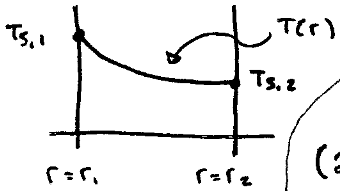
Feb. 11 / 19

<u>Note:</u> (1) Check the resulting temp. profile given by eq (2-88)



$T(r=r_1) = ?$

$T(r=r_2) = ?$

(2) $T(r)$ here in the above formulation is non-linear (logrithmic)

In order to determine the heat transfer rate $\dot{Q}_r$, Fourier's law of conduction is used as follows:

(2-89)a $\dot{Q}_r = -kA_r \left(dT/dr\right)\Big|_{r=r_1 \text{ or } r=r_2}$

$= -k(2\pi r L)\, dT/dr$

differentiating eq (2.88) then substituting (2-89)a, gives

$$\dot{Q}_r = -k(2\pi L)\frac{(T_{s,2} - T_{s,1})}{\ln(r_2/r_1)} \quad \text{or :}$$

(2-89)b $$\dot{Q}_r = k(2\pi L)\frac{(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Rearrange this eq'n; we get,

(2-90) $$\dot{Q}_r = \frac{(T_{s,1} - T_{s,2})}{\left(\ln(r_2/r_1)/2\pi k L\right)}$$

(ΔT)

R_{cond} for a cylinder

$T_{s,1} > T_{s,2}$

$$R_{cond} = \left[\frac{\ln(r_2/r_1)}{2\pi k L}\right]$$

Recall, $\dot{Q} = \dfrac{\Delta T}{R_{cond}}$

Fig 2-13 : Thermal circuit for steady-state 1-D conduction $(k = \text{const},\ \dot{E}_{gen} = 0)$

2.3.2 : Conduction Heat Transfer with Internal Heat Generation in Radial Systems

Case : very <u>long</u> cylinder

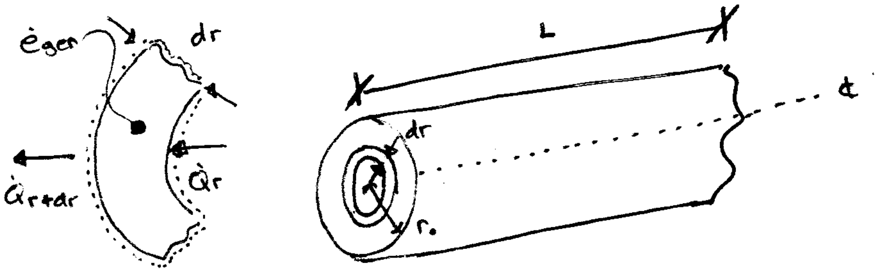


Fig (2-18)

Application of Energy Balance for the shown diff. Volume, gives

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \cancel{\dot{E}_{ST}} \rightarrow 0$$

(2-125) $$\underbrace{\dot{Q}_r}_{\dot{E}_{in}} + \dot{E}_{gen} = \underbrace{\dot{Q}_{r+dr}}_{\dot{E}_{out}}$$

Recall,

(2-126) $$\dot{Q}_{r+dr} = \dot{Q}_r + \frac{d\dot{Q}_r}{dr}dr$$

and

(2-127) $$\dot{E}_{gen} = \dot{e}_{gen}\, d\forall$$

where,

(2-128) $$d\forall = A_r dr \,, \quad A_r = 2\pi r L$$

Using Fourier's Law of Conduction, given by

(2-129)a $$\dot{Q}_r = -k A_r \, dT/dr \big|_{r=r}$$

Sub the foregoing results in eq (2-125), gives

(2-129)b $$-d/dr \left(-\underbrace{k\, 2\pi r L}_{const} \frac{dT}{dr} \right) \quad \dot{e}_{gen} = (2\pi r L)\,dr = 0$$

Simplifying and rearranging gives

(2-130) $\dot{e}_{gen} = -k\left(\frac{dT}{dr} + r\frac{d^2T}{dr^2}\right)$ or

(2-131) $\dot{e}_{gen}\, r = -k\frac{d}{dr}\left(r\frac{dT}{dr}\right)$

Integrating eq (2-131), once

(2-132) $\frac{\dot{e}_{gen} r^2}{2} = -kr\left(\frac{dT}{dr}\right) + C_1$

1st BC $\left.\frac{dT}{dr}\right| = 0$ (symmetry at the ℄ type of BC)

In order to satisfy this BC

(2-133) $\rightarrow C_1 = 0$

Integrating Eq (2-132), gives

(2-134) $T(r) = \frac{-\dot{e}_{gen} r^2}{4k} + \underbrace{C_2}_{?}$

Applying the 2nd BC, which is

Finally, sub. eq (2-136) in (2-134), gives

(2-137) $$\boxed{T(r) = T_0 + \frac{\dot{e}_{gen} r_0^2}{4k}\left[1 - \left(\frac{r}{r_0}\right)^2\right]}$$

(2-135) $T(r=r_0) = T_0$

(2-136) $\rightarrow C_2 = \frac{\dot{e}_{gen} r_0^2}{4k} + T_0$ (oops)

$T(r=0) = T_{max}$, sub $r = 0$ in eq. (2-137), gives T_{max} as

(2-138) $T_{max} = T_0 + \frac{\dot{e}_{gen} r_0^2}{4k}$

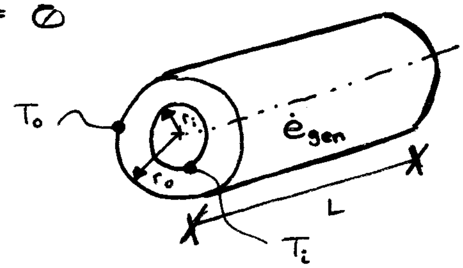
Remarks:

1 eq'n (2-137) can be written in dimensionless form as:

(2-139) $$\frac{T(r) - T_0}{T_{max} - T_0} = 1 - \left(\frac{r}{r_0}\right)^2$$

2 Conduction heat transfer in a hollow cylinder with $\dot{e}_{gen} \neq 0$

Fig (2-19):



2 B.C.'s :

(2-140)a (a) $T(r = r_i) = T_i$

(2-140)b (b) $T(r = r_o) = T_o$

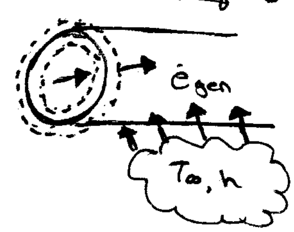
The solution for this case is given by

(2-141) $$T(r) = T_o + \frac{\dot{e}_{gen}}{4k}(r_o^2 - r^2) + \frac{\ln(r/r_o)}{\ln(r/r_i)}\left[\frac{\dot{e}_{gen}}{4k}(r_o^2 - r_i^2) + (T_o - T_i)\right]$$

3 1-D conduction heat transfer for a solid cylinder with $\dot{e}_{gen}$ emmersal in a fluid at T_∞ and h ($T(r=r_o) = T_o$ unknown)

Applying energy balance at the surface $(r = r_o)$ (surface energy balance)

$\dot{Q}_{cond}\big|_{@r=r_o} = \underset{at\ r=r_o}{\dot{Q}_{conv}}$ $\qquad \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \cancel{\dot{E}_{st}} = 0$



$$-k A_{r_o} \frac{dT}{dr} = h A_s (T_o - T_\infty)$$

But, $A_{r_o} = A_s$ (for this case)

(2-142) $$-k \left. dT/dr \right|_{r=r_o} = h(T_o - T_\infty)$$

The solution for this case is given by

(2-143) $$T(r) = \frac{\dot{e}_{gen} r_o}{4h}\left\{2 + \frac{h r_o}{k}\left[1 - \left(\frac{r}{r_o}\right)^2\right]\right\} + T_\infty$$

(2-2)C: 1-D Steady-State Conduction Heat Transfer in a Spherical Shell w/ no Heat Generation

Consider 1-D, steady state conduction heat transfer in a spherical shell with no heat generation ($\dot{E}_{gen} = 0$), as shown:

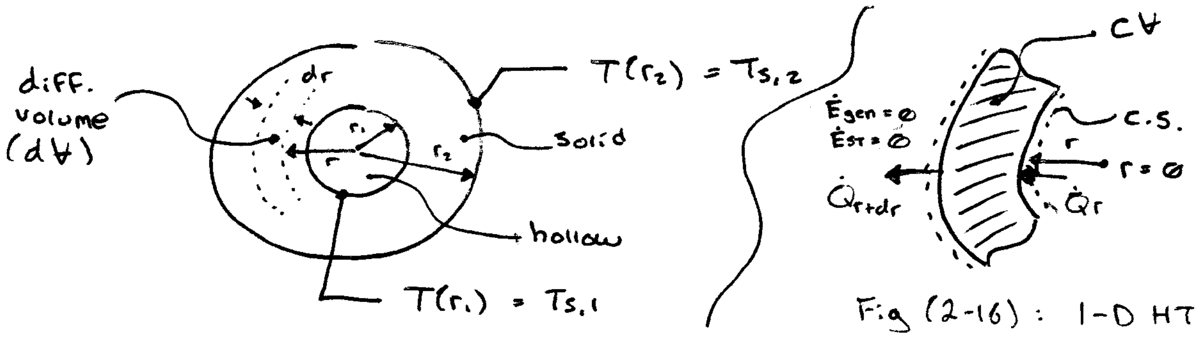


Fig (2-16): 1-D HT steady state in a spherical shell

Application of diff. volume energy balance over the above shown C.S., gives:

$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}^{\,0} = \cancel{\dot{E}_{st}}^{\,0}$$

$$\rightarrow \dot{E}_{in} = \dot{E}_{out}$$

(2-91) $$\dot{Q}_r = \dot{Q}_{r+dr}$$

Recall, the definition of a derivative, given by:

(2-92) $$\dot{Q}_{r+dr} = \dot{Q}_r + \frac{d\dot{Q}_r}{dr}\,dr$$

If we substitute this relation (eq. 2-92) back in eq. (2-91), we get

(2-93) $$\boxed{\frac{d\dot{Q}_r}{dr} = 0}$$

(2-96) $$\boxed{\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0}$$

Feb. 13/19

Recall Fourier's Law

$\dot{Q}_r = -kA_r \frac{dT}{dr}\Big|_{r=r}$

(2-95) $A_r = 4\pi r^2$ ---

(area $\perp$ to flow of $\dot{Q}_r$)

Sub (2-95) in (2-94) and then the resulting eq. back in (2-93), gives

(2-96) $\boxed{d/dr\,(r^2\, dT/dr) = 0}$ (verify)

Integrating once, gives

(2-97) $r^2 \frac{dT}{dr} = C_1$

Separation of Variables, T & r, gives

$dT\left(\frac{dr}{r^2}\right)\cdot C_1$

Integrating 2nd time, gives

$T(r) = -\frac{C_1}{r} + C_2$

(2-98)

In order to determine $T(r)$ (as a unique solution), C_1 & C_2 need to be evaluated using 2 BC's:

(2-99)a (a) $T(r_1) = T_{s,1}$

(2-99)b (b) $T(r_2) = T_{s,2}$

Application of the above 2 BCs back in eq (2-98) results in 2 equations with two unknowns (C_1 & C_2) Solving gives:

(2-100) $$\boxed{T(r) = \left[\left(\frac{r_2}{r_2 - r_1}\right)\left(1 - \frac{r_1}{r}\right)\right](T_{s,2} - T_{s,1}) + T_{s,1}}$$

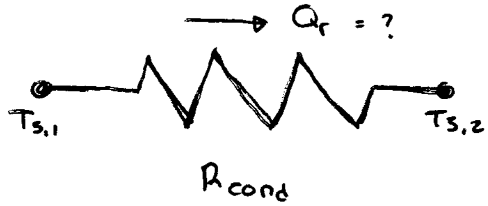
Remarks:

(1) Check $T(r_1) \stackrel{?}{=} T_{s,1}$ and $T(r_2) \stackrel{?}{=} T_{s,2}$

(2) The thermal circuit for this case

(sphere, 1-D, steady-state, $\dot{E}_{gen} = \emptyset$, $K = \text{const.}$)

$T_{s,1} > T_{s,2}$



$\dot{Q}_r = ?$

Using Fouriers Law,

(2-101) $\dot{Q}_r = -KA_r \frac{dT}{dr}\Big|_{r=r}$

Diff. eq(2-100), w.r.t. r and sub the result in (2-101) and then simplifying we get

(2-102) $$\dot{Q}_r = \frac{4\pi K r_2 r_1}{(r_2 - r_1)}\left(T_{s,1} - T_{s,2}\right)$$

Rearranging, gives:

(2-103)a $$\dot{Q}_r = \frac{\overbrace{(T_{s,1} - T_{s,2})}^{\Delta T}}{\underbrace{\left[\frac{(r_2 - r_1)}{4\pi K r_1 r_2}\right]}_{R_{cond}.}}$$

Recall,

(2-103)b $$(R_{cond})_{sphere} = \frac{(r_2 - r_1)}{4\pi K r_1 r_2}$$ OR

(2-103)c $$(R_{cond})_{sphere} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

(Review examples in Chapter 2)

Practice Problem
- [A] (2-34) (2-57) (2-59) (2-71) (2-72) (no heat gen)
- [B] (2-88) (2-97) (2-101) (2-109) (heat gen)

→ use <u>class</u> approach

Conduction heat transfer in a plane wall with variable thermal conductivity.

($k(T)$)

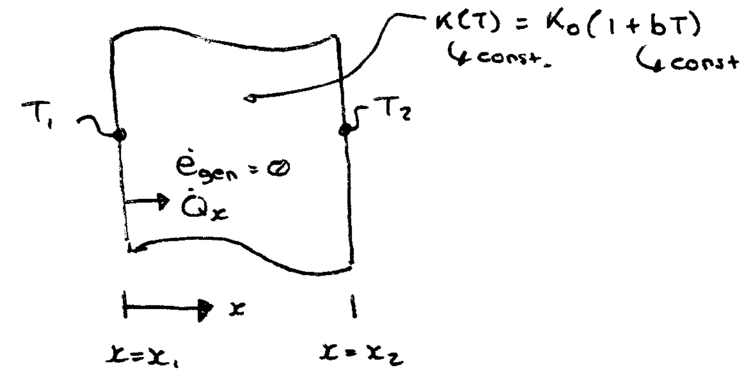
Consider the following plane wall

$T(x = x_1) = T_1$

$T(x = x_2) = T_2$

$T_1 > T_2$

$A = \text{const.}$



Recall Fourier's Law

$$\dot{Q}_x = -kA\frac{dT}{dx} \quad \cdots \quad (a)$$

Given that

$$k(T) = k_0(1 + bT) \quad \cdots \quad (b)$$

Sub eq (b) in (a), gives:

$$\dot{Q}_x = -k_0(1 + bT)A\frac{dT}{dx} \quad \cdots \quad (c)$$

{ Continue..., read Textbook, pg. 112 – 115