METH

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Feb. 28/19

Using Cramer's Rule:

$$a_1 x_1 + b_1 x_2 = c_1$$
$$a_2 x_1 + b_2 x_2 = c_2$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad ; \quad x_2 = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$$
$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = b_2$$
$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \quad ; \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

determinant of 3×3

$$x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$\boxed{\text{Example:}}$ Find $x_2$

$$0.3 x_1 + 0.52 x_2 + 1.00 x_3 = -0.01$$
$$0.5 x_1 + 1.00 x_2 + 1.90 x_3 = 0.67$$
$$0.1 x_1 + 0.30 x_2 + 0.50 x_3 = -0.44$$

$$D = \begin{vmatrix} 0.3 & 0.52 & 1 \\ 0.5 & 1 & 1.9 \\ 0.1 & 0.3 & 0.5 \end{vmatrix} = -0.0022$$

$$D_2 = \begin{vmatrix} 0.3 & -0.01 & 1 \\ 0.5 & 0.67 & 1.9 \\ 0.1 & -0.44 & 0.5 \end{vmatrix} = 0.0649$$

$$x = \frac{D_2}{D} = -29.5$$

**Example** (Gaussian elimination)

$$\begin{cases} 2x_1 + 2x_2 - 2x_3 = 8 \\ -4x_1 - 2x_2 + 2x_3 = -14 \\ -2x_1 + 3x_2 + 9x_3 = 9 \end{cases}$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 2 & -2 & 8 \\ -4 & -2 & 2 & -14 \\ -2 & 3 & 9 & 9 \end{array}\right] \xrightarrow[R_3 \times -1]{R_1/2,\ R_2 \times -1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 4 & 2 & -2 & 14 \\ 2 & -3 & -9 & -9 \end{array}\right]$$

$$\xrightarrow[R_3 - 2R_1]{R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -2 & 2 & -2 \\ 0 & -5 & -7 & -17 \end{array}\right] \xrightarrow[R_3 \times -1]{(R_2/2) \times -1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & 7 & 17 \end{array}\right]$$

$$\xrightarrow[R_3 - 5R_1]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 12 & 12 \end{array}\right]$$

thus, $12x_3 = 12$

$\hookrightarrow x_3 = 1$

$\rightarrow x_2 - x_3 = 1$

$\hookrightarrow x_2 = 2$

$\rightarrow x_1 + x_2 - x_3 = 4$

$\hookrightarrow x_1 = 3$

Total pivoting :

$$\left(\begin{array}{ccc|c} 2 & 2 & -2 & 8 \\ -4 & -2 & 2 & -14 \\ -2 & 3 & 9 & 9 \end{array}\right)$$

swap column 1 + 3

$$\rightarrow \left(\begin{array}{ccc|c} -2 & 3 & 9 & x_1 \\ -4 & -2 & 2 & x_2 \\ 2 & 2 & -2 & x_3 \end{array}\right) \xrightarrow{x_1 \leftrightarrow x_3} \left(\begin{array}{ccc|c} 9 & 3 & -2 & x_3 \\ 2 & -2 & -4 & x_2 \\ -2 & 2 & 2 & x_1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 9 & -2 & 3 & x_3 \\ 0 & -\frac{32}{9} & -\frac{8}{3} & x_1 \\ 0 & 0 & \frac{3}{2} & x_2 \end{array}\right) = \begin{pmatrix} 9 \\ -16 \\ 3 \end{pmatrix}$$