Solving for $T_m(x)$ yields

(5-46)b $T_m(x) = T_s - (T_s - T_{m,i}) \, EXP\left(\frac{-h P_w}{\dot{m} C_p} x \right)$

Note:

1 at $x = L \Rightarrow T_m(L) = T_{m,o}$

$\rightarrow T_{m,o} = T_s - (T_s - T_{m,i}) \, Exp\left(\frac{-h P_w L}{\dot{m} C_p} \right)$

But $A_s = P_w \cdot L$

(5-46)d $\therefore T_{m,o} = T_s - (T_s - T_{m,i}) \cdot EXP(-hA_s/\dot{m}C_p)$ (T_s = const. case)

(5-47)b 2 $\boxed{\dot{Q}_{conv} = \bar{h} A_s \Delta T_{lm}}$

where, $A_s = P_w \cdot L$, T_s = constant (case ii)

Remark (5): Req. Eqs (5-47)a&b to solve for $\dot{m}C_p$

Rearranging Eq. (5-46)d gives

$$\dot{m}C_p = - \frac{\bar{h} A_s}{\ln\left[(T_s - T_{m,o})/(T_s - T_{m,i})\right]} \quad \text{--- (a)}$$

Recalling Eq. (5-41)

$$\dot{Q} = \dot{m}C_p(T_{m,o} - T_{m,i}) \quad \text{--- (b)}$$

Recognizing that (from Eq (a))

$$\left.\begin{aligned} \Delta T_o &\equiv T_s - T_{m,o} && \text{(see eq (5-19)b} \\ \Delta T_i &\equiv T_s - T_{m,i} && \text{(see eq (5-19)c} \end{aligned}\right\} \quad \text{--- (c)}$$

and $\Delta T_o - \Delta T_i = (T_s - T_{m,o}) - (T_s - T_{m,i})$

or $\Delta T_i - \Delta T_o = (T_s - T_{m,i}) - (T_s - T_{m,o})$

$= T_{m,o} - T_{m,i}$ --- (d)

Sub (a), (c) & (d) in (b) gives:

$$\dot{Q} = \frac{-\bar{h} A_s}{\ln[\Delta T_o / \Delta T_i]} * (\Delta T_i - \Delta T_o)$$

or $$\dot{Q} = \bar{h} A_s \left[\frac{\Delta T_o - \Delta T_i}{\ln[\Delta T_o / \Delta T_i]} \right] \quad \text{--- (e)}$$

Recall the definition of ΔT_{lm} (5-19)a

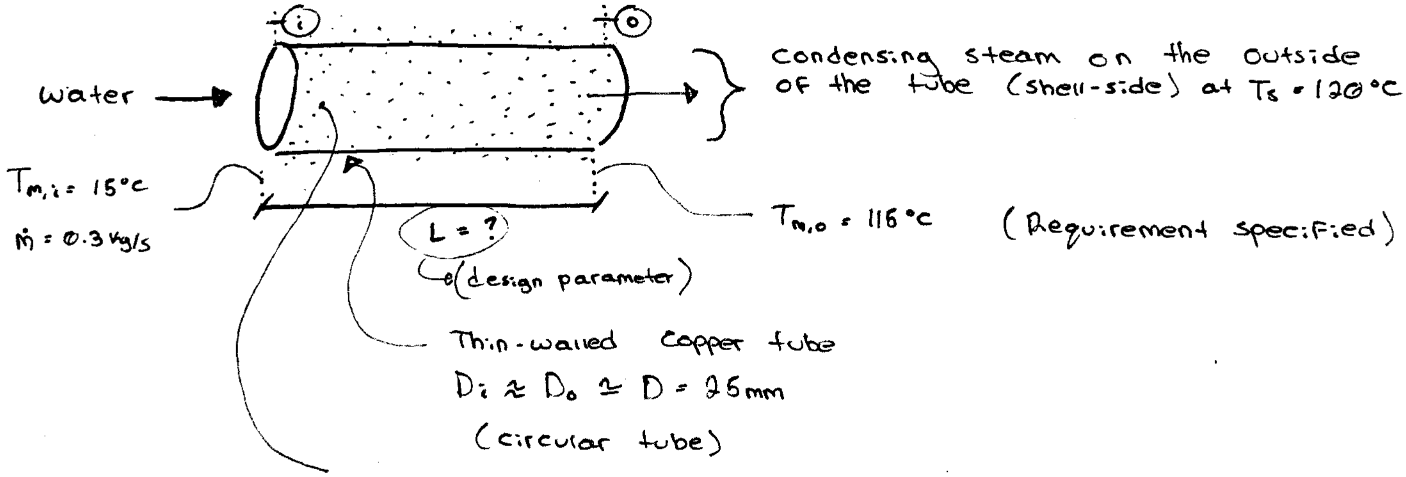
Eq. (e) becomes:

$$\boxed{\dot{Q} = \bar{h} A_s \Delta T_{lm}}$$ (which is Eq (5-47)b)

Example (5-1) : Heating Water in Tube Flow

See Ex (8-1) Text, p. 484

* Design the Length of a Heat Exchanger



The average HT coefficient of the internal flow is $\bar{h} = 800 \frac{W}{m^2 \cdot K}$

Assumptions:

* Steady-state operating conditions exist
* Fluid properties are constant
* $\bar{h}$ is constant
* The conduction thermal resistance across the thickness of the tube is negligible (since the tube wall is very thin - given) so that the inner surface temp. of the tube is ≈ the condensation temp. of the stream on the outer surface $= T_s = 120°C$

Analysis:

Recall Eq (5-47)b

$$\dot{Q} = \bar{h} A_s \Delta T_{lm} \quad --- \quad (1)$$

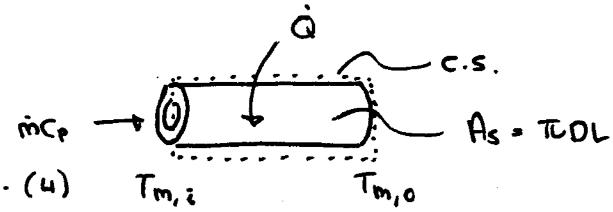
Since $A_s = P_w * L = \pi D L \quad --- \quad (2)$

Sub. (2) in (1) and solving for tube length gives

$$L = \frac{\dot{Q}}{\bar{h} \pi D \Delta T_{lm}} \quad --- \quad (3)$$

In order to solve for the required design of the tube length, we need to determine $\dot{Q}$ and ΔT_{lm}, as follows:

Recall (5-41)*



$\dot{Q} = \dot{m}C_p(T_{m,o} - T_{m,i})$ --- (4)

(The result of energy balance performed over the entire tube length as shown in the c.s.) The Specific heat of water C_p can be evaluated at the bulk mean temp. $\bar{T}_m = \dfrac{T_{m,i} + T_{m,o}}{2}$

$$\bar{T}_m = \frac{15 + 115}{2} = 65°C \rightarrow \bar{C}_p = 4187\ J/kg.K$$

Sub in Eq. (4) gives

$$\dot{Q} = 0.30 \times 4.187 \times (115 - 15) \approx 125.61\ kW$$

Recall Eq (5-19)a *

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)} \quad \text{---} (5)$$

$$\Delta T_o = T_s - T_{m,o} = 120 - 115 = 5°C$$

$$\Delta T_i = T_s - T_{m,i} = 120 - 15 = 115°C$$

Sub $\rightarrow$ $\Delta T_{lm} = \dfrac{5 - 105}{\ln(5/105)} \approx 32.85°C$

Now sub. back in Eq. (3) and solve for L, gives

$$L = \frac{125.61}{800 \times \pi \times 0.025 \times 32.85} \approx \boxed{61\ m}$$

Remark: The importance of using ΔT_{lm}

If one were to use the arithmetic mean temp.

$\Delta T_{am} = T_s - \bar{T}_m = 120 - 65 = 55°C$

Instead of using $\Delta T_{lm} = 32.85°C$, the results would give $L = 36\ m$, which is grossly in error

Flow Regimes in Internal Forces

For Flows inside pipes (circular or non-circular), a useful practical dimensionless number is used to characterize the flow type. This number is called Reynold's number defined as :

(5-8)a $$(Re)_{D_h} = \frac{V_{avg} D_h}{\nu} = \frac{\rho V_{avg} D_h}{\mu} \longrightarrow (V_{avg} = U_m = \bar{u})$$

D_h is the hydraulic diameter, defined previously (5-3) as:

(5-8)b $$D_h \equiv \frac{4A_f}{P_w} \longrightarrow$$ A_f = area in which flow takes place, i.e. A_c

(For circular tubes $D_h = D$, can be provided using Eq. (5-8)b above. For non-circular tubes, see previous remark)

Flows inside tubes can be

(a) Laminar $\longrightarrow$ $Re \le 2300$

(b) Fully turbulent $\longrightarrow$ $Re \gtrsim 10000$

(c) Transitional $\longrightarrow$ $2300 < Re < 10000$

In transitional flow, the flow switches between laminar & turbulent in a disorderly fashion.

Remark: V_{avg} (or U_m, $\bar{u}$) is defined by

(5-8)c ① $$V_{avg} = \int_{A_c} \frac{\rho\, u(r)\, dA_c}{\rho A_c}$$ ($u(r)$ is the velocity profile)

($A_c = \pi r^2$)

(5-8)d or $$V_{avg} = \frac{\int_0^R \rho\, u(r)\, \overbrace{2\pi r\, dr}^{dA_c}}{\rho \pi R^2}$$ (R = radius of tube)

(5-8)e $$\therefore V_{avg} = \frac{2}{R^2}\int_0^R u(r)\, r\, dr$$

② The fluid mean temp. T_m (or T_{avg}) is defined (in a circular tube) to be:

(5-8)e* $$T_m \equiv \frac{\int_{\dot{m}} C_p T(r)\, d\dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T(r) \rho\, u(r)\, 2\pi r\, dr}{\rho V_{avg} (\pi R^2) C_p}$$

NOTE: $d\dot{m}$! = ?

$\dot{m} = \rho V_{avg} A_c$

(*) $d\dot{m} = \rho V_{avg}\, dA_c$ = $d\dot{m} = \rho\, u(r)\, 2\pi r\, dr$

$$\therefore T_m = \frac{2}{V_{avg} R^2}\int_0^R T(r)\, u(r)\, r\, dr$$

Note:

above C_p & ρ are taken as constants

(3) As discussed:

(5-8) f $\quad \dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho\, u(r)\, dA_c$

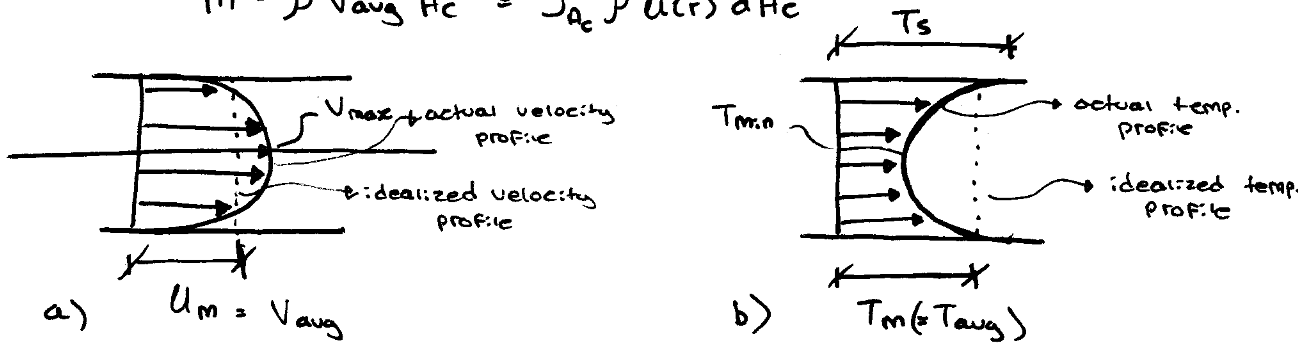


Fig (5-2) - Actual and Idealized Temp. and Velocity profiles (a) velocity, and (b) temp.

- Hydraulic Analysis (Pressure drop) For convection Internal Flow

A quantity of interest in the analysis of pipe flow is the pressure drop (loss) $[-\Delta P] = P_1 - P_2$, since it is directly related to the power requirements of the fan or pump to maintain flow in engineering piping systems.

* Introducing what is known as fluid pumping power $\dot{W}_L$ ($L \stackrel{losses}{=}$ to account for head losses)

Given by:

(5-13) $\quad \dot{W}_L = \dot{V} * |\Delta P_L|$

ΔP = pressure losses drop (ΔP_L is taken as +ve)

(5-12) b $\quad \boxed{\Delta P_L = f\left(L/D\right) \dfrac{\rho V_{avg}^2}{2}}$

NOTE: $\dot{m} = \rho \dot{V}$

$\dot{V} = \dot{m}/\rho$

ρ = density of fluid (kg/m³)

L = length of the tube (m)

f = Darcy Friction Factor

D_h = hydraulic diameter (m)

$\dot{V}$ = volume flow rate of the fluid (m³/s)

(a) For laminar fully developed flows

(5-12)c $$f_{lam} = \frac{64}{Re}$$

(b) For turbulent, fully-developed internal flows in smooth surface tubes, f is given by (Petukhov correlation)

(5-12)d $$f \cong (0.790 \ln Re - 1.64)^{-2} \quad [3000 < Re < 5 \times 10^6]$$

Remark: * An approximate explicit relation for f is given by Haalond, as: hydraulic diameter

(5-12)f $$\frac{1}{\sqrt{f}} \cong -1.8 \log \left[\frac{6.9}{Re} + \left(\frac{\varepsilon / D}{3.7} \right)^{1.11} \right]$$

2 ε / D in eqs (5-12)e & f is called the relative roughness of the tube surface. (Ratio of the mean height of the tube roughness to its diameter.)

Thurs. 12:30 → 1:00

4. Transient (Unsteady) Heat Conduction

In many engineering practices, heat transfer is transient (i.e. unsteady or time-dependent)

Examples are:

- heat treatment processes, such as annealing, quenching, etc.

In general, in an unsteady-state

(4-1)a $$T = f(\underbrace{x, y, z}_{\substack{3D \\ \text{Direction-dependent}}}, \underset{\text{time-dependent}}{t})$$

Or, for simple 1-D (e.g. x-direction) unsteady

(4-1)b problems → $T = f(x, t)$

Heat Transfer in unsteady-state applications, can be analyzed by a number of methods

1. Analytical Method
2. Approx. Method + Solu.
3. Graphical Method
4. Numerical Approach
5. Use of Trans. Heat Charts
6. Product Solu. Method

(*) There is a class of engineering problems that can be simplified by considering the temp. in a solid to be only a function of time. (i.e. $T(t)$) and that is spatially uniform throughout the solid at any instant of time. In these applications the temp. gradients within the solid are neg. (a crit. that has to be verified) ∴ internal comb. resist. is neg.