DOM

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Feb. 25/19

Rate of Velocity Change

$\hat{R}_p = \rho e^{j\theta}$

$\hat{V}_p = \frac{d\hat{R}_p}{dt} = \rho \frac{dj\theta}{dt} e^{j\theta} = j\rho\omega e^{j\theta}$

$\omega = d\theta/dt$

$j = e^{j90°}$

$\hat{A}_p = \frac{d\hat{V}_p}{dt} = j\rho\frac{d\omega}{dt} e^{j\theta} + j\rho\omega\frac{dj\theta}{dt}e^{j\theta} = j\rho\alpha e^{j\theta} - \rho\omega^2 e^{j\theta}$

$\hat{A}_p = \rho\alpha e^{j(\theta+90°)} + \rho\omega^2 e^{j(\theta+180°)}$
$= A_p^t + A_p^n$

→ tangential component – "t"

→ normal component – "n"

$\boxed{\text{Known:}}$ $\rho = 3\text{in}$, $\theta = 4t^3$ rad

Find: $\hat{V}_p$, $\hat{A}_p$, at $t = 1s$

$\boxed{\text{Solution:}}$

$\theta = 4(1)^3 = 4\text{ rad}$ or $\frac{4}{\pi}\times(180°) = 229.2° = 180° + 49.2°$

$V_p = j\rho\omega e^{j\theta}$

$\omega = (d\theta/dt) = 12t^2 = 12(1)^2 = 12$ rad/s

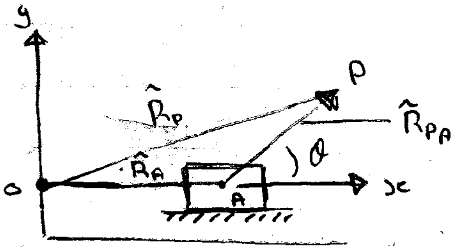
$\hat{V}_p = 3(12)e^{j(229.2°+90°)} = 36e^{j(319.2°)}$

$e^{j90°} = j\sin 90° = j$

$e^{j180} = \cos 180° = -1$

$\alpha = (d\omega/dt) = 24$ rad/s^2

$\hat{A}_p = j(3)24\, e^{j229.2°} - (3)(12)^2 e^{j229.2°}$
$= j72\, e^{j229.2°} - 432\, e^{j229.2°}$
$= 72(j\cos 229.2° - \sin 229.2°) - 432(\cos 229.2° + j\sin 229.2°)$
$= -j47.05 + 54.5 + 282.1 + j327.0$
$= 336.7 + j250$, $|\hat{A}_p| = 438$ in/s^2, $\beta = 0.694$ rad or $39.7°$



$$\hat{R}_P = \hat{R}_A + \hat{R}_{PA}$$

$$\hat{V}_P = \frac{d\hat{R}_P}{dt} = \frac{d\hat{R}_A}{dt} + \frac{d\hat{R}_{PA}}{dt}$$

$$= \hat{V}_A + \hat{V}_{PA} = \hat{V}_A + jp\omega e^{j\theta}$$

$$\hat{A}_P = \frac{d\hat{V}_A}{dt} + \frac{d\hat{V}_{PA}}{dt} = \hat{A}_A + \underbrace{\hat{A}^t_{PA} + \hat{A}^n_{PA}}_{\hat{A}_{PA}}$$

$\boxed{\text{Example:}}$

Known: $A_A = 10\ m/s^2$, $\theta = 4t^2$, $p = 0.5\ m$

Find: $\hat{A}_P$ at $t = 1$ sec

$\boxed{\text{Solution:}}$

$$\theta = 4(1)^2 = 4 \text{ rad} \quad \text{or} \quad 229.2^\circ$$

$$\omega = (d\theta/dt) = 2(4)t = 8(1) = 8 \text{ rad/s}$$

$$\alpha = (\partial^2\theta/dt^2) = 8 \text{ rad/s}^2$$

$$A_{PA} = j(0.5)(8)e^{j229.2^\circ} - 0.5(8)^2 e^{j229.2^\circ}$$

$$= 4(j\cos 229.2^\circ - \sin 229.2^\circ) - 32(\cos 229.2^\circ + j\sin 229.2^\circ)$$

$$= 24.02 + j21.53\ m/s^2$$

$$\hat{A}_P = 10 + 24.02 + j21.53 = 34.02 + j21.53$$

$$|\hat{A}_P| = 40.26\ m/s^2, \quad \beta = 32.3^\circ$$

Graphical solution:

$$\hat{R}_B = \hat{R}_A + \hat{R}_{BA} = \hat{R}_{O4} + \hat{R}_{BO4}$$

$$\frac{d\hat{R}_B}{dt} = \hat{V}_A + \hat{V}_{BA} = \hat{V}_B$$

$$= ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} = jc\omega_4 e^{j\theta_4}$$

$$\hat{A}_B = \hat{A}_A + \hat{A}_{BA}$$

$$\underbrace{\hat{A}^t_B} + \underbrace{\hat{A}^n_B}_{\omega_4} = \underbrace{\hat{A}^t_A}_{\alpha_2} + \underbrace{\hat{A}^n_A}_{\omega_2} + \underbrace{\hat{A}^t_{BA}} + \underbrace{\hat{A}^n_{BA}}_{\omega_3}$$

Known; $\theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4$

Find; α_3, α_4

$$A_A^t = a(\alpha_2) = 10(-10) = -100 \text{ cm/s}^2$$ Scaling: $100/50 = 2$ cm

$$A_A^n = a\omega_2^2 = 10(-5)^2 = -250 \text{ "}$$ $250/50 = 5$ cm

$$A_{BA}^n = b\omega_3^2 = 6(-4.2)^2 = 105.84 \text{ "}$$ $105.84/50 = 2.1$ cm

$$A_B^n = c\omega_4^2 = 8(-6.6)^2 = 348.48 \text{ "}$$ $348.48/50 = 7$ cm

Choose : 1cm = 50 cm/s²

$\boxed{\text{Example}}$ (pg. 8, Ch. 9)

Example (Chapter 7 - Page 18)
in powerpoint

→ $\overline{AB} = 2$ in

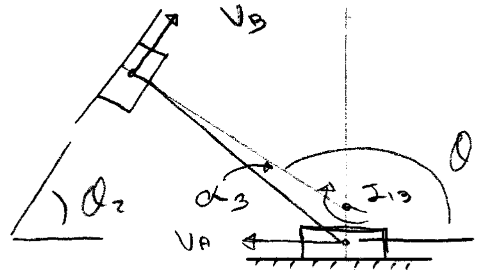
$\hat{V}_A = -10$ in/s

$A_A = -700$ in/s²

Solving, $\omega_3 = -11.55$ rad/s

$V_B = 16.42$ in/s

(1) Find ω_3, V_B, I_{13}



(3) $\hat{A}_B = \hat{A}_A + \hat{A}_{BA}$

$= \hat{A}_A + \hat{A}_{BA}^t + \hat{A}_{BA}^n$

→ Choose a scale

1mm = 10 in/s²

$\hat{A}_A = 700$ in/s

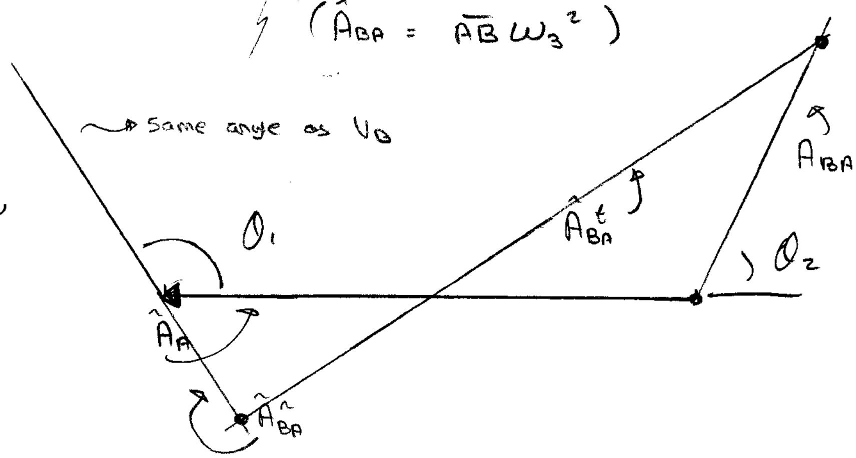
$\hat{A}_{BA} = 2(-11.55)^2 = 266.8$ in/s

$(\hat{A}_{BA} = \overline{AB}\,\omega_3^2)$

$A_{BA}^t = 95 \times 10 = 900$

$\alpha_3 = \frac{950}{2} = 475 \frac{rad}{s^2}$ cw

$A_B = 45 \times 10 = 450$ in/s



(2) $\hat{A}_B = \hat{A}_A + \hat{A}_{BA}^t + \hat{A}_{BA}^n$

$= -700 e^{j0°} + j2\alpha_3 e^{j128°} - 2(\omega_3)^2 e^{j128°}$

$= -700 + j1.23\alpha_3 - 1.576\alpha_3 + 164.3 - j210.25$

$\hat{A}_B = |\hat{A}_B| e^{j60°} = A_B \cos 60° + jA_B \sin 60°$

$A_B = |\hat{A}_B|$

Equate real parts :

$A_B \cos 60° = -700 - 1.576\alpha_3 + 164.3$ (1)

imag parts :

$A_B \sin 60° = 1.23\alpha_3 - 210.25$

A_B, α_3, solve...

$$\frac{②}{①} \quad \frac{\sin 60°}{\cos 60°} = \frac{-1.23\,\alpha_3 - 210.25}{-535.7 - 1.576\,\alpha_3} = 1.732$$

$$\alpha_3 = -478.4 \text{ rad/s}^2$$

② use ① $A_B = \dfrac{-1.23(-478.4) - 210.25}{\sin 60°} = 426.8 \text{ in/s}^2$

→ Slide 19:

$\hat{V}_B = \hat{V}_A + \hat{V}_{B/A}$

Differentiate:

$\hat{A}_B = \hat{A}_A + \hat{A}_{BA}$

$\hat{A}_B^t + \hat{A}_B^n = \hat{A}_A^t + \hat{A}_A^n + \hat{A}_{BA}^t + \hat{A}_{BA}^n$

$jc\alpha_4 e^{j\theta_4} - c\omega_4^2 e^{j\theta_4} = ja\alpha_2 e^{j\theta_2} - a\omega_2^2 e^{j\theta_2} + jb\alpha_3 e^{j\theta_3} - b\omega_3^2 e^{j\theta_3}$

$e^{j\theta} = \cos\theta + j\sin\theta$

Equate real parts:

$c\sin\theta_4\,\alpha_4 - b\sin\theta_3\,\alpha_3 = a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta - c\omega_4^2\cos\theta_4$

Equate imag parts:

$c\cos\theta_4\,\alpha_4 - b\cos\theta_3\,\alpha_3 = a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta + c\omega_4^2\sin\theta_4$

$$\begin{bmatrix} A & -B \\ D & -E \end{bmatrix}\begin{bmatrix} \alpha_4 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} C \\ F \end{bmatrix} \qquad \begin{bmatrix} \alpha_4 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} A & -B \\ D & -E \end{bmatrix}^{-1}\begin{bmatrix} C \\ F \end{bmatrix}$$

Remember: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{(ad - cb)}$