1. Energy Balance Formulations Involving Heat Transfer Mechanisms

→ Recall the energy conservation principle (1st law of thermodynamics)

> Energy can neither be created nor destroyed during a process ; it can only convert (transform) from one form of energy to another form.

Remark : The conservation of energy principle is also known as the energy balance.

(A) Energy Balance Formulations for a closed system

Consider a closed system (i.e. a system across which there is no exchange of mass with the surroundings), as follows

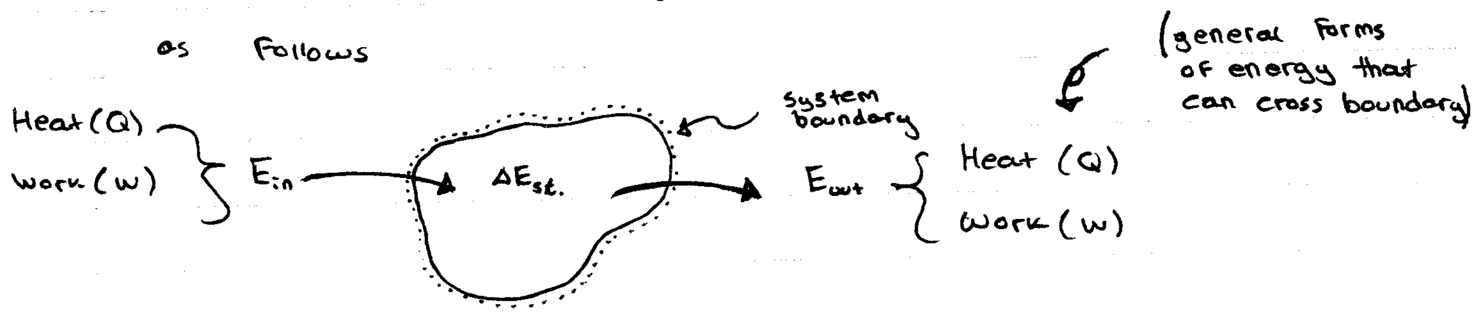

Fig (1-1): Energy balance for a closed system

Energy balance yields :

$$\underbrace{E_{in} - E_{out}}_{\text{net energy transfer by heat and work}} = \underbrace{\Delta E_{ST}}_{\text{change in internal, kinetic, potential, etc. energies (i.e. total energy stored)}} \quad (J \text{ or } kJ) \qquad (1-1)$$

OR, in a RATE FORM, gives

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat and work}} = \underbrace{\dot{E}_{st.}}_{\text{rate of change in internal, kinetic, potential, etc., energies (i.e. rate of stored total energy)}} \quad (W \text{ or } kW) \qquad (1-2)$$

Remarks : special

(1) For the case :

(Heat in) Transfer

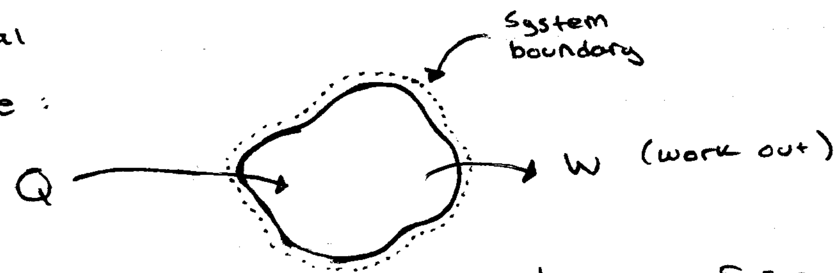

Fig (1-2): Energy balance for a closed system involving Q_{in} and W_{out} only.

Eqn. (1-1) reduces to:

(1-3) $$\underbrace{Q}_{in} - \underbrace{W}_{out} = \Delta E_{st.} \quad (\text{J or KJ})$$

or in a rate form Eqn. (1-2) reduces to:

(1-4) $$\underbrace{\dot{Q}_{in}}_{\text{rate of heat trans}} - \dot{W}_{out} = \dot{E}_{st.} \quad (\text{W or KW})$$

$\dot{E}_{st.} \rightarrow (dE_{st}/dt)$

(2) For the term $\Delta E_{st.}$, it can be expressed as:

(1-5)a $$\Delta E_{st.} = \underbrace{\Delta U}_{\text{change in internal energy}} + \underbrace{\Delta KE}_{\text{change in kinetic energy}} + \underbrace{\Delta PE}_{\text{change in potential energy}} + \ldots$$

(1-5)b where, $KE = \frac{1}{2} m v^2$

(1-5)c $PE = mgz$

(3) For steady-state systems → (no change in energy w.r.t. time.) but may change with location

(1-6) $$dE_{sys.}/dt = \dot{E}_{st} = 0$$

In heat transfer analysis, the forms of energy that can be transferred as a result of a temperature difference (i.e heat Q) are usually of interest. Therefore, it is convenient to write a heat balance and to treat the conversion of mechanical, chemical, nuclear, and electrical energies into thermal energy as heat generation (E_{gen}) as follows:

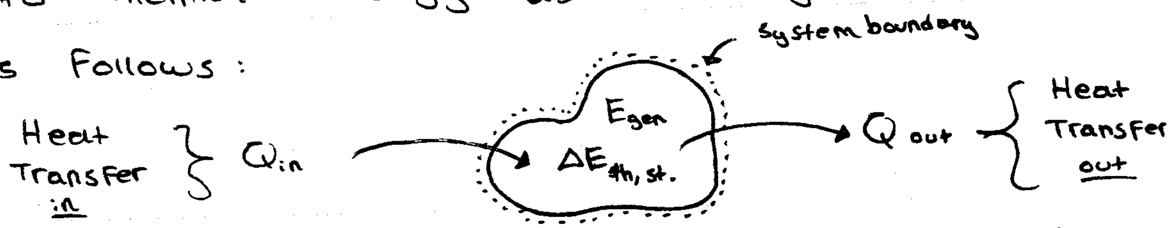

Fig: (1-3) Heat Balance for a closed system

So, Eq (1-1) can be modified to give:

(1-7) $$\underbrace{Q_{in} - Q_{out}}_{\text{net heat transfer}} + \underbrace{E_{gen}}_{\text{heat generation}} = \underbrace{\Delta E_{th,st}}_{\text{change in thermal energy of a system}} \quad (J \text{ or } kJ)$$

or in a rate form, gives ✱

(1-8) $$\boxed{\dot{Q}_{in} - \dot{Q}_{out} + \dot{E}_{gen} = \dot{E}_{th,st}} \quad (W \text{ or } kW)$$

Remarks: ① For a steady-state system

(1-9) $$\dot{E}_{th,st.} = \frac{dE_{th,st}}{dt} = 0$$

& Eq (1-8) reduces to:

(1-10) $$\dot{Q}_{in} - \dot{Q}_{out} + \dot{E}_{gen} = 0$$

② For the case $\underbrace{\dot{E}_{th,st} = 0}_{\text{steady-state}}$ & $\underbrace{\dot{E}_{gen} = 0}_{\text{no heat generation}}$

Eqn (1-8) becomes

(1-11) $$\dot{Q}_{in} = \dot{Q}_{out}$$

(B) Energy Balance Formulation for an <u>open system (control volume)</u>

Consider an arbitrary control volume as shown:

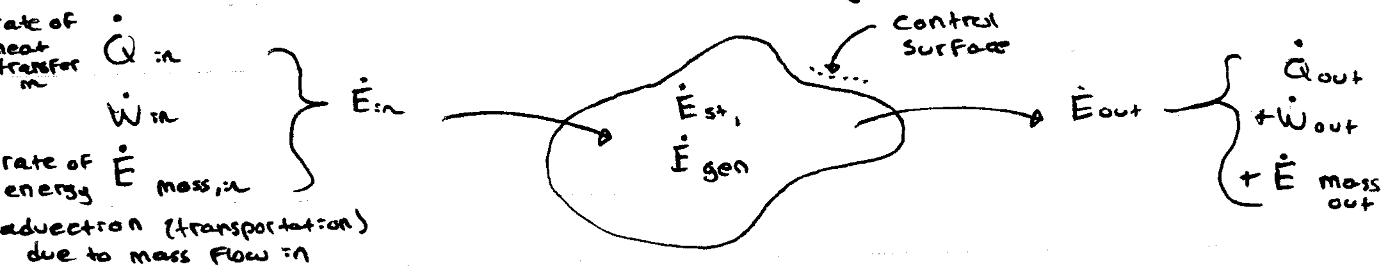

Fig (1-4) Energy Balance for the open

The energy balance for the above open system can be expressed as:

(1-12) $\left[\ \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st.} \right.$ (W or kW)

or

(1-13) $\left[\ E_{in} - E_{out} + E_{gen} = \Delta E_{st} \right.$ (J or kJ)

(1-14)a Where

$$\dot{E}_{in} = \overbrace{\dot{Q}_{in} + \dot{W}_{in} + \dot{E}_{mass,in}}^{\text{the sum of it all}}$$

(1-14)b $\dot{E}_{out} = \dot{Q}_{out} + \dot{W}_{out} + \dot{E}_{mass,out}$

↳ check out Ch. 2 (Textbook)

Heat Generation:

In heat transfer applications, the heat generation term might be involved in these applications. Typically, in heat conduction analysis, the conversion of mechanical, electrical, nuclear, or chemical energy into heat is characterized as heat generation, given by $\dot{E}_{gen}$.

In general,

(1-15)a $$\dot{E}_{gen} = \int_{\forall} \underbrace{\dot{e}_{gen}}_{\text{energy per unit volume}} \, d\overbrace{\forall}^{\text{volume}} \quad (W \text{ or } kW)$$

Therefore, energy generation is a volumetric phenomenom. It occurs within the control volume (c.v.) and is generally proportional to the magnitude of the volume.

For the case that $\dot{e}_{gen}$ = const. with respect to the volume it is acting within, it yields

(1-15)b $$\dot{E}_{gen} = \dot{e}_{gen} V \quad \text{or} \qquad \left(\dot{e}_{gen} \text{ in } \frac{W}{m^3} \text{ or } \frac{kW}{m^3}\right)$$

(1-15)c $$\underbrace{\dot{e}_{gen}}_{\text{rate of heat generation per unit volume}} = \dot{E}_{gen} / V \quad (W/m^3)$$

(Volumetric heat generation term)

→ Textbook, page 72 (Chapter 2)

Heat Transfer Mechanisms:

In heat transfer applications, there are three fundamental heat transfer mechanisms that might take place in the formulation of energy balance. These mechanisms were studied in detail in Heat Transfer I. A summary is given here:

Recall,

1. Conduction of Heat Transfer

Fourier's Law Governs conduction heat transfer.

It is generally given by:

(1-16) $$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \quad (W)$$

partial derivative

thermal conductivity (dependent on material)

negative because heat flows in a decreasing temp. gradient (from high to low temp.)

In rectangular (cartesian) coordinates, the heat conduction vector can be expressed in terms of its components as:

(1-17) $$\vec{\dot{Q}}_n = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k}$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ are the unit vectors, and $\dot{Q}_x$, $\dot{Q}_y$ and $\dot{Q}_z$ are the magnitudes of the heat transfer rates in x, y, and z directions. These can be determined using Fourier's law as.

(1-18)a,b,c $$\left.\begin{aligned} Q_x &= -kA_x\frac{\partial T}{\partial x} \\ Q_y &= -kA_y\frac{\partial T}{\partial y} \\ Q_z &= -kA_z\frac{\partial T}{\partial z} \end{aligned}\right\} \quad (W)$$

where A_x, A_y, A_z are heat conduction areas normal to the x-, y-, and z-directions, respectively.

2. Convection Heat Transfer

need fluid for convection + fluid must be moving

Convection heat transfer is governed by Newton's Law of cooling (NLC).

Recall (from previous Heat Transfer course),

(1-19) $$\boxed{\dot{Q}_{conv} = hA_s(T_s - T_\infty)} \quad (W)$$

temp. at surface

upstream free fluid temp.

3. Radiation Heat Transfer

Radiation heat transfer is governed by Stefan-Boltzman Law, recall

(1-20) $$\boxed{\dot{Q}_{rad} = \varepsilon \alpha A_s (T_s^4 - T_{surr}^4)} \quad (W)$$

(K) should be

The Heat Diffusion Equation

Differential Formulation :

- cartesian coordinate

Consider a homogeneous medium within which the temperature field is expressed in Cartesian Coordinate, i.e. $T(x, y, z)$.

A differential control volume $d\forall = dx \cdot dy \cdot dz$ is considered here for formulation, as follows :

$$\boxed{d\forall = dx \cdot dy \cdot dz}$$

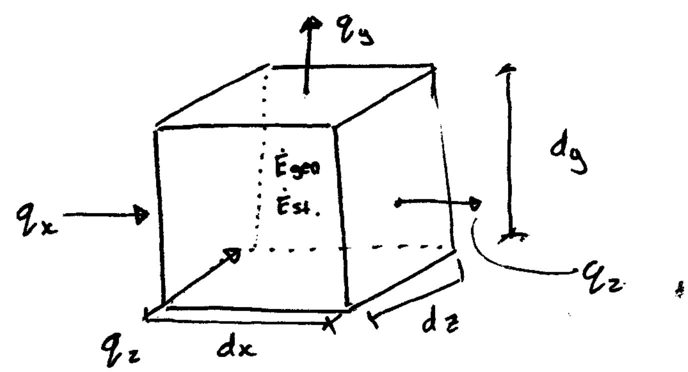

Fig (1-2)