The $\dot{Q}_{t,\text{no Fins}}$ is given by:

(2-43)b $$\dot{Q}_{t,\text{no Fins}} = hA_{\text{no Fins}}\theta_b$$

$\uparrow$ Surface area with no Fins attached to the surface

Sub. back in Eq(2-43)a above, gives:

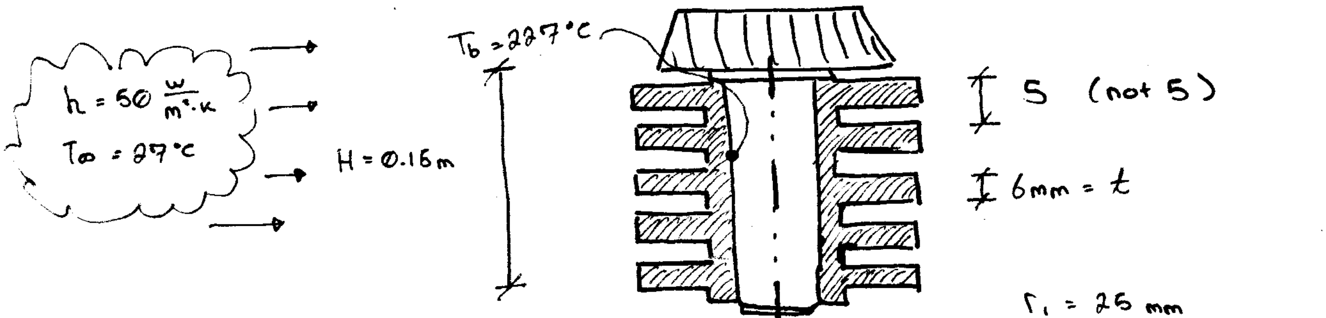
$$\varepsilon_o = \frac{h(N\eta_{Fin}A_{Fin} + A_b)\theta_b}{hA_{\text{no Fins}}\theta_b} = \frac{N\eta_{Fin}A_{Fin} + A_b}{A_{\text{no-Fins}}}$$

NOTE: For $N=1$, Eq. (2-43)c loads to Eq(2-35)

it checks ✓

$\boxed{\text{Example}}$ Fin array application

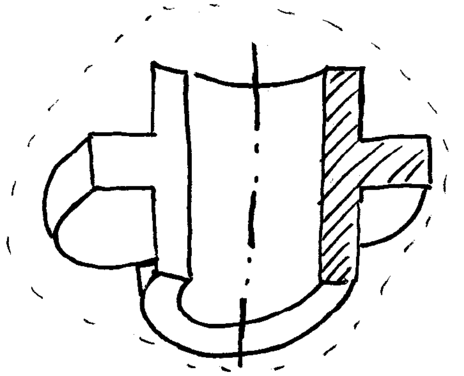
Consider the engine cylinder of a motorcycle as shown:



$r_1 = 25$ mm

$r_2 = 45$ mm

$L = r_2 - r_1 = 20$mm



3D-sketch of single Fin of the array

Fig (Ex 2-2): A Finned motorcycle cylinder engine

→ Given: The engine is made of 2024-T6 aluminum alloy and of height of 15cm and outside diameter of 5cm. under steady-state operation of the engine, the outer surface of the engine is at 227°C, which is exposed to air convection with 27°C and coefficient of 50 W/m²·K

→ There are five annular Fins integrally cast with the cylinder to enhance heat transfer to the surroundings. The Fins are equally spaced and each Fin has a thickness of 6mm and length of 20mm

Required : Estimate the total heat transfer from the cylinder ⓐ Finned ⓑ not Finned
(what is the increase in HT ?)

Assumptions :
- operating conditions and performance of the finned cylinder is at steady-state
- 1-D radial conduction in fins
- uniform properties
- uniform h over the outer surface with and without the fins
- negligible radiation heat-transfer exchange with the surroundings

Analysis :

Properties (Table A-3) at $\bar{T} = \frac{(227+273)+(27+273)}{2} = 400\text{K}$

$$K = 186 \frac{W}{M \cdot K}$$

(a) Recall : $\dot{Q}_t = N\dot{Q}_{Fin} + \dot{Q}_b$ ① HT from the unfinned / prime surface

But, $\dot{Q}_{Fin} = \eta_{Fin} * \dot{Q}_{Fin,max} = \eta_{Fin} * (hA_{Fin}\theta_b)$ ②

& $\dot{Q}_b = hA_b\theta_b$ ③ & ② in ①, gives :

$\dot{Q}_t = N[\eta_{Fin}(hA_{Fin}\theta_b)] + hA_b\theta_b$ ④

in order to calculate $\dot{Q}_t$, η_{Fin} has to be fund

In Eq (4), $A_b = 2\pi r_1 * H - N(2\pi r_1 * t)$

$\therefore A_b = 2\pi r_1 (H - Nt)$ ⑤

$A_{Fin} \equiv$ entire surface area of indiv. Fin

$$\therefore A_{Fin} = 2 * (\pi r_2^2 - \pi r_1^2) + 2\pi r_2 \times t$$
$$= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 * t$$
$$= 2\pi[(r_2^2 - r_1^2) + r_2 + t]$$ ⑥

$\theta_b = T_b - T_\infty$ ⑦

Now, η_{Fin} can be determined (estimated) using : Fig (3-44) of textbook, P.179. In order to use this Fig. we need to determine ξ (zeta) of the ratio (r_{2c}/r_1), As Follows : $\xi = L_c^{3/2}(h/KA_p)^{1/2}$ ⑧ (From Fig.)

where, $L_c = L + t/2$ ⑨

& $A_p = L_c * t$; $r_{2c} = r_2 + t/2$ ⑩ & ⑪

Numerical solutions give the following results

$L_c = 0.023\ m$; $A_p = 0.000138\ m^2$; $r_{2c} = 0.048\ m$

$\rightarrow \xi = (0.023)^{3/2} \left[\frac{50}{186 * 0.000138} \right]^{1/2} \overset{\sim}{=} 0.154$

using Fig. (3-44) text, gives (For $r_{2c}/r_1 = 1.92$)

$\eta_{Fin} \overset{\sim}{=} 0.95$

using eqs (5), (6), & (7) gives

$A_b = 0.01885\ m^2$, $A_{Fin} = 0.0105\ m^2$, $\underbrace{\theta_b}_{}$ (as ΔT) $= 200°c$ (= 200 K)

Sub the Foregoing results in eq. (4), gives:

$\dot{Q}_t = 5 \{ 0.95 (50 * 0.0105 * 200) \} + 50 * 0.01885 * 200$

$\boxed{\therefore \dot{Q}_t \overset{\sim}{=} 687.25\ W}$ ← total HT From the engine cylinder with the Fin array

(b) $\dot{Q}_{no,Fin} = h(\underbrace{2\pi r_1 * H}_{\text{surface area of cylinder without Fins} = A_{noFins}}) * \theta_b$ ⑫

Direct sub gives:

$\dot{Q}_{noFin} = 50 (2 * \pi * 0.025 * 0.15) * (200)$

$\overset{\sim}{=} 235.62\ W$

$\rightarrow \Delta\dot{Q} = \dot{Q}_{with\ Fins} - \dot{Q}_{noFins} = 687.25 - 235.62$

$\overset{\sim}{=} 451.63$ (~192 %)

Remarks:

(1) A_{Fin} was calculated using eq. (6) as an alt. A_{Fin} can be calc. as presc. in T(3-3) Text., being $A_{Fin} = 2\pi(r_{2c}^2 - r_1^2)$ which gives identical result For $A_{Fin} = 0.0105\ m^2$

(2) The overall. eff. of the Finned engine cylinder in this ex. can be est. using eq (2-38) b or eq (2-41). both give $\eta_o \overset{\sim}{=} 0.963$ (96.3 %)

(3) The ε_o of the Finned cylinder can be est. using the derived relationship in eq(2-43)c

$$\left. \begin{aligned} \varepsilon_o &= \frac{5 * 0.95 * 0.0105 + 0.01885}{2 * \pi * 0.026 * 0.015} \end{aligned} \right\} \text{or simply } \varepsilon_o = \frac{\dot{Q}_{t,with\ Fins}}{\dot{Q}_{t,without\ Fins}} = \frac{687.25}{235.62}$$

$\varepsilon_o \overset{\sim}{=} 2.92$

That is, around 3-Fold increase in HT is achieved by using the fin array on the engine cylinder.

Table (3-3), textbook, p. 177 provide some solutions for $\eta_{fin.}$ in some of therm (special case) Solution is given in terms of the modified Bessel Functions I & K (their values are given in T 3-4) p.178

Study, DO example (3-12), textbook, pg. 185,186. (in detail)

Thermal Analysis For Forced Convection - Internal Flow

For practical applications, we define a temp. difference called the log-mean-temperature difference ΔT_{lm}, given by:

(5-19)a $$\boxed{\Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}} \rightarrow \text{NOTE: in textbook, } \underset{\text{outlet}}{T_o} = \underset{\text{exit}}{T_e}$$

(5-19)b where, $\Delta T_o = T_s - T_{m,o}$

(5-19)c $\Delta T_i = T_s - T_{m,i}$

as shown (Fig. 5-1)

Surface temp. = T_s

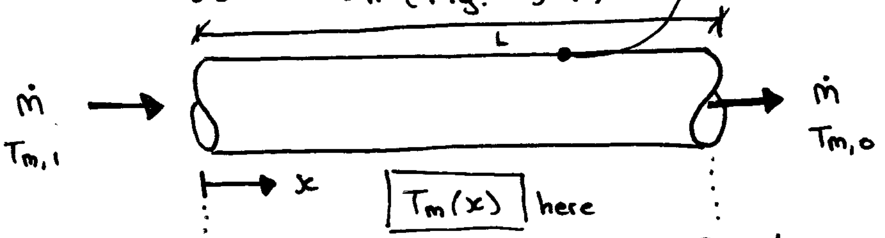


Fig. (5-1)

@ x = 0 $\underbrace{T_m = T_{m,i}}$ mean fluid temp. at inlet of tube

@ x = L $\underbrace{T_m = T_{m,o}}$ mean fluid temp. at outlet of tube

$\dot{m} = \rho U_m A_c$ = const. (mass conservation)

$A_c = (\pi/4)D^2$ (for circular tube)

- the convection heat transfer rate, $\dot{Q}_{conv.}$ for the entire tube (for laminar or turbulent flow) is given by:

(5-41)* $$\boxed{\dot{Q}_{conv} = \dot{m}C_p(T_{m,o} - T_{m,i})} \quad (W)$$

↳ Avg. specific heat capacity for the fluid.

→ For example:

First finding $I_0(0.3443) = ?$

Table (3-4)

@ $x = 0.2 \rightarrow e^{-0.2} I_0(0.2) \cong 0.8269$

@ $x = 0.4 \rightarrow e^{-0.4} I_0(0.4) \cong 0.6974$

Interpolating between the above for $x = 0.3443$

@ $x = 0.3443 \rightarrow e^{-0.3443} I_0(0.3443) \approx 0.73347$

$\therefore I_0(0.3443) = \dfrac{0.73347}{e^{-0.3443}} = 1.0350$

For example Assigned

Now, finding $I_1(0.3443)$:

@ $x = 0.2 \rightarrow e^{-0.2} I_1(0.2) = 0.0823$

@ $x = 0.4 \rightarrow e^{-0.4} I_1(0.4) = 0.1368$

Interpolating between the above . . .

- The convection heat transfer rate, $\dot{Q}_{conv}$ for this entire tube (for laminar or turbulent flow) is given by:

(5-41) $\boxed{\dot{Q}_{conv} = \underbrace{\dot{m} C_p}_{} (T_{m,o} - T_{m,i})}$ (From energy balance)

Avg. specific heat capacity for the fluid

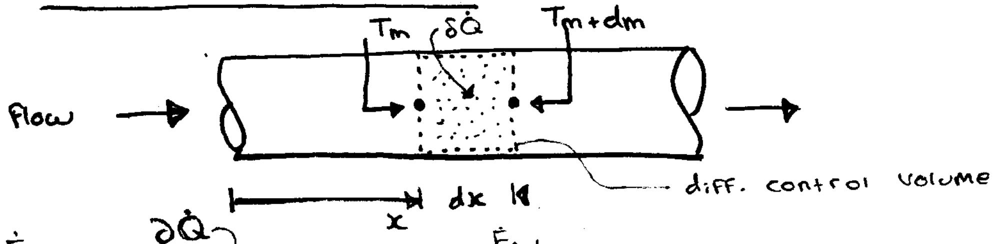
- The differential Eq. governing the variation of T_m as a function of x is given by

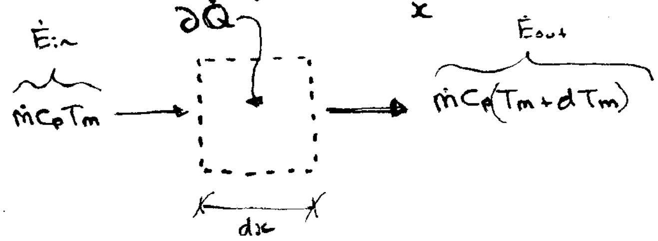
(5-42)a $$\frac{dT_m}{dx} = \frac{q''_s P_w}{\dot{m} C_p}$$ wetted perimeter (For circular tube $P_w = \pi D$)

($\dot{q}_s$ in text) where, q''_s = convection heat flux on the tube surface (W/m²)

called surface heat flux

Differential analysis:





Energy balance, yields :

$\dot{E}_i + \partial \dot{Q} = \dot{E}_o$ - - - (a)

where, $\partial \dot{Q} = \underset{\text{surface heat flux}}{q_s''} \times dA_s = q_s'' \times \overbrace{(P_w\, dx)}^{dA_s}$ - - - (b)

$\dot{E}_i = \dot{m} c_p T_m$ & $\dot{E}_o = \dot{m} c_p (T_m + dT_m)$ - - - (c)

sub (b) & (c) in (a) gives :

$\dot{m} c_p T_m + q_s'' (P_w\, dx) = \dot{m} c_p (T_m + dT_m)$

simplifying → $q_s'' (P_w\, dx) = \dot{m} c_p [(\cancel{T_m} + dT_m - \cancel{T_m})]$

(5-42)a Dividing both sides by $(\dot{m} c_p) dx$ → $\frac{dT_m}{dx} = \frac{q_s'' \cdot P_w}{\dot{m} c_p}$

(5-42)b where $q_s'' = \frac{\delta \dot{Q}_{conv}}{dA_s} = h(T_s - T_m)$

So that Eq. (5-42)a * becomes

(5-42)c $\boxed{\frac{dT_m}{dx} = \frac{P_w}{\dot{m} c_p} \underbrace{h(T_s - T_m)}_{= q_s''}}$

↳ First order ODE → Need 1 B.C.

$T_m(x)$ can be determined by integrating Eq (5-42)c depending on the surface thermal conditions (i.e. BC's) of the tube.

* Two special cases of interest are :

Case (i) q''_s (or q_s) = constant (and known)
(specified heat flux at the tube surface)
(for this case T_s will vary with x)

OR Case (ii) T_s = const. (for this case q''_s varies with x)
(specified surface temp)

Note : T_s must change (not const.) when q''_s = const. (and vice versa)

∴ The solutions for Eq (5-42) are given by

Case (i) for q_s'' = const. case, the solution to (5-42)c

(5-43) $\boxed{T_m(x) = T_{m,i} + \frac{q_s'' P_w}{\dot{m} c_p} x}$

$$\frac{dT_m}{dx} = \frac{P_w}{\dot{m}C_p} q_s''$$

$$\rightarrow \quad dT_m = \left(\frac{P_w q_s''}{\dot{m}C_p}\right) dx$$

Integrating both sides, gives

$$\int_{T_{m,i}}^{T_m} dT_m = \int_{x=0}^{x=x} \left(\frac{P_w q_s''}{\dot{m}C_p}\right) dx$$

$$T_m(x) - T_{m,i} = \frac{P_w q_s''}{\dot{m}C_p}\int_0^x dx = \frac{P_w q_s''}{\dot{m}c_p} x$$

OR

$$T_m(x) = T_{m,i} + \frac{P_w q_s''}{\dot{m}c_p} x \qquad (5\text{-}43)^*$$

And for
(ii) T_s = const. case, the solution to (5-42)c is

(5-46)a*

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \text{Exp}\left(\frac{-P_w x}{\dot{m}C_p}\bar{h}\right)$$

→ Avg. heat transfer coefficient.

(5-46)b* or

$$\boxed{T_m(x) = T_s - (T_s - T_{m,i})\,\text{Exp}\left(-\frac{P_w x}{\dot{m}C_p}\bar{h}\right)}$$

(Remark 4) → Regarding (5-46) a*, b*

so, we have (5-42) c* (the diff. eq.)

$$\frac{dT_m}{dx} = \frac{P_w}{\dot{m}C_p} h(T_s - T_m)$$

OR

$$\dot{m}C_p\, dT_m = hP_w(T_s - T_m)dx \quad --- (a)$$

$$\rightarrow \quad \frac{dT_m}{T_m - T_s} = \frac{-hP_w}{\dot{m}C_p} dx \quad --- (b)$$

Since T_s = const. For case (ii), write:

$$dT_m = dT_m - dT_s$$
$$= d(T_m - T_s)$$
$$\therefore dT_m = -d(T_s - T_m)$$

Sub (c) in (b), gives:

$$\frac{-d(T_s - T_m)}{(T_m - T_s)} = \frac{-hP_w}{\dot{m}C_p} dx$$

OR $$\frac{d(T_s - T_m)}{(T_s - T_m)} = \frac{-h\,\ell_w}{\dot{m}\,C_p} \quad --- \; (d)$$

Now, Integrating Eq. ⓓ from $x = 0$ (tube inlet where $T_m = T_{m,i}$) to x (where $T_m = T_m(x)$)

$$\ln \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \frac{-h\,\ell_w}{\dot{m}C_p}\,x$$

(5-46)a OR $$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \mathrm{EXP}\left[\frac{-h\,\ell_w}{\dot{m}C_p}\,x\right] \quad --- \; (e)$$