$\boxed{\text{Example}}$ → linear / Quadratic interpolation to eval $\ln(2)$

$\ln(2) = 0.693147$

x_i	$\ln(x_i)$
1	0
4	1.386294
6	1.791759

linear: $x = 1, 4$; quadratic: $x = 1, 4, 6$

$$f(x) = b_1 + b_2(x - x_1)$$

$$\begin{cases} x_1 = 1 \\ x_2 = 4 \end{cases}$$

$$b_1 = f(x_1) = 0$$

$$b_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{1.386294 - 0}{4 - 1} = 0.462098$$

$$f_1(x) = 0 + 0.462098(x-1)$$

if $x = 2 \rightarrow f_1(2) = 0.462098(2-1) = 0.462098$

$$\varepsilon_t = \left| \frac{0.693147 - 0.462098}{0.693147} \right| \times 100\% = 33.3\%$$

$$f_2(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2)$$

$$b_3 = \left[\frac{\dfrac{f(x_3) - f(x_2)}{x_3 - x_2} - \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1} \right]$$

$$= \left[\frac{\dfrac{1.791759 - 1.386294}{6 - 4} - \dfrac{1.386294 - 0}{4 - 1}}{6 - 1} \right] = -0.0618731$$

$$f_2(x) = (0.462098)(x-1) - (0.0618731)(x-1)(x-4)$$

$$f_2(2) = 0.5658444$$

$$\varepsilon_t = \left| \frac{0.693147 - 0.5658444}{0.693147} \right| \times 100\% \rightarrow \varepsilon_t = 18.4\%$$

Example - Construct divided difference table

$$f(x) = x^3 + 7x^2 + 1$$
$$x = 1,\ 2,\ 3,\ 4,\ 5$$

k	x_k	$f(x_i)$	First-order $(n/1)$	Second order $(n/2)$	Third order $(n/3)$
0	1	9			
			28		
1	2	37		$\frac{54-28}{3-1} = 13$	
			54		$\frac{16-13}{4-1} = 1$
2	3	91		$\frac{86-54}{4-2} = 16$	
			86		$\frac{19-16}{4-1} = 1$
3	4	177		$\frac{124-86}{5-3} = 19$	
			124		
4	5	301			

$(n/4)$

Fourth order

$= \left(\frac{1-1}{4}\right) = 0$

Example - Find Newton Form for data

x	0	1	2
$f(x)$	1	2	3

order not given, use 2nd order (because 3 data points)

k	x_k	$f(x_k)$	1st	2nd
0	0	1		
			$\frac{2-1}{1-0} = 1$	
1	1	2		$\frac{1/2 - 1}{3-0} = -1/6$
			$\frac{3-2}{3-1} = \frac{1}{2}$	
2	3	3		

$$f_2(x) = 1 + 1(x-0) - (1/6)(x-0)(x-1)$$
$$f_2(x) = 1 + x - (1/6)(x^2 - x)$$

$$f_2(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1)$$

$\boxed{\text{EXAMPLE}}$ - $f(x) = \ln(x+2)$ interval $0 \le x \le 3$

k	x_k	$f(x_k)$	1st order	2nd order
0	0	0.6932	0.4055	-0.0589
1	1	1.0986	0.2877	-0.0323
2	2	1.3863	0.2232	
3	3	1.6094		

3rd order
0.0089

$$f_3(x)\ f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) \cdots$$
$$+ f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2)$$

$$f_3(x) = (0.6932) + (0.4055)(x-x_0) + (-0.0589)(x-x_0)(x-x_1) \cdots$$
$$+ (0.0089)(x-x_0)(x-x_1)(x-x_2)$$

$$\ln(3.5) = \ln(\underbrace{1.5}_{x}+2) = f_3(1.5) = 1.2587$$

$f_3(x)$ (Lagrangian order)

x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_2)$
x_4	$f(x_3)$

third order... $(n-1)$

$$L_1 = \left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right)\left(\frac{x-x_4}{x_1-x_4}\right)$$

$$L_2 = \left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right)\left(\frac{x-x_4}{x_2-x_4}\right)$$

$$L_3 = \left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right)\left(\frac{x-x_4}{x-x_4}\right)$$

$$L_4 = \left(\frac{x-x_1}{x_4-x_1}\right)\left(\frac{x-x_2}{x_4-x_2}\right)\left(\frac{x-x_3}{x_4-x_3}\right)$$

$$f_3(x) = L_1 f(x_1) + L_2 f(x_2) + L_3 f(x_3) + L_4 f(x_4)$$

$\boxed{\text{Example}}$ construct interpolating polynomial...

$$f_2(x) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2) \quad (*)$$

x	$f(x)$
0	2
3	4
7	19

$$L_0 = \left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right) = \frac{(x-3)}{(0-3)}\cdot\frac{(x-7)}{(0-7)}$$

$$= (1/21)(x^2 - 10x + 21)$$

$$L_1 = \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right) = \frac{(x-0)}{(3-0)}\cdot\frac{(x-7)}{(3-7)}$$

$$= (-1/12)(x^2 - 7x)$$

$$L_2 = \left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right) = \frac{(x-0)}{(7-0)}\cdot\frac{(x-3)}{(7-3)}$$

$$= (1/28)(x^2 - 3x)$$

Substitute L_0, L_1, L_2 into $(*)$ $\longrightarrow$

$f_2(*) = (1/21)(x^2 - 10x + 21)(2) + (-1/12)(x^2 - 7x)(4) + (1/28)(x^2 - 3x)(19)$

$f_2(*) = (1/84)(37x^2 - 55x + 168)$

$f(4) = 6.4286$ (should be between values) cont'd ... ?

$L_0(4) + L_1(4) + L_2(4) = ?$

↳ would equal 1 for 2, 4, 19

$f_2(4)$

$f_1(4)$

EXAMPLE

x	1	2	3	5	6
$f(x)$	7	4	5.5	40	82

Calculate $f(4)$ through the lagrange

$f_4(4)$

$$f_1(x) = \left(\frac{x - x_2}{x_1 - x_2}\right) f(x_1) + \left(\frac{x - x_1}{x - x_1}\right) f(x_2)$$

$$f_1(x) = \left(\frac{x-5}{3-5}\right)(5.5) + \left(\frac{x-3}{5-3}\right)(40)$$

$$\boxed{f_1(4) = 17.25}$$

$$f_2(x) = \frac{(x-5)(x-2)}{(3-5)(3-2)}(5.5) + \frac{(x-3)(x-2)}{(5-3)(5-2)}(40) \ldots$$

$$\ldots + \frac{(x-5)(x-3)}{(2-5)(2-3)}(4)$$

$$\boxed{f_2(4) = 17.5}$$

$$f_3(x) = \frac{(x-3)(x-5)(x-6)}{(2-3)(2-5)(2-6)}(4) + \frac{(x-2)(x-5)(x-6)}{(3-2)(3-5)(3-6)}(5.5) + \ldots$$

$$\ldots \frac{(x-2)(x-3)(x-6)}{(5-2)(5-3)(5-6)}(40) + \frac{(x-2)(x-3)(x-5)}{(6-2)(6-3)(6-5)}(82)$$

$$\boxed{f_3(4) = 16}$$

$$f_4(x) = \frac{(x-2)(x-3)(x-5)(x-6)}{(1-2)(1-3)(1-5)(1-6)}(7) + \frac{(x-1)(x-3)(x-5)(x-6)}{(2-1)(2-3)(2-5)(2-6)}(4) + \ldots$$

$$\ldots \frac{(x-1)(x-2)(x-5)(x-6)}{(3-1)(3-2)(3-5)(3-6)}(5.5) + \frac{(x-1)(x-2)(x-3)(x-6)}{(5-1)(5-2)(5-3)(5-6)}(40) + \ldots$$

$$\ldots \frac{(x-1)(x-2)(x-3)(x-5)}{(6-1)(6-2)(6-3)(6-5)}(82)$$

$f_4(x) =$

$\boxed{\text{EXAMPLE}}$ The vertical stress...

$$a = 4.6\ m,\quad b = 14\ m,\quad z = 10\ m \longrightarrow m = 4.6/10 = 0.46,\quad n = 14/10 = 1.4$$

(pulled from table)

m	$n = 1.4$
0.3	0.08561
0.4	0.10941
0.46 $\rightarrow$	? $\rightarrow$ = 0.122162
0.5	0.13003
0.6	0.14749

$$f(x) = 0.0033x^3 - 0.163x^2 + 0.3509x - 0.0051$$

$$q = \frac{100}{(4.6)(14)} = 1.552795$$

$$\sigma_z = q\,f(x) = 1.552795\,(0.122162) = 0.18963$$

$\boxed{\text{EXAMPLE}}$

v (m³/kg)	0.10377	0.1144	0.12540
S (kJ/kg·K)	6.4147	6.5453	6.7664

$$S = f_2(v) = L_1 f(v_1) + L_2 f(v_2) + L_3(v_3)$$

$$f_2(v) = L_1(6.4147) + L_2(6.5453) + L_3(6.7664)$$

$$L_1 = \frac{(v-v_2)(v-v_3)}{(v_1-v_2)(v_1-v_3)}(6.4147) \longrightarrow L_1 = \frac{(v-0.11144)(v-0.1254)}{(1.659\times10^{-4})}$$

$$L_2 = \frac{(v-0.10377)(v-0.1254)}{(-1.071\times10^{-4})}$$

$$L_3 = \frac{(v-0.10377)(v-0.1144)}{(3.0195\times10^{-4})}$$

$$f_2(v) = -38.91v^2 + 25.178v + 4.22$$

a) $f_2(0.108) = ?$

b) $S = 6.6$

$f(v) = 6.6 \longrightarrow v_s = ?$

$$-38.91v^2 + 25.178v + 4.22 - 6.6 = 0$$

$v = 0.114945$ ✓

$v = 0.5321$ ✗ (not between values)