

For the endurance limit, S_e, we only use k_a, k_b, and k_c for bending and account for the axial load factor by dividing the alternating axial stress by 0.85.

In the common case of a shaft with bending stresses, torsional shear stresses and axial stresses, the von Mises stress is:

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{\frac{1}{2}}$$

And therefore:

$$\sigma_a' = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2\right\}^{1/2} \quad (6\text{–}55)$$

$$\sigma_m' = \left\{\left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2\right\}^{1/2} \quad (6\text{–}56)$$

For first-cycle localized yielding,

First add the axial and bending alternating and midrange stresses to obtain σ_{max}

Second add the alternating stress to the midrange shear stresses to obtain τ_{max}

Then substitute σ_{max} and τ_{max} into the von-mises stress equation

A simpler and more conservative method is to add σ_a' and σ_m' to find σ_{max}'

$$\sigma_{max}' = \sigma_a' + \sigma_m' = \frac{S_y}{n}$$

Reading Assignment:
Example 6.14

The fatigue factor of safety is then found using one of the following:

Modified Goodman and Langer Failure Criteria

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

For Gerber and Langer Failure Criteria

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \left(\frac{\sigma_e}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \quad ; \quad \sigma_m > 0$$

For ASME Elliptic and Langer Failure Criteria

$$n_f = \sqrt{\frac{1}{(\sigma_m/S_e)^2 + \left(\sigma_m/S_y\right)^2}}$$

For First-cycle failure

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{S_y}{\sigma_{max}}$$

In the case of pure shear, it is convenient to use the maximum shear-stress theory and replacing σ by τ in the above equations and S_{ut} by S_{su} from Equation 6.54:

$$S_{su} = 0.67 S_u$$

And:

$$n_f = \frac{\frac{S_y}{2}}{\tau_{max}}$$

Section 6.17 pp 338-341 provides a good summary for fatigue failure.

Surface Endurance Shear (Buckingham Wear Factor)

When two surfaces roll, slide, or roll and slide against each other with sufficient force, a pitting failure will occur after a certain number of cycles of operation. To determine the surface strength of mating materials, Buckingham conducted many tests which were later extended by Talbourdet. Based on the data obtained, and using Heart constant stresses equations, Buckingham defined a wear factor, which is also known as load-stress factor, as follow:

Hearty equations for contacting cylinders are:

$$b = \left[\frac{2F}{\pi l}\frac{\frac{(1-v_1^2)}{E_1} + \frac{(1-v_2^2)}{E_2}}{\left(\frac{1}{d_1}\right) + \left(\frac{1}{d_2}\right)}\right]^{\frac{1}{2}}$$

$$P_{max} = 2F/\pi bl$$

Where:
$b =$ half width of rectangular contact area
$F =$ contact force
$w\ or\ l =$ width of cylinders (length of contact)
$v =$ poisson's ratio
$E =$ modulus of elasticity
$d =$ cylinder diameter

Replacing d by $2r$, l by w and using an average value of 0.3 for v_1 and v_2 to get:

$$b^2 = 1.16\left(\frac{F}{w}\right)\frac{\left(\frac{1}{E_1}\right)+\left(\frac{1}{E_2}\right)}{\left(\frac{1}{r_1}\right)+\left(\frac{1}{r_2}\right)}$$

Defining surface strength as the maximum pressure at which surface fatigue failure starts to get:

$$S_e = \frac{2F}{\pi bw}$$

This is also known as the contact strength, the contact fatigue strength, or the Hertigan endurance strength.

Substituting b and rearranging to get:

$$2.857S_e^2\left(\frac{1}{E_1}+\frac{1}{E_2}\right) = \frac{F}{w}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

Buckingham's load-stress factor k_1 is defined as:

$$k_1 = 2.857S_c^2\left(\frac{1}{E_1}+\frac{1}{E_2}\right)$$

The design equation for surface fatigue strength is then:

$$k_1 = \frac{F}{w}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

In the presence of a factor of safety n this equation is written as:

$$\frac{k_1}{n} = \frac{F}{w}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

The Hertigian endurance strength of steels for 10^8 cycles of repeated contact stress is obtained from the following equation:

$$S_e = \begin{cases} 0.4H_B - 10kpsi \\ 2.76H_B - 70\ MPa \end{cases}$$

Where H_B is the Brinnel hardness number.