DOM

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Feb. 4/19

Three cases

- velocity of a point on a link in a pure rotation

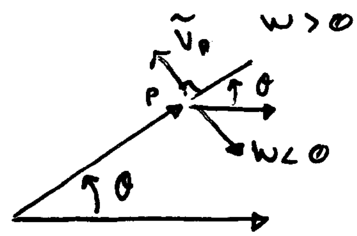
$\hat{R}_p = ae^{j\theta}$

where a is distance from A to P (radius)

$\hat{V}_p = dR_p/dt = a\, de^{j\theta}/dt = ae^{j\theta}\frac{dj\theta}{dt} = aje^{j\theta}\, d\theta/dt$

$\tilde{V}_p = jae^{j\theta}\cdot\omega = a\omega e^{j(\theta+90^\circ)}$

Consider :



Known: $a = 1.5$ m

$\theta = 30^\circ$

$\omega = 4$ rad/s

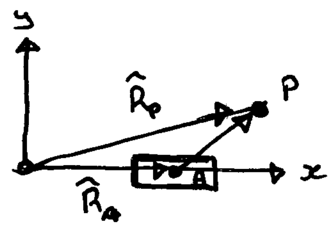
$V_p = 1.5(4)e^{j(30^\circ+90^\circ)}$

$= 6e^{j120^\circ}$

$|\hat{V}_p| = 6$ m/s

@ $\theta = 120^\circ$

- Velocity of a point on link pivoted to a moving sider



$\hat{R}_p = \hat{R}_A + \hat{R}_{P/A}$

$\hat{V}_p = d\hat{R}_p/dt = d\hat{R}_A/dt + d\hat{R}_{PA}/dt = \hat{V}_A + \hat{V}_{PA}$

$\hat{V}_{PA}$ = Velocity diff

$\hat{R}_{PA} = ae^{j\theta}$

$d\hat{R}_{PA}/dt = a\omega e^{j(\theta+90^\circ)}$

Consider

Known : $V_A = 3$ m/s , $a = 1.5$m

$\theta = 30^\circ$, $\omega = -4$ rad/s

$\hat{V}_p = 3e^{j30} + 1.5(-4)e^{j(30^\circ+90^\circ)}$

$= 3 + 3 - j5.195$

$= 6 - j5.195$ . . .

$= 7.936^{j(-40.9)}$

$V_A = 3$

$\theta = 30t^2$ degree

$\hat{V}_p$ at $t = 1$s

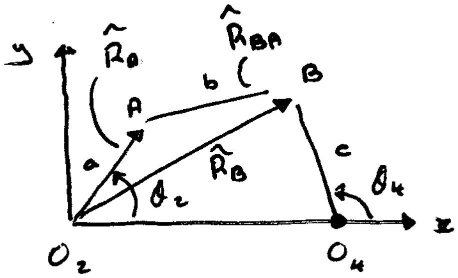
$\theta = 30(1)^2 = 30^\circ$

$\omega = d\theta/dt = 2(30)t = 60^\circ$

$\hat{V}_{PA} = a\omega e^{j(\theta+90^\circ)}$

$= 1.5\left(\frac{60}{180}\right)\pi e^{j(30^\circ+90^\circ)}$

- Velocity of a coupler point



$\theta_2, \theta_3, \theta_4$ known

give ω_2, Find ω_3, ω_4

$$\hat R_B = \hat R_A + \hat R_{B/A} = ae^{j\theta_2} + be^{j\theta_3}$$

$$\hat V_B = d\hat R_B/dt = d\hat R_A/dt + d\hat R_{B/A}/dt$$

$$= a\omega_2 e^{j(\theta_2+90)} + b\omega_3 e^{j(\theta_3+90)}$$

$$\hat R_B = \hat R_{O_4} + \hat R_{BO_4} = de^{j0^\circ} + ce^{j\theta_4}$$

$$d\hat R_B/dt = 0 + c\omega_4 e^{j(\theta_4+90^\circ)}$$

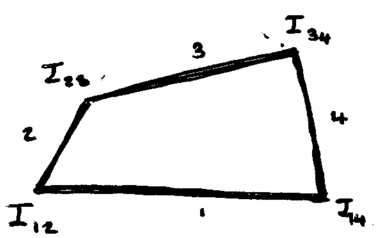
Instantaneous Centre of Velocity (IC)

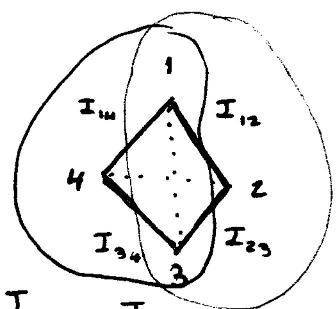
→ linkage has n links

No. of IC $= \dfrac{n(n-1)}{2}$

when $n=4$ (4 links)

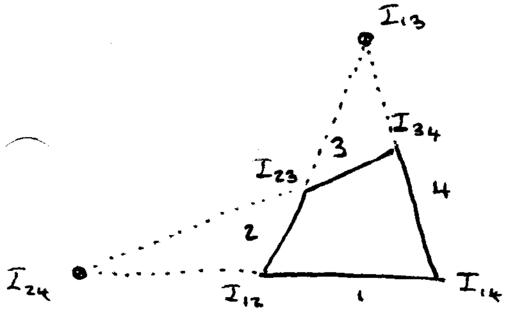
↳ $\dfrac{(4)(4-1)}{2}$ → 6 IC





+ I_{13}, I_{24}

Kennedy's Rule : Any three bodies in plane motion will have exactly three instant centres, and they will lie on the same straight line.



Example

Draw linkage to a proper scale

1) Choose 1:5

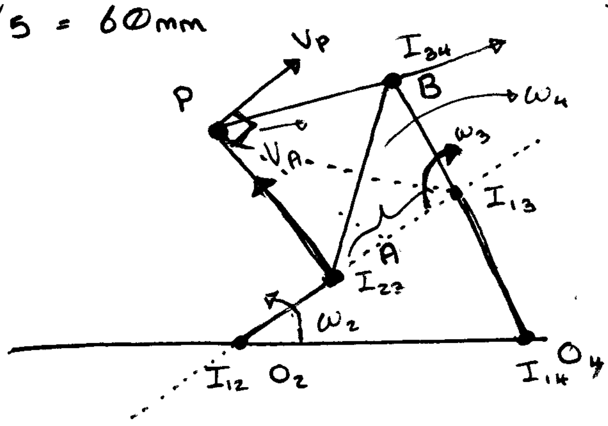
2) identify I_{12}, I_{23}, draw line

$d = 300$ mm

$300/5 = 60$mm

3) identify I_{14}, I_{34}, draw line

4) $V_A = a\omega_2 = 100$ cm/s



5) measure $I_{13}A = 3.5$

$V_A = \overline{I_{13}A} \cdot \omega_3 = 3.6(5)\,\omega_3 = 100$

$\omega_3 = \frac{100}{3.5(5)} = 5.88$ rad/s (cw)

6) $V_B = \overline{I_{13}B} \cdot \omega_3$

$= (6)(5)(5.88)$

$V_B = C \cdot \omega_4 = 45\,\omega_4$

$\omega_4 = \frac{(6)(5)(5.88)}{45} = 3.92$ rad/s (cw)

7) V_P

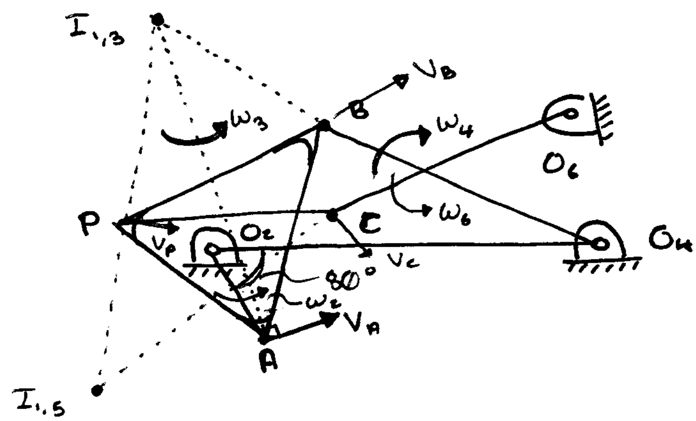
Example

$\omega_3 = 10/0.035$ ccw or cw?

$V_B = \omega_3(0.03536) \ \cdots$

Example – Stephenson's Sixbar

$\omega_2 = 10$ rad/s ($\therefore$ ccw)



$V_A = O_2A\,\omega_2$

$= I_{13}A\,\omega_3$

$\omega_3 = \dfrac{O_2A\,\omega_2}{I_{13}A}$

$V_P = I_{13}P \cdot \omega_3$

$= I_{15}P\ \omega_5$

$\omega_5 = \dfrac{I_{13}P\,\omega_3}{I_{15}P}$ cw

Find $\omega_3, \omega_4, \omega_5, \omega_6$

$V_B = \overline{I_{13}B} \cdot \omega_3$

$\omega_B = V_B / O_{4B}$

$V_C = \overline{I_{15}C} \cdot \omega_5$

$= O_6C \cdot \omega_6 = V_C$

$\omega_6 = \dfrac{\overline{I_{15}C} \cdot \omega_5}{\overline{O_6C}}$ ccw