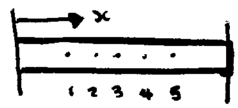
METH

1

MAR. 5/19

Example



$i = 1$ $$\frac{T_2 - 2T_1 + T_0}{\Delta x^2} + h'(T_a - T_1) = 0$$

$i = 2$ $$\frac{T_3 - 2T_2 - T_1}{\Delta x^2} + h'(T_a - T_2) = 0$$

$i = 3$ $$\frac{T_4 - 2T_3 - T_2}{\Delta x^2} + h'(T_a - T_3) = 0$$

$i = 4$ $$\frac{T_5 - 2T_4 - T_3}{\Delta x^2} + h'(T_a - T_4) = 0$$

$$-T_0 + 2.04\,T_1 - T_2 = 0.8 \longrightarrow 2.04T_1 - T_2 = 0.8 + T_0$$
$$-T_1 + 2.04\,T_2 - T_3 = 0.8$$
$$-T_2 + 2.04\,T_3 - T_4 = 0.8$$
$$-T_3 + 2.04\,T_4 - T_5 = 0.8 \longrightarrow -T_3 + 2.04\,T_4 = 0.8 + T_5$$

$$\underbrace{\begin{bmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}}_{T} = \underbrace{\begin{bmatrix} 0.8 + T_0 \\ 0.8 \\ 0.8 \\ 0.8 + T_5 \end{bmatrix}}_{B} \qquad T = A \backslash B$$

↳ tridiagonal matrix

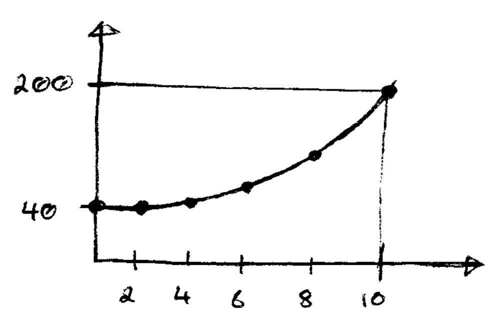
→ using MATLAB

$T_1 = 65.9698$

$T_2 = 93.7785$

$T_3 = 124.5382$

$T_4 = 159.4792$



LU Factorization

$[A]\{x\} = \{B\}$

$[L][U]\{x\} = \{B\}$

$$[A] = LU = \overset{(L)}{\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}} \overset{(U)}{\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} U_{11}^{=a_{11}} & U_{12}^{=a_{12}} & U_{13}^{=a_{13}} \\ L_{21}U_{11}^{=a_{21}} & L_{21}U_{12} + U_{22}^{=a_{22}} & L_{21}U_{13} + U_{23}^{=a_{23}} \\ L_{31}U_{11}^{=a_{31}} & L_{31}U_{12} + L_{32}U_{22}^{=a_{32}} & L_{31}U_{13} + L_{32}U_{23} + U_{33}^{=a_{33}} \end{bmatrix}$$

$\boxed{\text{Example}}$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.003 & -0.2933 \\ 0 & 0 & 10.012 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{0.1}{3} & 1 & 0 \\ \frac{0.3}{3} & -0.02713 & 1 \end{bmatrix}$$

$$[L]\underbrace{[U]\{x\}}_{\{d\}} = \{B\}$$

$[L]\{d\} = \{B\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.033 & 1 & 0 \\ 0.1 & -0.02713 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix} \longrightarrow \begin{aligned} d_1 &= 7.85 \\ d_2 &= -19.5617 \\ d_3 &= 70.0843 \end{aligned}$$

$[U]\{x\} = \{d\}$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.003 & -0.2933 \\ 0 & 0 & 10.012 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.5617 \\ 70.0843 \end{Bmatrix} \longrightarrow \begin{aligned} x_1 &= 3 \\ x_2 &= -2.5 \\ x_3 &= 7.0003 \end{aligned}$$

* KNOW HOW TO PERFORM FOR 3x3 FOR EXAM

Cholesky Factorization

- on symmetric matrices only.

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \longrightarrow \begin{array}{lcl} ax_1 & = & b_1 \\ bx_2 & = & b_2 \\ cx_3 & = & b_3 \end{array}$$

$\boxed{\text{Example}}$ (Gauss Elimination)

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ -1 & 0 & -2 & -5 \\ -3 & -5 & 1 & -4 \end{array}\right] \xrightarrow[R_3+3R_1]{R_2+R_1} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 2 & -2 & -2 \\ 0 & 1 & 1 & 5 \end{array}\right]$$

$$\xrightarrow[\text{eventually:}]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \longrightarrow [I]\{x\} = \begin{Bmatrix} -1 \\ 2 \\ 3 \end{Bmatrix}$$

$\boxed{\text{Example}}$ (inverse of matrix technique)

$$2x + 3y = 1$$
$$2x + 2y = 7$$

$$\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{Bmatrix} 1 \\ 7 \end{Bmatrix}$$

$$|A| = 4-6 = 2$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 2 & -2 \\ -3 & 2 \end{bmatrix}^T$$

$$= \frac{1}{-2}\begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3/2 \\ 1 & -1 \end{bmatrix}$$