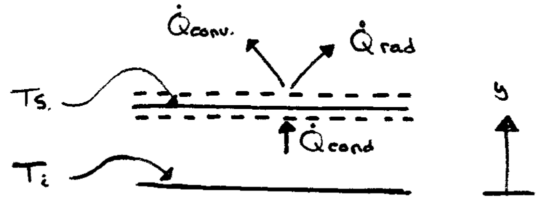


$T_i > T_s$

- Chose the Free surface so heat in (conduction) could be compared to heat out (radiation, convection)
-
- Steady / unsteady? no volume – no storage – doesn't matter.

Analysis:

Performing surface energy balance, gives

$\dot{E}_{in} = \dot{E}_{out}$ - - - 1

In this case,

$\dot{E}_{in} = \dot{Q}_{cond}$ & $\dot{E}_{out} = \dot{Q}_{rad} + \dot{Q}_{conv.}$

$\Rightarrow \dot{Q}_{cond} = \dot{Q}_{rad} + \dot{Q}_{conv.}$ - - - 2

Using the laws of heat transfer, yields for conduction (1-D in y-direction only)

$\dot{Q}_{cond} = -k A_y \frac{dT}{dy}$ - - - 3

Integrating (separation of Variables) gives

$\int_T dT = \int_y \left(\frac{1}{-kA_y}\right) \times \dot{Q}_{cond} \times dy$

Since $\dot{Q}_{cond}$, k, A_y are constant in y-direction

$\Rightarrow \Delta T = \left[-\frac{1}{kA_y}\right] \Delta y$

Rearranging, gives

$\dot{Q}_{cond} = -kA_y \frac{\Delta T}{\Delta y} = -kA_y\left(\frac{T_s - T_i}{L}\right)$

OR $\dot{Q}_{cond} = kA_y\left(\frac{T_i - T_s}{L}\right)$ - - - 4

Newton's Law of Cooling, Gives:

$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$ - - - 5

Stefan-Boltzmann Law, Gives:

$\dot{Q}_{rad} = \varepsilon_s \sigma . A_s (T_s^4 - T_{surr}^4)$ - - - 6

For mathematical convenience, eq. 6 can be written as:

$\dot{Q}_{rad} = \varepsilon_s \sigma . A_s (T_s^2 + T_{surr}^2)(T_s + T_{surr})(T_s - T_{surr})$

(1-22) Define: $\boxed{h_r = \varepsilon_s \sigma (T_s^2 + T_{surr}^2)(T_s + T_{surr})}$ --- (8)

(1-23) so that $\boxed{\dot{Q}_{rad} = h_r A_s (T_s - T_{surr})}$ --- (a)

Now sub (4), (5) and (a) in (2), yields

$$K(A_y)\,\frac{(T_i - T_s)}{L} = h \cdot A_s \cdot (T_s - T_\infty) + h_r A_s (T_s - T_{surr})$$

Since (in this case) $A_y = A_{s,conv.} = A_{s,rad}$

so, $K\left(\frac{T_i - T_s}{L}\right) = h(T_s - T_\infty) + h_r(T_s - T_{surr})$

Rearranging this eqn (10) with $\boxed{T_\infty = T_{surr}}$ --- (10) (in this ex.)

and solving for T_s, gives:

$$T_s = \frac{\frac{KT_i}{L} + (h + h_r)T_\infty}{\frac{K}{L} + (h + h_r)} \quad \text{(in K)} \quad \text{--- (11)}$$

see eqn (8)

* use Kelvin

NOTE: T_s is also embedded in h_r in the RHS of eqn (11). This eqn can be solved by trial-and-error procedure, as follows:

Let, $T_s = 305$ K $\longrightarrow$ RHS ≈ 307.2 K $(\sim 34\,^\circ C)$

2nd-trial: using the new value $T_s = 307.2$ K

$\Rightarrow$ RHS ≈ 307.2 K o.k.

∴ The skin temperature $= T_s = 307$ K $(\approx 34\,^\circ C)$

The rate of heat loss can be found using Eq (2), when $\dot{Q}_{cond} = \dot{Q}_{loss,skin} = \dot{Q}_{rad} + \dot{Q}_{conv.}$

But, eq (4) gives

$$\dot{Q}_{cond} = \dot{Q}_{loss,\,skin} = 0.3 \times 1.8 \times \left(\frac{(308 - 307.2)}{0.003}\right) \cong \boxed{146\ W}$$

Remarks:
1. $\dot{Q}_{conv} \cong 37$ W (from Eqn (5))
2. $\dot{Q}_{rad} \cong 109$ W (eqn (6) or (7))

3. Exercise

Re-do the previous example assuming that the fluid over the skin is water at $T_\infty = 297$ K and $h = 200\ \frac{W}{m^2 K}$ (instead of air)

Final answers:

$T_s \cong 300.7$ K (or $28\,^\circ C$)

$\dot{Q}_{loss,s} = 1320$ W

Section 2-2, pg. 73 (related to notes)

1-D Heat Transfer

2.1 Conduction heat transfer in a plane wall

→ Let's consider 1-D steady state conduction heat transfer in the plane wall as shown in Figure (2-1)a

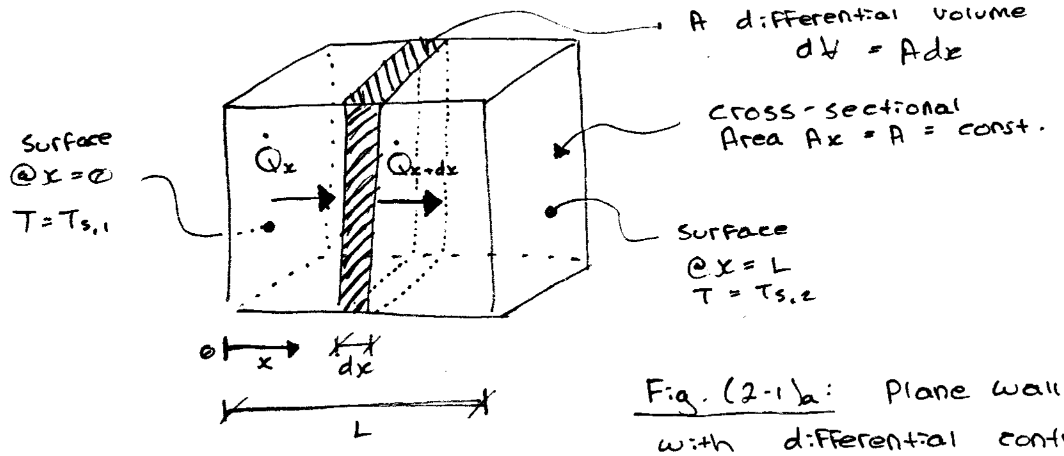


Fig. (2-1)a: Plane wall system with differential control volume ($d\forall$)

Mathematical Modeling & Formulations

- Here we develop mathematical equations that represent the heat transfer within the plane wall. In this problem, there are no temp. gradients in y or z directions, so that the formulations for this class of heat cond. prob becomes 1D in x-dir only.
- we also consider S.S. condition that leads to
$$\frac{\Delta E_{st}}{\Delta t} = 0 \quad (\dot{E}_{st} = 0)$$
- Applying energy balance to the differential element $d\forall$, we get:

$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}^{0} = \cancel{\dot{E}_{st}}^{0} \quad \text{---} \quad (2\text{-}1)a$$
$$\dot{E}_{in} = \dot{E}_{out} \quad \text{---} \quad (2\text{-}1)b$$
$$\dot{Q}_x = \dot{Q}_{x+dx} \quad \text{---} \quad (2\text{-}2)$$

The relationship between $\dot{Q}_x$ & $\dot{Q}_{x+dx}$ can be obtained using the definition of a derivative, given by:

$$\frac{d\dot{Q}_x}{dx} = \lim_{\Delta x \to 0} \frac{\dot{Q}_{x+dx} - \dot{Q}_x}{\Delta x} \quad \text{---} \quad (2\text{-}3)$$

Since $(\Delta x \to dx)$

$$\frac{d\dot{Q}_x}{dx} = \frac{d\dot{Q}_{x+dx} - \dot{Q}_x}{dx} \quad --- \quad (2-4)$$

Rearranging this eqn, yields

$$\dot{Q}_{x+dx} = \dot{Q}_x + \frac{d\dot{Q}_x}{dx}\,dx \quad --- \quad (2-5)$$

Using (2-5) in (2-2) gives

$$\boxed{\frac{d\dot{Q}_x}{dx} = 0} \quad --- \quad (2-6)$$

Remark : Eqn (2-6) here suggests that $\dot{Q}_x$ is not a function of x.

$\therefore$ $\dot{Q}_x$ = const.

- S.S.
- 1-D
- $\dot{E}_{gen} = 0$

- next step to obtain temp distribution (T_x)

Next step in this formulation is to obtain (or formulate) temperature distribution $T(x)$. This is done as follows:

using Fourier's Law of Conduction for $\dot{Q}_x$, as:

(2-7) $\dot{Q}_x = -KA_x\left(\frac{dT}{dt}\right)$

Sub. eqn (2-7) in (2-6), gives:

(2-8) $\boxed{[d/dx]\left(-KA_x\frac{dT}{dx}\right) = 0}$

under the previous fixed conditions; namely, A_x = const., and considering K = uniform

$-KA\, d/dx(dT/dx) = 0$, eq (2-8) gives

$d/dx(\partial T/dx) = 0$, or

(2-9) $\boxed{\frac{d^2T}{dx^2} = 0}$

Here, eq(2-9) is 2nd-order ordinary differential eq. (ODE). The formulation is completed by specifying the boundary conditions (BC's), (a boundary condition is a mathematical statement pertaining to the behavior of the dependent variable T at the system boundary. In this case, we need 2 BC's since the ODE is a 2nd order diff. eq.

(2-10) 1^{ST} - BC: @ $T(x=0) = T(0) = T_{s,1}$

(2-11) 2^{ND} - BC: @ $T(x=L) = T(L) = T_{s,2}$

In order to solve the DE (Heat conduction diff eqn) recall,

Eq. $d/dx\,(dT/dx) = 0$

Integrating once, gives:

$dT/dx = C_1$

Integrating again, gives:

$T(x)\ \cancel{dT/dx} = C_1x + C_2$

(2-12) $\boxed{T(x) = C_1x + C_2}$

Here, C_1 & C_2 are constants that can be evaluated using BC's

(2-13) $T(x=0) = T_{s,1} = C_1(0) + C_2 \Rightarrow \boxed{C_2 = T_{s,1}}$

Similarly,

(2-14) $T(x=L) = T_{s,2} = C_1(L) + C_2$

Sub eq (2-13) in (2-14), yields:

(2-15) $C_1 = \dfrac{T_{s,2} - T_{s,1}}{L}$

Now, sub eqn's (2-15) & (2-13) back in (2-12), we get

(2-16) $\boxed{T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1}}$

It would be useful to re-write eq (2-16) in a dimensionless form, given by

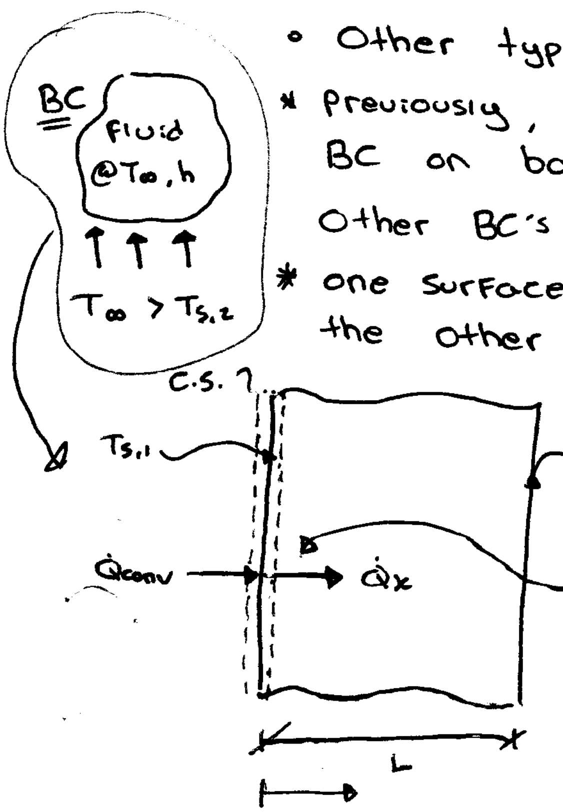
(2-17) $\boxed{\dfrac{T - T_{s,1}}{T_{s,2} - T_{s,1}} = \dfrac{x}{L}}$

- Other types of Standard Boundary Conditions

* Previously, the BC type was constant-temp. BC on both surfaces.

Other BC's could be:

* one surface is exposed to convection HT and the other is at const. temp.



$T_{s,2}$ = Given or Fixed as a BC

NOTE:
at $x=0$, $\dot{Q}_{conv} = \dot{Q}_{cond}$

Figure (2-2): 1-D steady-state HT in a plane-wall with convective BC and no heat generation.

Application of energy balance over the shown $\underset{x=0}{CS}$, we get:

(2-21)a $\quad \dot{Q}_{cond}\big|_{@x=0} = \dot{Q}_{conv.}\big|_{@x=0}$

using Fourier's Law of Conduction at $x = 0$, gives

(2-21)b $\quad \dot{Q}\big|_{x=0} = -KA \frac{dT}{dx}\Big|_{x=0}$

Recall, Newton's Law of Cooling

(2-21)c $\quad \dot{Q}_{conv.} = hA_s(T_\infty - T_{s,1})$

<u>Note:</u> in this case, (plane wall) $A_s = A_x = A$

Sub for $\dot{Q}_{cond}$ & $\dot{Q}_{conv}$ in the surface energy balance, we obtain

$\dot{Q}_{conv}\big|_{@x=0} = \dot{Q}_{cond}\big|_{@x=0}$

$* \; hA(T_\infty - T_{s,1}) = -KA(dT/dx)\big|_{x=0}$

(2-22) <u>OR</u> $\quad hA(T_{s,1} - T_\infty) = KA(dT/dx)\big|_{x=0} \quad$ (1st BC)

$\hookrightarrow$ but $T_{s,1} = ?$

(2-23) $\quad$ 2nd <u>BC</u> : $T(x=L) = T_{s,2}$ = known or specified

(2-24) $\quad$ Recall: $T(x) = C_1 x = C_2$

(Note: Still, it is the same general solution for $T(x)$, but with different C_1 & C_2)

C_1 & C_2 can now be determined using eqs (2-22) & (2-23). In this case;

(2-25) $\quad$ - Using the $\overset{(2-24)}{\text{second}}$ BC: $\quad T_{s,2} = C_1 L + C_2$

(2-26) $\quad$ - Using the $\overset{(2-22)}{\text{First}}$ BC: $\quad KC_1 = h(T_{s,1} - T_\infty)$