Example

Use the bisection method...

$$f(x) = x^3 - 2x - 1 = 0$$

$$\left.\begin{aligned} x_1 &= 1.5 \longrightarrow f(1.5) = -0.625 \\ x_2 &= 2 \longrightarrow f(2) = 3 \end{aligned}\right\} f(x)\,f(x+1) < 0$$

$$x_r = \frac{1.5 + 2}{2} = 1.75$$

$$F(x_r) = f(1.75) = 0.859375$$

$$f(x_L)\,f(x_r) < 0 \Rightarrow x_u = x_r = 1.75$$

$$x_r = \frac{1.5 + 1.75}{2} = 1.625$$

$$f(1.625) = 0.041056$$

$$f(x_L)\,f(x_r) < 0$$

$$x_u = 1.625$$

n	lower x_L	upper x_u	x_r	$f(x_r)$	$\lvert \varepsilon_a (\%) \rvert$
1	1.5	2	1.75	0.859375	—
2	1.5	1.75	1.625	0.041056	7.69 %
3	1.5	1.625	1.5625	-0.3103027	
4	1.5625	1.625	1.5937	-0.1393127	
5	1.5937	1.625	1.609375	-0.0503273	
6					

Example: use the bisection method:

$x = \sqrt[4]{18}$

$f(x) = x - \sqrt[4]{18} = 0$ (not the most basic form)

$[2, 2.5]$ = Domain (picked because one side is -ve, one is positive)

→ $f(2) = -2$
$f(2.5) = 21.0621$ → $f(x_l)f(x_u) < 0$

where $f(2) = (2) - \sqrt[4]{18} = -2$

but $f(2.5) = (2.5) - \sqrt[4]{18} \neq 21.0621$

→ $x^4 = 18$ (most basic form) *

$x^4 - 18 = 0$

where $f(2) = (2)^4 - 18 = -2$
$f(2.5) = (2.5)^4 - 18 = 21.0621$ → $f(x_l)f(x_u) < 0$ ∴ there's a root

$$x_r = \frac{2 + 2.5}{2} = 2.25$$

I $f(x_r) = f(2.25) = (2.25)^4 - 18 = 7.6289$

$x_u = x_r = 2.25$

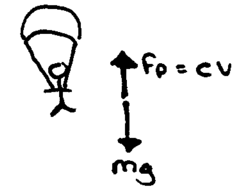
II $x_r = \frac{2 + 2.25}{2} = 2.125$ → $f(2.125) = 2.3909$

$x_u = x_r = 2.125$

III $x_r = \frac{2 + 2.125}{2} = 2.0625$ → $f(2.0625) = 0.0957$

$x_u = x_r = 2.0625$

$$\boxed{2 < \text{root} < 2.0625}$$



$\boxed{\text{Example}}$ Use bisection method:

$$V = \frac{mg}{c}\left(1 - e^{-\left(\frac{c}{m}\right)t}\right) \quad \text{(from earlier notes)}$$

$$\left\{ \begin{array}{l} m = 68.1 \text{ kg} \\ V = 40 \text{ m/s} \\ t = 10 \text{ sec} \\ g = 9.81 \text{ m/s}^2 \end{array} \right. \quad \Rightarrow \quad (40) = \frac{(68.1)(9.81)}{c}\left(1 - e^{-\left(\frac{c}{68.1}\right)10}\right)$$

$$f(c) = \frac{(68.1)(9.81)}{c}\left(1 - e^{-\left(\frac{c}{68.1}\right)(10)}\right)$$

For guesses:

$$\left.\begin{array}{l} f(12) = 6.11 \\ f(16) = -12.2 \end{array}\right\} \text{root exists}$$

I $\quad x_r = \dfrac{12+16}{2} = 14$

$f(14) = 1.611$ (positive, use x_L)

$f(12)\,f(14) = 2.86 > 0 \longrightarrow x_L = x_r = 14$

II $\quad x_r = \dfrac{14+16}{2} = 15$

$f(x_r) = f(15) = -0.384$

$f(14)\,f(15) < 0 \longrightarrow x_u = x_r = 15$

III $\quad x_r = \dfrac{14+15}{2} = 14.5$

$\circ \quad \circ \quad \circ$

Example Use bisection and false position to locate the root of:

$f(x) = x^{10} - 1$

between $x = 0$ / $x = 1.3$

$$\varepsilon_t = \left| \frac{0.65 - 1}{0.65} \right| \times 100\%$$

Iteration	x_L	x_u	x_r	ε_a (%)	ε_t (%)
1	0	1.3	0.65	~~100~~	35
2	0.65	1.3	0.975	33.3	25
3	0.975	1.3	1.1375	14.3	13.8
4	0.975	1.375	1.0565	7.7	5.6
5	0.975	1.0565	1.01525	4	1.6

$$\varepsilon_a = \left| \frac{0.975 - 0.65}{0.975} \right| \times 100\%$$

< 2% good enough