Jan. 8TH / 18

Midterm : Feb. 14TH / 18 (25 %)

Final (45%) $\longrightarrow$ Assignments / Project (30 %)

Two projects $\rightarrow$ midterm (10 or 5 %)

Final (the rest ?)

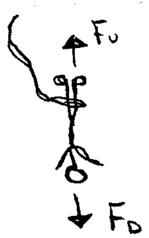
ATAC-4019 (Lab with computers w/ matlab)

Wed. 10:30 $\rightarrow$ 12 (Office hours) $\rightarrow$ ATAC-5008

Model Function

Dependent Variable = f (independent variables , parameters , forcing functions)

$\rightarrow$ Model Function Example



$F = ma$

$F = F_D + F_U$

$F_D = mg$

$F_U = -\overset{\text{drag coeff.}}{c}v$

$a = F/m$ $\quad dv/dt = (mg - cv)/m$

$$\boxed{\frac{dv}{dt} = g - \frac{c}{m}v} \rightarrow \text{solution} \rightarrow v = f(t)\ ?$$

Solution can be found via :

Analytical solution

Numerical solution

$$\left.\begin{array}{l} \dfrac{dv}{dt} = g - \dfrac{c}{m}v \\ t = 0 \\ v = 0 \end{array}\right\} \quad v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

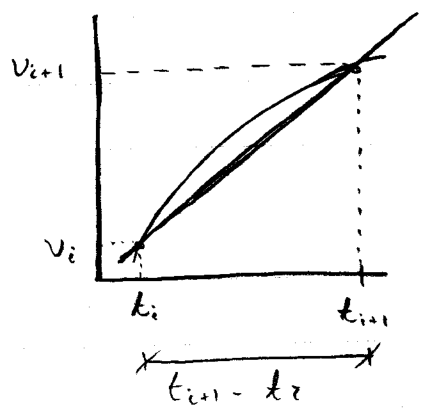
Numerical solution:

$$\frac{dv}{dt} \simeq \frac{\Delta V}{\Delta t} = \frac{v(t_{i+1}) - V(t_i)}{t_{i+1} - t_i}$$ (the slope, basically)

$$\frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{V(t_2) - V(t_1)}{t_2 - t_1}$$

$((\Delta t << 1))$

$$= \frac{V(t_{i+1}) - V(t_i)}{t_{i+1} - t_i}$$



$$\frac{dV}{dt} \cong \frac{\Delta V}{\Delta t}$$ → Euler method

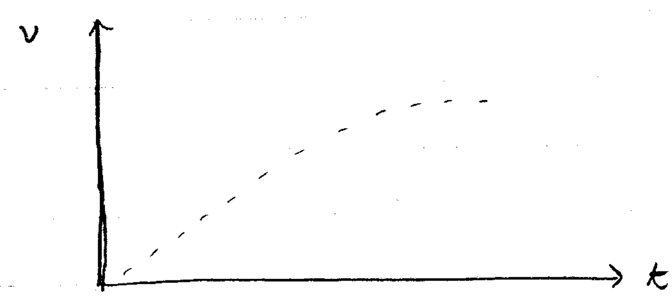
<u>mathematical model</u>

$$V(t_{i+1}) = V(t_i) + \left(g + \frac{c}{m} V(t_i)\right)(t_{i+1} - t_i)$$

$V(0) = 0$

$\Delta t = 1 \text{ sec}$

i	t_i	$V(t_i)$	$V(t_{i+1})$
0	0	0	g
1	1	g	$g + (g - \frac{c}{m} g)(1)$
2	2	$\underbrace{g + (g - \frac{c}{m} g)}_{\alpha}$	$\alpha + (g - \frac{c}{m} \alpha)(1)$
⋮	⋮		⋮



→ solution can be improved by using smaller steps.

Example 1

"newton's law of cooling says..."

$$\frac{dT}{dt} = -k(T - T_a)$$

$T = 70\,^\circ C$

$t = 0$ to 20 min

Step size of 2 min

$T_a = 20\,^\circ C$

$k = 0.017$

Solution :

$$\frac{dT}{dt} = -k(T - T_a)$$

$$\int \frac{dT}{T - T_a} = \int -k\,dt \Rightarrow \ln|T - T_a| = -kt + c$$

$$\Rightarrow T - T_a = e^{-kt} + C_1$$

$$\Rightarrow \boxed{T = T_a + e^{-kt} + C_1}$$

@ $t = 0$, $T = 70\,^\circ C$: $70 = 20 + e^{-(0.017)(0)} + C_1$, $C_1 = 49$

$$\boxed{T = T_a + e^{-kt} + 49}$$ analytical solution

t	0	2	4
T	70	...	...

Numerical Method

$$\frac{dT}{dt} = -k(T - T_a)$$

$$\frac{\Delta T}{\Delta t} = \frac{T(t_{i+1}) - T(t_i)}{t_{i+1} - t_i}$$

$$\Rightarrow \frac{T(t_{i+1}) - T(t_i)}{t_{i+1} - t_i} = -k(T(t_i) - T_a)$$

$\Delta t = 2$ min

$$T(t_{i+1}) = T(t_i) - k(T(t_i) - T_a)(t_{i+1} - t_i)$$

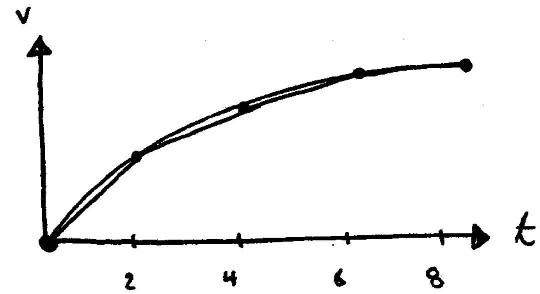
$$T(t_2) = 70 - (0.019)(70 - 20)(2)$$
$$T(t_3) = 68.1 - (0.019)(68.1 - 20)(2)$$

i	t_i	T
1	0	70
2	2	68.1
3	4	66.27
$\vdots$	$\vdots$	$\vdots$
11	20	53.94

$$\frac{dV}{dt} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} \cong \underset{\Delta t \ll 1}{\frac{\Delta V}{\Delta t}} = \frac{V(t_2) - V(t_1)}{t_2 - t_1}$$

$$\frac{dV}{dt} \cong \frac{V(t_{i+1}) - V(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} V(t_i)$$

$$V(t_{i+1}) = V(t_i) + \left(g - \frac{c}{m} V(t_i)\right)(t_{i+1} - t_i)$$



$\boxed{\text{Example 2}}$: $\frac{dV}{dt} = -KA$

$t = 0$ to 10 min

$\Delta t = 0.25$ min

$K = 0.08$ mm/min

radius $= 2.5$ mm

Volume of sphere:

$(4/3)\pi r^3 \longrightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$

$A = 4\pi r^2$

$A = 4\pi \left(\frac{3V}{4\pi}\right)^{2/3}$

@ $t = 0$, $r = 2.5$ mm

$\Rightarrow V = \frac{4\pi (2.5)^3}{3}$

$V = 65.44985$ mm^3

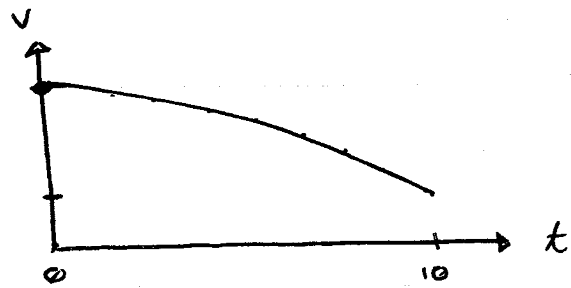
$$\frac{dV}{dt} = -K 4\pi \left(\frac{3V}{4R}\right)^{2/3}$$

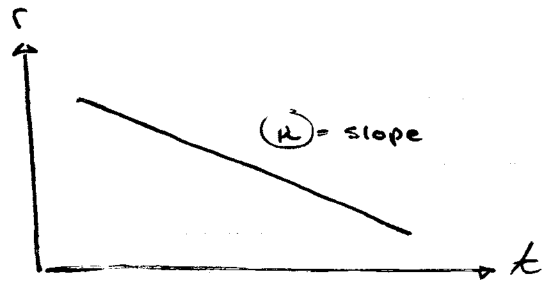
$$\frac{V(t_{i+1}) - V(t_i)}{t_{i+1} - t_i} = -K 4\pi \left(\frac{3V}{4R}\right)^{2/3}$$

$$V(t_{i+1}) = V(t_i) + \left[-K 4\pi \left(\frac{3V(t_i)}{4R}\right)^{2/3}\right](t_{i+1} - t_i)$$

$\longrightarrow \frac{V_{t_{i+1}} - V_{t_i}}{t_{i+1} - t_i}$

i	t (min)	V (mm^3)	dV/dt =
1	0	65.44985	—
2	0.25	63.879	-6.283
3	0.5	62.333	-6.1822
4	0.75	60.812	-6.08212
⋮	10	20.2969	





$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r(10) = \sqrt[3]{\frac{3(20.2969)}{4\pi}}$$

$$r(10) = 1.692182$$

$$k = \frac{2.5 - 1.692182}{10}$$

$$k = 0.08078 \ \text{mm/min}$$

Example 3 : Change in volume = in flow − out flow

$$\left.\begin{aligned} dV/dt &= 3Q\sin^2(t) - Q \\ V &= Ay \end{aligned}\right\} \quad \frac{d(Ay)}{dt} = 3Q\sin^2(t) - Q$$

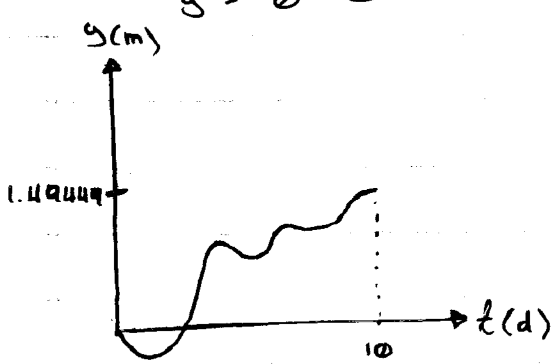
$$\boxed{\frac{dy}{dt} = \frac{3Q}{A}\sin^2(t) - \frac{Q}{A}}$$

$$\frac{y(t_{i+1}) - y(t_i)}{t_{i+1} - t_i} = \frac{3Q}{A}\sin^2(t_i) - \frac{Q}{A}$$

$$y(t_{i+1}) = y(t_i) + \left[\left(\frac{3Q}{A}\right)\sin^2(t_i) - \frac{Q}{A}\right](t_{i+1} - t_i)$$

$$\Delta t = 0.5$$

$$\left.\begin{aligned} t &= 0 \\ y &= 0 \end{aligned}\right\} \text{initial conditions}$$



i	t(day)	y(metre)
1	0	0
2	0.5	−0.18
3	1	−0.23508
4	1.5	−0.03352
5	2	0.32278
6	2.5	0.59026
⋮ (something)	10	1.49449