(2-27) • using eq'n (2-24) → $T(x=0) = T_{s,1} = C_2$

Now, sub. eq'n (2-27) in (2-26), gives

(2-28) $KC_1 = h(C_2 - T_\infty)$

Solving for C_1 & C_2 (finally), gives

(2-29)a $\boxed{T(x) = \left(\frac{L-x}{L+(k/h)}\right)(T_\infty - T_{s,2}) + T_{s,2}}$ verify!

Remarks:

i.e. sub for $x=0$, $T|_{@x=0} = T_{s,1}$

- $T_{s,1}$ can be determined using eq(2-27)

(2-29)b • $T_{s,1} = \left(\frac{L}{L+k/h}\right)(T_\infty - T_{s,2}) + T_{s,2}$

Table (2-1): Standard BC's for 1-D, steady-state, heat cond.

1 convection BC (each equation is obtained by performing energy balance over the surface)

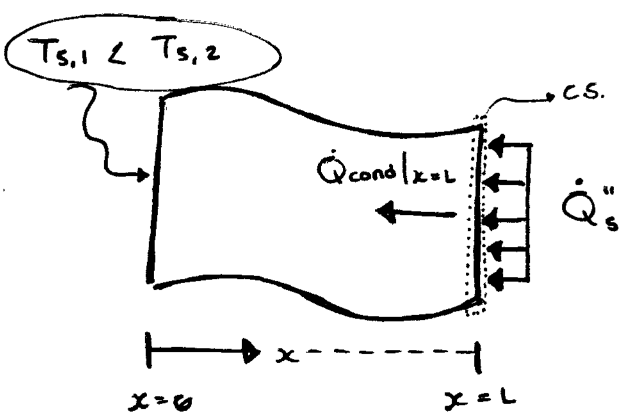
(2-37) @ $x=0$ $\qquad -K\, dT/dx|_{x=0} = h(T_\infty - T_{s,1})$

(2-38) @ $x=L$ $\qquad -K\, dT/dx|_{x=L} = h(T_{s,2} - T_\infty)$

2 Specified heat flux $\qquad (\dot{Q}/A = \dot{Q}''_s$ (+ve in x-dir)) double prime

(2-39) @ $x=0$ $\qquad \dot{Q}''_s = -K\, dT/dx|_{x=0}$

(2-40) @ $x=L$ $\qquad -K\, dT/dx|_{x=L} = \dot{Q}''_s$



$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}^{0} = \cancel{\dot{E}_{st}}^{0}$

$\dot{E}_{in} = \dot{E}_{out}$

$\dot{Q}_s = \dot{Q}_{cond}|_{x=L}$

$= -KA\frac{dT}{dx}|_{x=L}$

$\dot{Q}''_s \cancel{A} = -K\cancel{A}\frac{dT}{dx}|_{x=L}$

→ $\dot{Q}''_s = -K\frac{dT}{dx}|_{x=L}$

3 Insulated Surface (ideal)

(2-41) @ $x = 0$ $\qquad 0 = -k\left(\frac{dT}{dx}\right)\Big|_{x=0}$

$\dot{Q}_{in} = \dot{Q}_{out}$ (surface energy balance at $x = 0$)

(2-42) @ $x = L$ $\qquad -k\left(\frac{dT}{dx}\right)\Big|_{x=L} = 0$

4 Black-body radiative HT

(2-43) @ $x = 0$ $\qquad \sigma \varepsilon_s \left(T_{surr}^4 - T_{s,1}^4\right) = -k \, dT/dx\big|_{x=0}$

emissivity of the surface

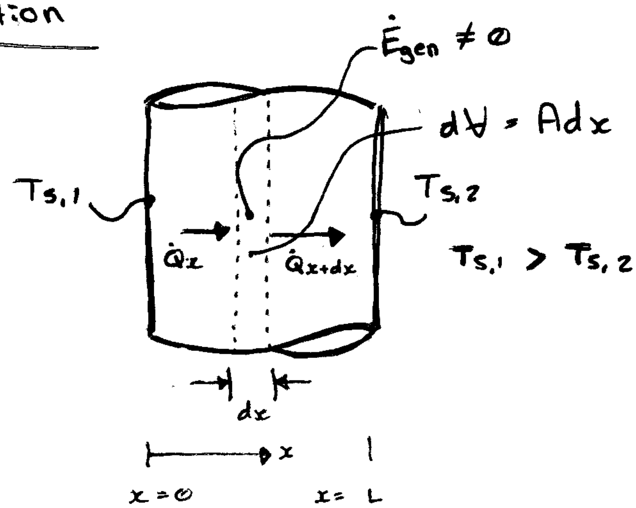
(2-44) @ $x = L$ $\qquad -k \, dT/dx\big|_{x=L} = \sigma \varepsilon_s \left(T_{s,2}^4 - T_{surr}^4\right)$

2.3 Conduction Heat Transfer with Internal Energy Generation

2.3.1:

Conditions (assumptions):

- 1-D (in x-dir)
- Steady state HT
- k = const.



{ Fig (2-17)a : Conduction w/ $\dot{E}_{gen} \neq 0$

Formulations:

Application of energy balance for the shown diff. volume ($d\forall$) gives:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \underbrace{\cancel{\dot{E}_{st}}}_{= 0 \text{ (steady-state)}}$$

(2-110)a $\quad \dot{Q}_x - \dot{Q}_{x+dx} + \dot{E}_{gen} = 0$

Recall, the def'n of a derivative

(2-110)b $\quad \dot{Q}_{x+dx} = \dot{Q}_x + d\dot{Q}_x/dx$

sub eq'n (2-110)b in (2-110)a, gives

(2-111) $\quad -d\dot{Q}_x/dx \; dx + \dot{E}_{gen} = 0$

Recall,

(2-112) $\dot{e}_{gen} = \dfrac{\dot{E}_{gen}}{dV} = \dfrac{\dot{E}_{gen}}{A\,dx}$

Fourier's Law of Conduction

(2-113) $\dot{Q}_x = -KA(dT/dx)$

Sub eqs (2-112) & (2-113) in (2-111), gives

$-d/dx\,(-KA\,dT/dx)\,dx + \dot{e}_{gen}\,dV = \emptyset$

$KA(d^2T/dx^2)\,dx + \dot{e}_{gen}\,(A\,dx) = \emptyset$

Rearranging gives

(2-114) $$\boxed{\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{K} = \emptyset}$$

(2-115)a 2 BC's : $*\,T(x=\emptyset) = T_{s,1}$

(2-115)b $*\,T(x=L) = T_{s,2}$

$\dfrac{d}{dx}\left(\dfrac{dT}{dx}\right) = -\dfrac{\dot{e}_{gen}}{K}$

Integrating once : $dT/dx = -(\dot{e}_{gen}/K)x + C_1$

(2-116) Integrating twice : $T(x) = -(\dot{e}_{gen}/2K)x^2 + C_1x + C_2$

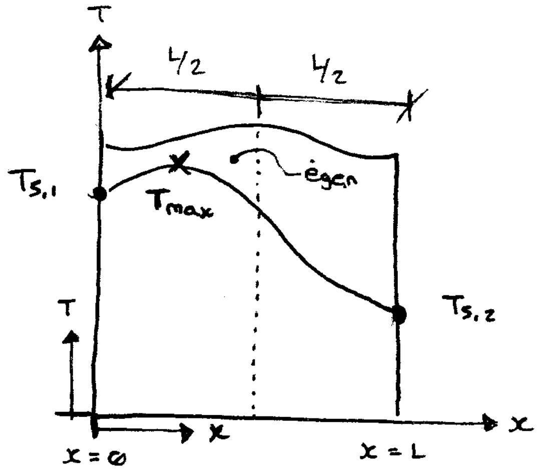
C_1 & C_2 can be determined by applying 2 BC's as follows:

(2-117)a 1st BC: $C_2 = T_{s,1}$

(2-117)b 2nd BC: $C_1 = \frac{\dot{e}_{gen} L}{2k} + \frac{(T_{s,2} - T_{s,1})}{L}$ Verify!

Sub C_1 & C_2 using eq's (2-117)a,b in eq (2-116), gives

(2-118) $$\boxed{T(x) = \frac{-\dot{e}_{gen} x^2}{2k} + \frac{\dot{e}_{gen} L x}{2k} + \frac{T_{s,2} - T_{s,1}}{L} x + T_{s,1}}$$



↳ Verify w/ $T(0)$, $T(L)$

$T(0) = T_{s,1}$, $T(L) = T_{s,2}$

$T_{s,1} > T_{s,2}$

- non-linear profile

$\dot{e}_{gen}$ = const. = uniform

* Temp. profile for the case ($T_{s,1} > T_{s,2}$) → given

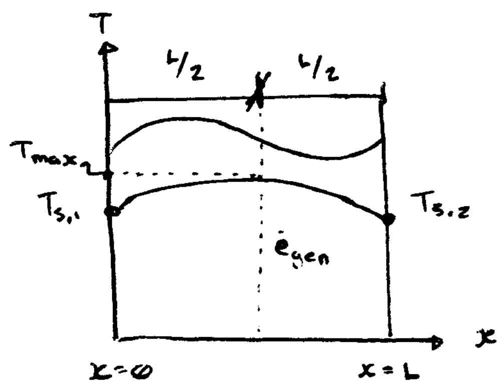
(*) Eq. (2-118) can also be simplified for the case $\boxed{T_{s,1} = T_{s,2}}$ to

(2-119) $$\boxed{T(x) = \frac{\dot{e}_{gen} L^2}{2k} \left[\left(\frac{x}{L}\right) - \left(\frac{x}{L}\right)^2 \right] + T_{s,1}}$$

↳ Just sub. $T_{s,1} = T_{s,2}$ – rewritten (2-118)

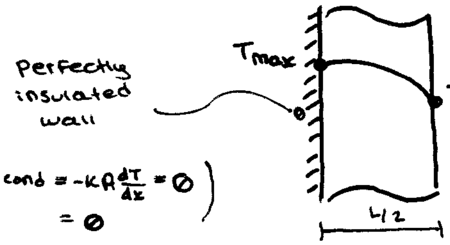
(*) NOTE when $\dot{e}_{gen} = 0$, eq (2-118) reduces to the case previously dealt with ($\dot{e}_{gen} = 0$)

* The temp. profile for the case $T_{s,1} = T_{s,2}$ is parabolic and symmetric about the centerline



$\dot{e}_{gen}$ = constant or uniform throughout

(*) NOTE that (for the above special case) at the centerline ($L/2 = x$), $T = T_{max}$ and $dT/dx = 0$ as if the plane wall looks like:



$(\dot{Q}_{cond} = -kA\frac{dT}{dx} = 0) = 0$

Fig (2-17)d

T_{max} is given by ($x = L/2$, $T_{s,1} = T_{s,2}$)

(2-120) $$\boxed{T_{max} = \frac{\dot{e}_{gen} L^2}{8k} + T_{s,1}}$$

T_{max} with symmetric B.C. ($T_{s,1} = T_{s,2}$)

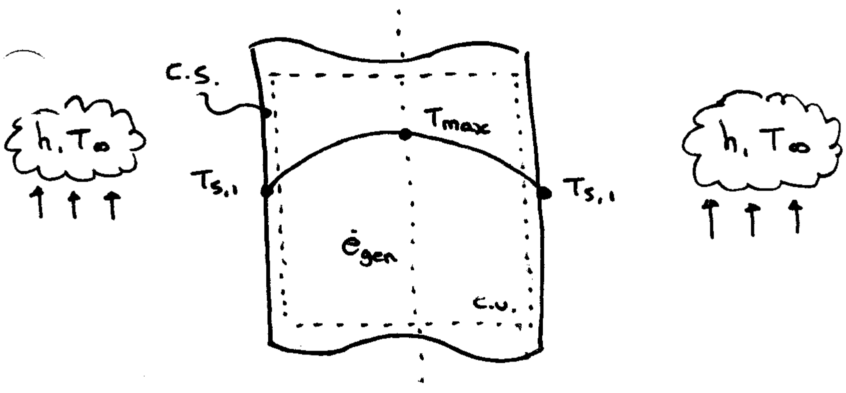


Fig (2-17)e

Application of Energy Balance for the shown C.V., for this case

(2-122) $$\dot{E}_{gen} = \dot{E}_{out}$$

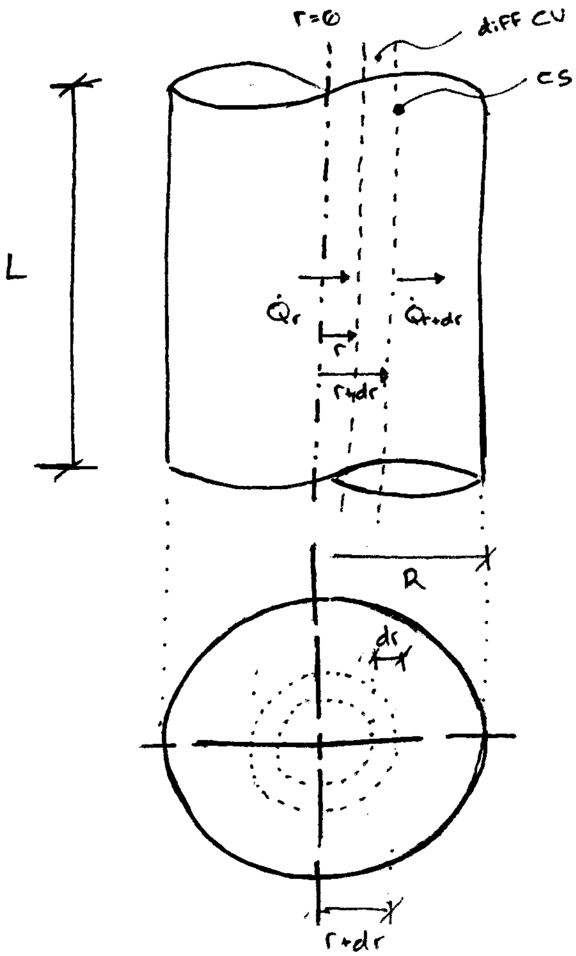
$$\dot{e}_{gen}(\cancel{A}L) = h\cancel{A}(T_{s,1} - T_\infty)$$

Solving for $T_{s,1}$ gives

(2-123) $$T_{s,1} = \frac{\dot{e}_{gen} L}{h} + T_\infty$$

(2-2)B Conduction Heat Transfer in Radial Walls (cylinder)

Consider steady state 1-D (in r-dir) heat conduction in a radial wall with no heat generation ($\dot{E}_{gen} = \emptyset$), as shown:



$$\boxed{A_r = 2\pi r L}$$

$$d\forall = A_r dr$$
$$= 2\pi r \, dr L$$

Fig. (2-11)

Formulations

- $T(r) = ?$
- $\dot{Q}_r = ?$

Applying energy balance for the shown diff. C$\forall$ (previously), gives

$$\dot{E}_{in} - \dot{E}_{out} + \underbrace{\cancel{\dot{E}_{gen}}}_{=\emptyset \text{ given}} = \underbrace{\cancel{\dot{E}_{ST}}}_{=\emptyset \text{ steady state}}$$

$$\dot{E}_{in} - \dot{E}_{out} = \emptyset$$

$$\dot{E}_{in} = \dot{E}_{out}$$

(2-77) $\dot{Q}_r = \dot{Q}_{r+dr}$

Using the definition of a derivative

(2-78) $\dot{Q}_{r+dr} \equiv \dot{Q}_r + \dfrac{d\dot{Q}_r}{dr}\,dr$

Sub eq(2-78) in (2-77) gives

(2-79) $\boxed{\dfrac{d\dot{Q}_r}{dr} = 0}$

- This indicates that $\dot{Q}_r \neq f(r)$ = const.

  using Fourier's Law of Conduction, we get $\hookrightarrow$ (eq 2-79)

(2-80) $\dot{Q}_r = -kA_r \dfrac{dT}{dr}\Big|_{r=r}$

$\dfrac{d}{dr}\big(\underbrace{-kA_r \dfrac{dT}{dr}}_{\dot{Q}_r}\big) = 0$

- consider that k = const.

(2-81) $d/dr\,(A_r\, dT/dr) = 0$

(2-82) $d/dr\,(2\pi r L\, dT/dr) = 0$

NOTE: $(A_r = 2\pi r L)$

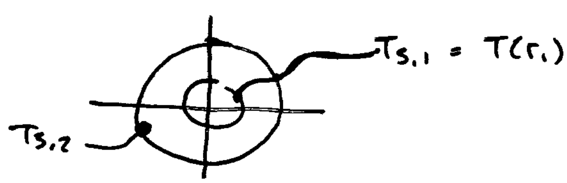
(2-83) $\boxed{d/dr\,(r\, dT/dr) = 0}$

The result is valid only for steady-state, $\dot{E}_{gen} = 0$, 1-D

Next step is to obtain $T(r)$ by solving the diff. eq'n given by eq(2-83)

Eq(2-83) is 2nd-order ODE, we need 2 B.C.'s

Consider a hollow thick cylinder as shown:



(2-84)a 1st BC $\Rightarrow$ $T(r=r_1) = T_{s,1}$

(2-84)b 2nd BC $\Rightarrow$ $T(r=r_2) = T_{s,2}$

Integrating eq. (2-83) once, gives

(2-85)a $$r\frac{dT}{dr} = C_1$$

(2-85)b $$dT = \frac{C_1}{r}\,dr$$

Integrating again, gives

(2-86) $$T(r) = C_1 \ln r + C_2$$

Applying the 2 B.C.'s in this eq'n (2-86 gives):

(2-87)a $$T(r=r_1) = T_{s,1} = C_1 \ln r_1 + C_2$$ and

(2-87)b $$T(r=r_2) = T_{s,2} = C_1 \ln r_2 + C_2$$

Solving for C_1 & C_2 and then sub the results in eq (2-86), gives:

(2-88) $$\boxed{T(r) = \frac{T_{s,2} - T_{s,1}}{\ln(r_2/r_1)} \ln(r/r_2) + T_{s,2}}$$

verify!